Systematics of the excitation energy of the 1^+ scissors mode and its empirical dependence on the nuclear deformation parameter

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From the data on dipole excitations obtained in photon scattering experiments we extract and tabulate the mean values of the excitation energy of the 1^+ scissors mode in nuclei of the mass region $130<A<200$. The centers of gravity of the observed $M1$ strength distributions are always close to 3 MeV including several moderately deformed transitional nuclei. In particular, the data exhibit a weak dependence of the scissors mode energy on the deformation parameter. Including roughly the deformation dependence of pairing effects we could modify an earlier estimate based on a schematic random phase approximation and get a simple formula which yields qualitative agreement with the data. [S0556-2813(98)01207-2]

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I. INTRODUCTION

One of the most exciting findings in nuclear structure in the last decade is the observation of strong low-lying magnetic dipole excitations in deformed nuclei, which are frequently referred to as a *scissors mode*. In a geometrical picture [1] the scissors mode is visualized as a counterrotational oscillation of the deformed proton body against the deformed neutron body in the intrinsic frame of reference. The scissors mode is, thus, expected to be collective and of orbital character. For a recent review on this topic see Ref. [2]. In even-even nuclei the total magnetic dipole excitation strength of the scissors mode closely correlates to the collective $E2$ excitation strength of the 2_1^+ state [3–5] and, thus, depends quadratically on the nuclear deformation parameter [6–8]. We focus here on the systematics and, in particular, on the deformation dependence of the excitation energy of the scissors mode in nuclei of the rare-earth region.

Since the prediction of the scissors mode [1] in the late 1970's and its discovery in 1984 in a high-resolution electron scattering experiment [9] on ^{156}Gd in Darmstadt, many nuclei in the rare-earth region have been systematically investigated by electron scattering [10] and by photon scattering experiments. Photon scattering, frequently called nuclear resonance fluorescence (NRF), is particularly well suited to study dipole excitations with high-energy resolution γ spectroscopy [11]. Recent NRF experiments with enhanced sensitivity have provided data on the scissors mode for nuclei, for which only a small strength is expected. They include weakly deformed nuclei [12] and transitional nuclei with deformations that are considered to be not axially symmetric [13–19].

The wealth of data which has been accumulated during the last decade allows the systematical analysis of the properties of the scissors mode as a function of other nuclear characteristics, e.g., the mass number A or the nuclear deformation. The established phenomenological correlations between the properties of the scissors mode and other nuclear attributes can serve as important tools to judge the predictive power of different models for the description of the scissors mode and can lead to their further improvement.

The experimental systematics of the total $M1$ excitation strength obtained from photon scattering on even-even rare-earth nuclei has been discussed, for instance, in Ref. [5]. Figure 1 shows the data for the total $M1$ strength (left) and the low-lying $E2$ strength (right) versus the mass number A. One observes a clear correlation between the total $M1$ strength and the low-lying collective $E2$ strength. This correlation also holds true for transitional nuclei without axial symmetry [18] and it proves the collective origin of the $M1$ strength which is fragmented among several 1^+ states.

Besides the excitation strength, another basic property of

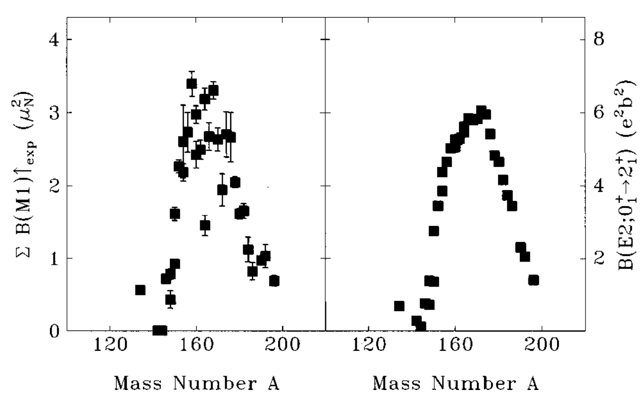

FIG. 1. Total low-lying $M1$ strength (left) and low-lying $E2$ strength (right) plotted versus the mass number. Both quantities exhibit collective behavior: the strengths are small near shell closures, maximum at midshell, and vary smoothly in between.

