π-a_1 mixing at intermediate energies

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We describe the mixing of the $q\bar{q}$ pseudoscalar channel with the longitudinal $q\bar{q}$ axial-vector channel, making use of a generalized Nambu–Jona-Lasino model that includes a model of confinement. In addition to the pion, we find $J^P=0^-$ states at 1.18, 1.36, 1.47, 1.63, and 1.68 GeV. The first three of these states are in the region of the $\pi(1300)$ excitation that is assigned a mass of 1300 ± 100 MeV and a width of 200–600 MeV in the data tables. Our work, therefore, suggests the $\pi(1300)$ may not represent a single state of $q\bar{q}$ character. [S0556-2813(98)01809-3]

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In a series of papers, we have been developing an effective field theory for quarks based upon the Nambu–Jona-Lasino (NJL) model [1], supplemented with a relativistic model of confinement [2–6]. In the present work we extend our considerations to the mixing between pseudoscalar states and longitudinal axial-vector states, a phenomenon that is usually called "π-a_1 mixing." A novel feature of the present study is that we are able to study this mixing in the energy region $0\leqslant P^2\leqslant 3.0\ \mathrm{GeV}^2$. We are particularly interested in the region where one finds the $\pi(1300)$. In the data tables the energy is 1300 ± 100 MeV, while the width is given as 200–600 MeV [7].

In the following we first review our treatment of the vacuum polarization diagrams that play an important role in the NJL model. The confinement model eliminates cuts in the P^2 plane that would appear when the quark and antiquark both go on mass shell. Therefore, the vacuum polarization integrals $J(P^2)$ are real if we do not take into account decay into open channels, such as $\pi+\gamma$, $\rho+\pi$, etc.

Next, we study the $q\bar{q}$ T matrix that describes the coupling of the $q\bar{q}$ pseudoscalar channel to the longitudinal component of the $q\bar{q}$ axial-vector channel. Singularities of the T matrix correspond to resonant states of the system. Following this, we present the results of numerical calculations.

For the purposes of this work, we consider the Lagrangian with SU(2)-flavor symmetry

$$\mathcal{L}=\bar{q}(i\partial-m^0)q+\frac{G_s}{2}[(\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2] -\frac{G_V}{2}[(\bar{q}\gamma^\mu\vec{\tau}q)^2+(\bar{q}\gamma^\mu\gamma_5\vec{\tau}q)^2]+\mathcal{L}_{\mathrm{conf}}, \tag{1}$$

where $m^0=\mathrm{diag}(m_u^0,m_d^0)$. We use Lorentz-vector confinement with

$$\mathcal{L}_{\mathrm{conf}}(x)=\int d^4y\,\bar{q}(x)\gamma^\mu q(x)V^c(x-y)\bar{q}(y)\gamma_\mu q(y). \tag{2}$$

Here, $V^c(r)=\kappa r\exp(-\mu r)$, where κ is the "string tension" and μ is a small parameter introduced to soften the singularities of the Fourier transform of $V^c(r)$. We find

$$V^c(\vec{k}-\vec{k}')=-8\pi\kappa\left[\frac{1}{[(\vec{k}-\vec{k}')^2+\mu^2]^2}-\frac{2\mu^2}{[(\vec{k}-\vec{k}')^2+\mu^2]^3}\right], \tag{3}$$

in the case where we neglect energy transfer via the confining field. In this work we have taken $\mu=0.020$ GeV.

We begin our analysis by defining the polarization integrals

$$-iJ^{PP}(P)=(-1)n_cn_f\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}[i\gamma_5 iS(P/2+k)i\bar{\Gamma}_5 \times iS(-P/2+k)], \tag{4}$$

$$-iJ_\mu^{PA}(P)=(-1)n_cn_f\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}[iS(P/2+k)i\bar{\Gamma}_5 \times iS(-P/2+k)\gamma_\mu\gamma_5], \tag{5}$$

$$-iJ_\mu^{AP}(P)=(-1)n_cn_f\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}[iS(P/2+k)\bar{\Gamma}_\mu \times iS(-P/2+k)i\gamma_5], \tag{6}$$

and

$$-iJ_{\mu\nu}^{AA}(P)=(-1)n_cn_f\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}[iS(P/2+k)\bar{\Gamma}_\mu \times iS(-P/2+k)\gamma_\nu\gamma_5]. \tag{7}$$