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a nuclear excitation is the excitation energy. It is the aim of this paper to extract the excitation energies of the scissors mode from the photon scattering data on heavy even-even nuclei with masses $130<A<200$ and to compare the data with theoretical predictions.

II. SURVEY OF THE CENTERS OF GRAVITY OF THE EXPERIMENTALLY OBSERVED *M*1 STRENGTH DISTRIBUTIONS

From NRF experiments on heavy nuclei [7,12–28] it is known that the scissors mode is fragmented into several 1^+ states. We define the mean excitation energy of the scissors mode as the center of gravity of the $M1$ strength distributed among the low-lying $J^\pi=1^+$ states

$$E_{\rm sc}\equiv\frac{\Sigma_i E_i B(M1)_i}{\Sigma_i B(M1)_i}. \tag{1}$$

The sums are evaluated in the energy intervals around 3 MeV given in Table I, which have been chosen according to Ref. [5]. The observed 1^+ states are considered as the fragments of the scissors mode. The selection of states for which no parity was measured was done in accordance with Ref. [5]. The definition of the mean excitation energy of the scissors mode from Eq. (1) has already been used in previous papers [6,29] for Sm nuclei (where slightly different energy ranges have been used). It is appropriate for a comparison of the data to microscopically calculated 1^+ energies [29].

The data for the mean excitation energy of the scissors mode in the even-*A* nuclei of the $Z=50-82$ major shell are presented in Table I. The errors are of the order of 1%. They are calculated by standard error propagation from the errors of the experimental $B(M1)_i$ values. More than 66% of the data lie between 2.9 and 3.2 MeV. The mean excitation energies from Table I are shown in Fig. 2 where they are plotted versus the mass number A.

In analogy to the well-known collective isovector $E1$ mode, the giant dipole resonance (GDR), it has been proposed that the excitation energy of the $M1$ scissors mode should roughly be proportional to the reciprocal nuclear radius, which would result in a mass dependence of $A^{-1/3}$ [30–35]. The solid curve in Fig. 2 corresponds to this simple proportionality. This curve roughly describes the mass dependence of the scissors mode energy in the large mass region $A=130-200$. A fit to the data using the function $E_{\rm sc}=aA^{-b}$ actually yields an exponent $b\approx0.3$. However, due to the scatter of the data, the χ^2 value for that least squares fit is very large ($\chi^2_{\rm red}\approx3\times10^2$), which may hint at a dependence of the $M1$ mode energy on an additional quantity besides the mass and at the failure of the use of collective or average properties on this level of accuracy.

TABLE I. For the nuclei in the mass region $130<A<200$, where photon scattering data on low-lying 1^+ states are available, we give the following quantities: the deformation parameter δ calculated from the $B(E2;0_1^+\rightarrow2_1^+)$ value [38], the excitation energy of the scissors mode defined in Eq. (1) as the center of gravity of the $M1$ strength distribution, which was observed in the energy range given in the fourth row. For a discussion on the identification of the fragments of the scissors mode see Ref. [5].

Nucleus	δ	$E_{\rm sc}$ [MeV]	Energy range [MeV]	Ref.
^{134}Ba	0.139	2.989(15)	<3.5	[15]
^{144}Nd	0.114	3.213(1)	2.7–3.7	[12]
^{146}Nd	0.131	3.469(10)	2.7–3.7	[7]
^{148}Nd	0.168	3.367(17)	2.7–3.72	[7]
^{150}Nd	0.223	3.037(7)	2.68–3.7	[7]
^{148}Sm	0.123	3.069(6)	2.7–3.7	[23]
^{150}Sm	0.161	3.132(14)	2.7–3.7	[23]
^{152}Sm	0.236	2.993(4)	2.7–3.7	[23]
^{154}Sm	0.257	3.200(14)	2.7–3.7	[23]
^{156}Gd	0.256	3.060(7)	2.7–3.7	[26]
^{158}Gd	0.262	3.143(8)	2.7–3.7	[26]
^{160}Gd	0.265	3.176(10)	2.7–3.7	[48]
^{160}Dy	0.255	2.870(5)	2.7–3.7	[27]
^{162}Dy	0.257	2.956(4)	2.7–3.7	[49]
^{164}Dy	0.262	3.143(2)	2.7–3.7	[21,50]
^{164}Er	0.253	2.901(34)	2.4–3.7	[20]
^{166}Er	0.258	2.961(26)	2.4–3.7	[20]
^{168}Er	0.256	3.206(10)	2.4–3.7	[20]
^{170}Er	0.255	3.220(17)	2.4–3.7	[20]
^{172}Yb	0.251	3.031(47)	2.4–3.7	[24]
^{174}Yb	0.248	3.146(48)	2.4–3.7	[24]
^{176}Yb	0.238	2.960(57)	2.4–3.7	[24]
^{178}Hf	0.220	3.110(14)	2.4–3.7	[19]
^{180}Hf	0.216	2.952(15)	2.4–3.7	[19]
^{182}W	0.200	3.103(25)	2.4–3.7	[22]
^{184}W	0.190	3.308(54)	2.4–3.7	[22]
^{186}W	0.183	3.195(29)	2.4–3.5	[22]
^{190}Os	0.148	2.897(17)	2.4–3.7	[17]
^{192}Os	0.140	3.011(30)	2.4–3.7	[16]
^{196}Pt	0.114	2.680(18)	<3.7	[13,14]

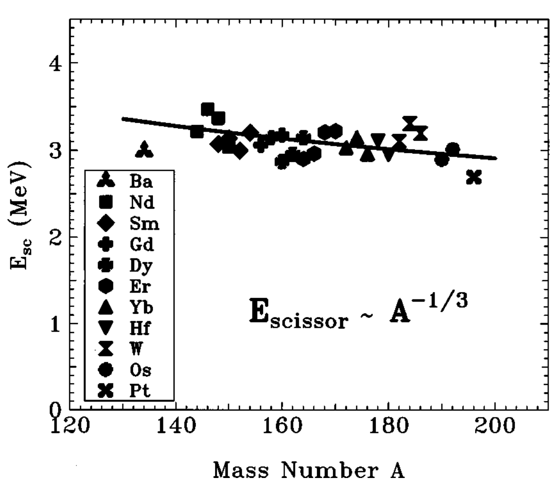

FIG. 2. Mean excitation energy of the scissors mode $E_{\rm sc}$ plotted versus the nuclear mass number. $E_{\rm sc}$ has been calculated from the data according to Eq. (1).

III. EMPIRICAL DEFORMATION DEPENDENCE

Apart from the mass dependence, the excitation energy of the scissors mode can be expected to be also a function of the nuclear deformation. For well deformed, axially symmetric nuclei the excitation energy of a low-lying, scissorlike $M1$ mode has been predicted in different theoretical approaches to be in leading order proportional to the deformation parameter [1,30–35]. This linear deformation dependence has been used earlier for a comparison to the data on strongly deformed nuclei [2,10,11,36]. In the literature the nuclear deformation is frequently parametrized by the Nilsson deformation parameter δ, defined [37] by the relations

$$\omega_\perp^2 = \omega_0^2\left(1+\frac{2}{3}\delta\right), \tag{2}$$

$$\omega_z^2 = \omega_0^2\left(1-\frac{4}{3}\delta\right) \tag{3}$$

for the oscillator frequencies perpendicular and parallel to the deformation axis, respectively. In order to compare the experimental data to the model predictions formulated in terms of δ, we determine values of δ from the measured $B(E2;0_1^+\rightarrow 2_1^+)$ values [38,39].

Several theoretical works predict the excitation energy of the scissors mode to be proportional to the deformation parameter δ. In the following we will focus on the formula derived by Bes and Broglia [32], which we will use for a comparison to our data. Within the framework of a schematic random phase approximation (RPA) Bes and Broglia derived the expression

$$E_{\rm sc}=\frac{E}{e}(2e)\sqrt{1+b_{\rm eff}}, \tag{4}$$

where $2e=\delta\hbar\omega_0=41\delta A^{-1/3}$ MeV is the energy of a particle-hole excitation in the schematic RPA and $E=\sqrt{e^2+\Delta^2}$ is the one-quasi-particle energy with the pairing gap parameter Δ. The parameter $b_{\rm eff}\approx 0.6$ has been deduced from the inclusion of a quadrupole-quadrupole interaction and from a coupling to the isovector quadrupole giant resonance [31,32,40]. The rescaling factor E/e accounts for pairing effects. The cranking formula yields for the moment of inertia