Here $S(P)=[\not{P}-m+i\epsilon]^{-1}$, with m being the constituent quark mass. Further, the number of flavors is $n_f=2$ and the number of colors is $n_c=3$. The $\bar{\Gamma}_5$ and $\bar{\Gamma}_\mu$ are vertex functions of the confinement potential which will be described shortly. We also define

$$J_\mu^{PA}(P)=iJ^{PA}(P^2)\frac{P_\mu}{\sqrt{P^2}}, \tag{8}$$

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$$J_\mu^{AP}(P)=iJ^{AP}(P^2)\,\frac{P_\mu}{\sqrt{P^2}},\tag{9}$$

and

$$J_{\mu\nu}^{AA}(P)=-\tilde{g}_{\mu\nu}(P)J_T^{AA}(P^2)-\frac{P_\mu P_\nu}{P^2}\,J_L^{AA}(P^2),\tag{10}$$

with $\tilde{g}_{\mu\nu}=g_{\mu\nu}-P_\mu P_\nu/P^2$. Note also that $J^{AP}(P^2)=-J^{PA}(P^2)$ and $P_\mu\tilde{g}^{\mu\nu}=\tilde{g}^{\mu\nu}P_\nu=0$.

The separation of transverse and longitudinal parts of the tensor $J_{\mu\nu}^{AA}(P)$ is appropriate since the transverse part may be treated separately. Thus, only $J^{PP}(P^2)$, $J^{PA}(P^2)$, $J^{AP}(P^2)$, and $J_L^{AA}(P^2)$ will appear in the coupled equations that describe π-a_1 mixing.

The confining field $V^c(\vec{k}-\vec{k}')$ is used to define two vertex functions. Our treatment of these functions has its origin in the method used to calculate the vacuum polarization integrals. These are calculated by using the relation

$$S(P)=\frac{m}{E(\vec{P})}\left[\frac{\Lambda^{(+)}(\vec{P})}{P^0-E(\vec{P})+i\epsilon}-\frac{\Lambda^{(-)}(-\vec{P})}{P^0+E(\vec{P})-i\epsilon}\right]\tag{11}$$

for each propagator in Eqs. (4)–(7) and then performing the integral in the complex k^0 plane. The use of Eq. (11) in Eq. (4), with $\vec{P}=0$, shows that only the elements $\Lambda^{(+)}(\vec{k})\overline{\Gamma}_5(P,k)\Lambda^{(-)}(-\vec{k})$ and $\Lambda^{(-)}(-\vec{k})\overline{\Gamma}_5(P,k)\Lambda^{(+)}(\vec{k})$ appear. Therefore, it is useful to define $\Gamma_5^{+-}(P,k)$

$$\Lambda^{(+)}(\vec{k})\overline{\Gamma}_5(P,k)\Lambda^{(-)}(-\vec{k})=\Gamma_5^{+-}(P,k)\Lambda^{(+)}(\vec{k})\gamma_5\Lambda^{(-)}(-\vec{k})\tag{12}$$

and $\Gamma_5^{-+}(P,k)$

$$\Lambda^{(-)}(-\vec{k})\overline{\Gamma}_5(P,k)\Lambda^{(+)}(\vec{k})=\Gamma_5^{-+}(P,k)\Lambda^{(-)}(-\vec{k})\gamma_5\Lambda^{(+)}(\vec{k}),\tag{13}$$

where $\Gamma_5^{+-}(P,k)$ and $\Gamma_5^{-+}(P,k)$ are ordinary functions with no Dirac matrix structure.

We may obtain equations for $\Gamma_5^{+-}(P,k)$ and $\Gamma_5^{-+}(P,k)$ starting with the equation for the matrix $\Gamma_5(P,k)$

$$\overline{\Gamma}_5(P,k)=\gamma_5-i\int\frac{d^4k'}{(2\pi)^4}\,[\gamma^\rho S(P/2+k)\overline{\Gamma}_5(P,k')\times S(-P/2+k')\gamma_\rho V^c(\vec{k}-\vec{k}')].\tag{14}$$

We find that if we neglect coupling between Γ^{+-} and Γ^{-+}, we have (for $\vec{P}=0$)

$$\Gamma_5^{+-}(P^0,|\vec{k}|)=1-\int\frac{d^3k'}{(2\pi)^3}\left[\frac{m^2-2E(\vec{k})E(\vec{k}')}{E(\vec{k})E(\vec{k}')}\right]\times\frac{\Gamma_5^{+-}(P^0,|\vec{k}'|)V^C(\vec{k}-\vec{k}')}{P^0-2E(\vec{k}')}.\tag{15}$$