$$\mathcal{J}=\left(\frac{e}{E}\right)^3\mathcal{J}_{\rm rig}, \tag{5}$$

where $\mathcal{J}_{\rm rig}=(2/5)AMR_0^2(1+\delta/3)$ is the moment of inertia of an axially symmetric rigid rotor [40]. This expression, already derived in Refs. [32,41], coincides with the one obtained when the deformed field is generated self-consistently, so as to avoid spurious rotational admixtures [42]. One may identify the moment of inertia calculated within the schematic QRPA with the experimental moment of inertia $\mathcal{J}=\mathcal{J}_{\rm expt}$, where the latter may be taken as the effective moment of inertia of the ground-state band $\mathcal{J}_{\rm expt}=3\hbar^2/E(2_1^+)$. The rescaling factor E/e may, thus, be obtained empirically through the expression

$$(e/E)^3=\mathcal{J}_{\rm expt}/\mathcal{J}_{\rm rig}. \tag{6}$$

A zeroth-order (in the deformation parameter) effect of the pairing interaction was taken into account by Bes and Broglia in estimating the rescaling factor E/e by the value

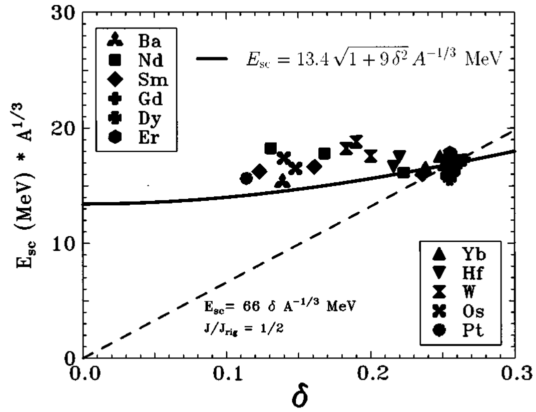

FIG. 3. Deformation dependence of the excitation energy of the scissors mode in the rare earth region. The product of the cubic root of the mass number and the mean excitation energies from Table I are plotted versus the deformation parameter δ calculated from the $B(E2;0_1^+\rightarrow 2_1^+)$ values [38]. The dashed straight line corresponds to a proportionality between the energy and the deformation parameter. The solid curve is obtained by inclusion of the deformation dependence of pairing effects (see text). The parameter-free error-weighted χ^2 value reduces from 1.1×10^5 for the dashed curve by an order of magnitude to 0.9×10^4 for the solid curve.

$(1/2)^{-1/3}$ from the moment of inertia ratio $\mathcal{J}_{\rm expt}/\mathcal{J}_{\rm rig}\approx 1/2$ for strongly deformed nuclei. Starting from Eq. (4) they obtained the expression

$$E_{\rm sc}=66\,\delta A^{-1/3}\ {\rm MeV}, \tag{7}$$

yielding a linear deformation dependence for the energy of the scissors mode [32]. We emphasize that a deformation dependence of the rescaling factor E/e has been neglected in this derivation. An analysis of the deformation dependence of the rescaling factor E/e will lead us below to a modified formula. Inserting a typical value of $\delta\approx 0.26$ for strongly deformed nuclei (e.g., for ^{156}Gd, ^{164}Dy, ^{168}Er) into Eq. (7), one obtains good agreement with the data. Figure 3 shows the deformation dependence of the energy of the scissors mode. However Eq. (7) does not account for the data from moderately deformed transitional nuclei, where the deformation parameter is smaller, because the rescaling factor E/e was not considered to be a function of the deformation in the derivation of Eq. (7).

In order to analyze a deformation dependence of the rescaling factor E/e we make the ansatz

$$\frac{E}{e}=\frac{1}{a\delta}\sqrt{1+(b\delta)^2}. \tag{8}$$

We consider a and b as free parameters and we will fix them by a fit to the experimental moments of inertia using Eq. (6). The expression from Eq. (8) is motivated by the definition of the single particle energy $e=\delta\hbar\omega_0/2$ and the quasiparticle energy $E=\sqrt{e^2+\Delta^2}$ in the schematic QRPA and by the assumption that the strong deformation dependence of e dominates the deformation dependence of the ratio E/e. If the pairing gap parameter Δ would be completely independent of the deformation and if the schematic QRPA would be exact, we would expect $b=a$.

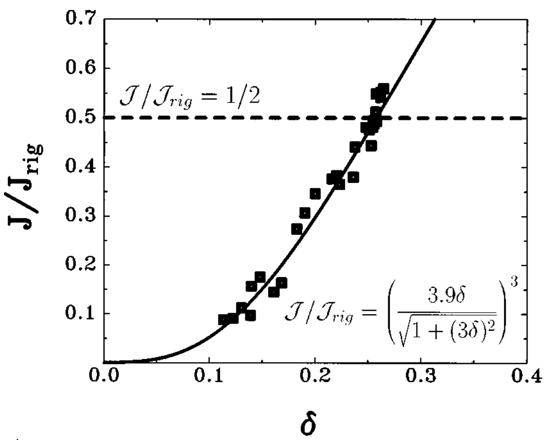

FIG. 4. Deformation dependence of the moment of inertia relative to the prediction of the axially symmetric rigid rotor for the nuclei listed in Table I. The curve is a two parameter fit to the data. The fit function is suggested from a schematic QRPA.

In Fig. 4 we show the deformation dependence of the moment of inertia for all nuclei from Table I. On the abscissa we plot the parameter δ extracted from the $B(E2;0_1^+\to 2_1^+)$ value. For strongly deformed nuclei one observes $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}} \approx 0.5$. However, the ratio $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}}$ shows a considerable deformation dependence [43], as can be seen from Fig. 4. It is obvious that the strong deformation dependence of $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}}$ makes the zeroth-order approximation for well-deformed nuclei $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}}=1/2$ used in the derivation of Eq. (7), not accurate enough to describe weakly deformed nuclei as well. The rescaling factor $E/e=(\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}})^{-1/3}$ itself depends on the deformation. Consequently, the strong deformation dependence of the prediction for the energy of the scissors mode will be weakened, as is necessary for a correct description of the data.

In order to compare these data to theoretical estimates, we use Eq. (4), derived by Bes and Broglia, but now with the inclusion of a deformation dependence of the rescaling factor E/e. This should phenomenologically take into account the deformation dependence of pairing effects in moderately deformed nuclei. To obtain the rescaling factor E/e as a function of deformation from experimental data, we fit the quantity $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}}$ as a function of δ. Employing Eqs. (6) and (8) we obtain the fit function

$$\left(\frac{\mathcal{J}_{\mathrm{expt}}}{\mathcal{J}_{\mathrm{rig}}}\right)(\delta)=\left(\frac{a\delta}{\sqrt{1+(b\delta)^2}}\right)^3, \tag{9}$$

which depends only on two parameters. From a least squares fit to the data with a χ^2 value 7×10^{-4}, we obtain the parameter values

$$a=3.9(1) \quad \text{and} \quad b=3.0(2). \tag{10}$$

As we see from Fig. 4 this two parameter fit works very well for the nuclei considered here which have widely varying deformation parameters $0.12<\delta<0.3$.

If the energy of the scissors mode from Eq. (4) is rescaled by the factor $E/e=\sqrt{1+(3\delta)^2}/(3.9\delta)$, one obtains the expression

$$E_{\mathrm{sc}}=13.4\sqrt{1+(3\delta)^2}\,A^{-1/3}\ \mathrm{MeV}. \tag{11}$$

This formula for the excitation energy of the scissors mode is included in Fig. 3. Of course, it describes the data from strongly deformed nuclei as well as Eq. (7). Additionally, it accounts for the data of weakly deformed nuclei. Consequently, using the schematic QRPA approach of Bes and Broglia, we find that the deformation dependence of the excitation energy of the scissors mode is no longer predicted to be linear. Just the opposite, the deformation dependence of the scissors mode energy is expected to be very weak, in agreement with the data.

IV. FINAL REMARKS

We note that, strictly speaking, all formulas adopted in this study are only approximately valid for well-deformed nuclei, because a Nilsson asymptotic basis, which neglects a spin-orbit term, has been adopted. The procedure, by which the rescaling factor has been obtained from the moment of inertia, is (only approximately) justified for deformed nuclei. It is remarkable that the fit formula for the moment of inertia, when extrapolated to weakly deformed nuclei, works for the nuclei considered in this paper as well as it does.