A similar analysis leads to

$$\Gamma_5^{-+}(P^0,|\vec{k}|)=1+\int\frac{d^3k'}{(2\pi)^3}\left[\frac{m^2-2E(\vec{k})E(\vec{k}')}{E(\vec{k})E(\vec{k}')}\right]\times\frac{\Gamma_5^{-+}(P^0,|\vec{k}'|)V^C(\vec{k}-\vec{k}')}{P^0+2E(\vec{k}')}.\tag{16}$$

For example, Eq. (15) is obtained if we complete the integral in the lower complex k_0' plane and pick up *only* the pole where the quark is on its positive mass shell [8]. The other pole in the lower-half k_0' plane corresponds to the antiquark being on its negative mass shell. [It plays a role when we obtain Eq. (16).] Note that when $P^0-2E(\vec{k})=0$, $\Gamma^{+-}(P^0,|\vec{k}|)=0$. This aspect of the confinement model removes the unphysical $q\bar{q}$ cuts that would otherwise appear in the vacuum polarization integrals $J(P^2)$. Using these results we find

$$J^{PP}(P^2)=-2n_cn_f\int\frac{d^3k}{(2\pi)^3}\left[\frac{\Gamma_5^{+-}(P^0,|\vec{k}|}{P^0-2E(\vec{k})}-\frac{\Gamma_5^{-+}(P^0,|\vec{k}|)}{P^0+2E(\vec{k})}\right].\tag{17}$$

Since the second term is small, except at low energy, where $\Gamma^{-+}(P^0,|\vec{k}|)$ is fairly close to unity, we will use the approximation

$$J^{PP}(P^2)=-2n_cn_f\int\frac{d^3k}{(2\pi)^3}\left[\frac{\Gamma_5^{+-}(P^0,|\vec{k}|)}{P^0-2E(\vec{k})}-\frac{1}{P^0+2E(\vec{k})}\right]\tag{18}$$

in the intermediate energy region. In the absence of confinement ($\kappa=0$) we put $\Gamma_5^{+-}(P^0,|\vec{k}|)=1$ in Eq. (18). [Note that $J^{PP}(P^2)$ is finite for $P^2=0$.]

We also need to introduce a longitudinal axial-vector vertex $\overline{\Gamma}_L^\mu$ in the calculation of $J_{\mu\nu}^{AA}(P)$. We write for $\vec{P}=0$

$$\Lambda^{(+)}(\vec{k})\Gamma_L^\mu(P,k)\Lambda^{(-)}(-\vec{k})=\frac{P^\mu}{\sqrt{P^2}}\,\Gamma_L^{+-}(P,k)\Lambda^{(+)}(\vec{k})\gamma_5\Lambda^{(-)}(-\vec{k}).\tag{19}$$

Now note that from Eq. (10) $P^2J_L^{AA}(P^2)=-P^\mu J_{\mu\nu}(P)P^\nu$ so that, including $\overline{\Gamma}_L^\mu$ at one vertex, we have

$$P^2J_L^{AA}(P^2)=-n_cn_fi\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}[S(P/2+k)P^\mu\overline{\Gamma}_\mu(P,k)\times S(-P/2+k)\not{P}\gamma_5].\tag{20}$$

Completing the integral in the lower k^0 plane and picking up the contribution of both poles of the propagators found there, we obtain

$$J_L^{AA}(P^2)=2n_cn_f\int\frac{d^3k}{(2\pi)^3}\frac{m}{E(\vec{k})}\left[\frac{\Gamma_L^{+-}(P^0,|\vec{k}|)}{P^0-2E(\vec{k})}-\frac{m/E(\vec{k})}{P^0+2E(\vec{k})}\right]. \quad (21)$$

Here we have neglected confinement in the second term of Eq. (21). In the absence of a confinement model ($\kappa=0$), $\Gamma_L^{+-}(P^0,|\vec{k}|)=m/E(\vec{k})$, so that

$$J_L^{AA}(P^2)=2n_cn_f\int\frac{d^3k}{(2\pi)^3}\left[\frac{m}{E(\vec{k})}\right]^2\frac{4E(\vec{k})}{(P^0)^2-[2E(\vec{k})]^2}, \quad (22)$$

for $P^0<2m$. As noted above, an important feature of our confinement vertex functions is that they are zero when the quark and antiquark both go on their (positive) mass shells. Therefore, expressions such as $\Gamma_5^{+-}(P^0,|\vec{k}|)/[P^0-2E(\vec{k})]$ and $\Gamma_L^{+-}(P^0,|\vec{k}|)/[P^0-2E(\vec{k})]$ are finite.

An equation for the longitudinal axial-vector vertex is obtained by starting with

$$\overline{\Gamma}_L^{\mu}(P,k)=\frac{P^\mu \not{P}}{P^2}\gamma_5-i\int\frac{d^4k'}{(2\pi)^4}\gamma^\rho S(P/2+k')\overline{\Gamma}_L^{\mu}(P,k')\times S(-P/2+k')\gamma_\rho V^C(\vec{k}-\vec{k}') \quad (23)$$

and using Eq. (19). If we neglect the coupling of Γ_L^{+-} to Γ_L^{-+} we find

$$\Gamma_L^{+-}(P^0,|\vec{k}|)=\frac{m}{E(\vec{k})}+\int\frac{d^3k'}{(2\pi)^3}\left[\frac{2E(\vec{k}')E(\vec{k})-m^2}{E(\vec{k})E(\vec{k}')}\right]\times\frac{\Gamma_L^{+-}(P^0,|\vec{k}|)}{P^0-2E(\vec{k})}V^C(\vec{k}-\vec{k}'). \quad (24)$$

Thus, in the absence of confinement $\Gamma_L^{+-}(P^0,|\vec{k}|)=m/E(\vec{k})$, as noted above.

Proceeding in a fashion analogous to our calculations of $J^{PP}(P^2)$ and $J^{AA}(P^2)$, we find

$$J^{PA}(P^2)=-2n_cn_f\int\frac{d^3k}{(2\pi)^3}\frac{m}{E(\vec{k})}\left[\frac{\Gamma_5^{+-}(P^0,|\vec{k}|)}{P^0-2E(\vec{k})}+\frac{1}{P^0+2E(\vec{k})}\right]. \quad (25)$$

As is well known, the integrals defining the vacuum polarization functions are divergent. Therefore, they are cut off by inserting a theta function $\theta(\Lambda_3-|\vec{k}|)$. We used $\Lambda_3=0.622$ GeV in our earlier work and we continue to use that value here.

The resonant states of the coupled pseudoscalar and longitudinal axial-vector fields may be found by studying the T matrix for $q\overline{q}$ scattering, including channel coupling terms. We write

$$\hat{T}=i\gamma_5T^{PP}(P^2)i\gamma_5+i\gamma_5iT^{PA}(P^2)\frac{\not{P}}{\sqrt{P^2}}\gamma_5+\frac{\not{P}\gamma_5}{\sqrt{P^2}}iT^{AP}(P^2)i\gamma_5+\frac{\not{P}\gamma_5}{\sqrt{P^2}}T_L^{AA}(P^2)\frac{\not{P}\gamma_5}{\sqrt{P^2}}. \quad (26)$$

This form serves to define $T^{PP}(P^2)$, $T^{PA}(P^2)=-T^{AP}(P^2)$ and $T^{AA}(P^2)$. We also define

$$\det D(P^2)=[1-G_SJ^{PP}(P^2)][1-G_VJ_L^{AA}(P^2)]+G_SG_V[J^{PA}(P^2)]^2, \quad (27)$$

where we have used the fact that $J^{AP}(P^2)=-J^{PA}(P^2)$.

We may solve for $T^{PP}(P^2)$, $T^{PA}(P^2)$, and $T^{AA}(P^2)$ to obtain

$$T^{PP}(P^2)=-\frac{G_S[1-G_VJ_L^{AA}(P^2)]}{\det D(P^2)}, \quad (28)$$

$$T^{PA}(P^2)=\frac{G_VG_SJ^{PA}(P^2)]}{\det D(P^2)}, \quad (29)$$

$$T^{AA}(P^2)=\frac{G_V[1-G_SJ^{PP}(P^2)]}{\det D(P^2)}, \quad (30)$$

with $T^{AP}(P^2)=-T^{PA}(P^2)$. Note that bound (or resonant) states correspond to the zeros of $\det D(P^2)$.

Now consider the T matrix of Eq. (28) in the frame where $\vec{P}=0$. We may write

$$\hat{T}=(i\gamma_5,\gamma^0\gamma_5)\begin{pmatrix}T^{PP}(P^2) & iT^{PA}(P^2)\\ iT^{AP}(P^2) & T_L^{AA}(P^2)\end{pmatrix}\begin{pmatrix}i\gamma_5\\ \gamma^0\gamma_5\end{pmatrix}, \quad (31)$$

which may be written as $\Phi^T T(P^2)\Phi$. Now we use the matrix

$$M(\theta)=\begin{pmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{pmatrix} \quad (32)$$

to bring $T(P^2)$ of Eq. (31) to diagonal form:

$$M(\theta)T(P^2)M^{-1}(\theta)=\begin{pmatrix}T_1(P^2) & 0\\ 0 & T_2(P^2)\end{pmatrix}. \quad (33)$$

TABLE I. Values of the mixing angle for various bound or resonant states.

Energy (GeV)	Channel	θ (rad)	θ (deg)
0.138	T_1	−0.059	−3.39°
1.18	T_1	0.990	53.0°
1.36	T_2	−1.61	−92.0°
1.47	T_2	−0.55	−31.8°
1.63	T_1	−0.048	−2.70°
1.68	T_1	1.08	61.6°

We put $G_S = 12.80\,\text{GeV}^{-2}$ and $G_V = 12.50\,\text{GeV}^{-2}$ and find that there are zeros of det $D(P^2)$ at $P^0 = 1.18$, 1.36, 1.47, 1.63, and 1.68 GeV, in addition to the zero at $P^0 = 0.138$ GeV. The data tables only list the $\pi(1300)$ in the energy region considered here. Our results suggest that the $\pi(1300)$, whose width is given as 200–600 MeV in the data tables, may actually be composed of two (or three) states. These states appear in either $T_1(P^2)$ or in $T_2(P^2)$. (See Table I.) In $T_1(P^2)$ we find resonances at $P^0 = 0.138$, 1.18, 1.63, and 1.68 GeV, while $T_2(P^2)$ has states at $P^0 = 1.36$ and 1.47 GeV.

There are six states listed in Table I. These have their origin in the $1S$, $2S$, and $3S$ states in the confining field. There are two sets of such states, corresponding either to the pseudoscalar vertex Γ_5^{+-} or to the longitudinal axial-vector vertex Γ_L^{+-} making for six states in all.

We note that if $G_S = G_V = 0$, the potential $V^C(\vec{k} - \vec{k}')$ acts and provides doublets at $P^0 = 1.20$, 1.49, and 1.69 GeV. One member of the doublet is a pseudoscalar state and the other is a (longitudinal) axial-vector state. When we turn on the NJL interaction the degeneracy is lifted. In large part, the $1S$ pseudoscalar state becomes the pion, moving down over 1 GeV from $P^0 = 1.20$ GeV to $P^0 = 0.138$ GeV. The axial-vector state is mixed with the pseudoscalar state (see Table I) and the mixed state is at 1.18 GeV, quite close to 1.20 GeV, the original position of the $1S$ states.

We next consider the $2S$ states that were at 1.49 GeV, when $G_S = G_V = 0$. With reference to Table I, we see that there is a state at $P^0 = 1.36$ GeV. The next (mixed) state is at 1.47 GeV, indicating almost no downward movement from the original position at 1.49 GeV.

Finally, we consider the two $3S$ states, which are at 1.69 GeV when $G_S = G_V = 0$. These states evolve into our states at 1.68 and 1.63 GeV. The first state has only moved down about 10 MeV from the original position at 1.69 GeV. This analysis suggests that the $\pi(3S)$ is the state we obtained at 1.63 GeV.

Note that the width of the $\pi(1300)$ is given as 200–600 MeV in Ref. [7]. Taking a value of 400 MeV for that quantity shows that our states at 1360 and 1180 MeV are encompassed in the energy region of the $\pi(1300)$, when we include the width of that state in our considerations. Therefore, we suggest that when we take the coupling to the decay channels into account, the two states at 1360 and 1180 MeV, will obtain significant widths and will overlap such that the resulting structure may be identified as the $\pi(1300)$. Further details of the work presented here may be found in Ref. [9].

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