In the fit function of Eq. (9) we have considered a and b as free parameters. This consideration is equivalent to the simple ansatz $\Delta^2=\Delta_0^2(1-c\delta^2)$ for a possible deformation dependence of the pairing gap parameter with the free parameters $\Delta_0=\hbar\omega_0/2a$ and $c=a^2-b^2$. We emphasize that the constraint $b=a$ or equivalently $c=0$ does not allow for such a good fit of the data for $\mathcal{J}_{\mathrm{expt}}/\mathcal{J}_{\mathrm{rig}}$. The χ^2 value of the fit would be three times larger. The freedom of $b\neq a$ allows for enough flexibility of the fit function (9) to provide a satisfactory fit of the moment of inertia. Within the framework of the schematic QRPA the difference $c=6.2(14)$ could be interpreted as a small correction term caused by a deformation dependence of the pairing gap parameter Δ. From the fitted value $c>0$ one may conclude a hint at a decrease of the pairing gap parameter Δ as a function of deformation in transitional nuclei. Such a tendency can be observed in microscopic calculations where the pairing gaps are calculated [29,44,45]. In rough agreement to these calculations our ansatz results in reasonable values for the pairing gap parameter Δ ranging from 1.1 to 0.6 MeV for nuclei with masses $130<A<200$ and deformation parameters $0.1<\delta<0.3$. However, different values of a and b, which we obtain by the fitting procedure by Eq. (9) do not necessarily mean a decrease of the pairing gap with increasing deformation. This is because Eq. (9) is an approximation, which has been obtained in the framework of a schematic QRPA, and which can hardly be used for judging the deformation dependence or the mass dependence of the pairing gap parameter. We were interested in a simple parametrization of the rescaling factor E/e. The deformation dependence of the rescaling factor $E/e=\sqrt{e^2+\Delta^2}/e$ is dominated by the linear deformation dependence of $e=\delta\hbar\omega_0/2$. Consequently, the least squares fit to the moment of inertia is not very sensitive to the detailed deformation dependence of Δ, leading to a rather large error bar of about 20% for the parameter c, as compared to the errors for the fit parameters a and b of less than 7%. Of course, Δ may be a more complicated function of the deformation or of other quantities, such as the mass, and must be expected to depend on the details of the shell structure. These may be considerations for future studies. We choose our simple ansatz for E/e, because it led us to a simple two-parameter fit formula for the moment of inertia, which works well and is sufficient for our goals.

The improvement of the estimate for the scissors mode energy results from the consideration of the deformation dependence of the rescaling factor E/e responsible for the inclusion of pairing effects. We note that the importance of this deformation dependence in predicting the excitation energy and excitation strength of the scissors mode has already been pointed out by Hamamoto and Magnusson [29]. In addition, microscopic calculations for the scissors mode have been carried out for some nuclei with smaller deformations which can be compared to similar results for well deformed rotors in the rare-earth region (see, e.g., Refs. [46,47]). In these calculations, pairing was included and the scissors mode turned out to lie at about 3 MeV excitation energy. This is in agreement with the relation from Eq. (11), which has been obtained in the framework of the schematic approach by Bes and Broglia and basically assumes a scissors mode as the fundamental excitation responsible for the observed collective $M1$ excitations.

Finally, we wish to point out the consistency of the present results with the δ^2 law known for the deformation dependence of the total $M1$ strength. By obtaining the rescaling factor E/e as a function of deformation from a fit to $\mathcal{J}_{\rm expt}/\mathcal{J}_{\rm rig}$, we have produced a parametrization of the moment of inertia $\mathcal{J}_{\rm expt}$. The same schematic QRPA which provides the basic formulas for our approach predicts (in leading order) the total $M1$ strength of the scissors mode in terms of the moment of inertia of the ground-state and of the excitation energy of the scissors mode, namely,

$$B(M1;0_1^+\rightarrow 1_{\rm sc}^+)=\frac{3}{16\pi}\,\mathcal{J}E_{\rm sc}g_{\rm eff}^2\mu_N^2. \tag{12}$$

As the $B(M1)$ values found experimentally scale with δ^2, we must answer the following question: Are the parametrizations of the moment of inertia and of the scissors mode energy used in our analysis consistent with the δ^2 law? Inserting $E_{\rm sc}$ from Eq. (11) and $\mathcal{J}/\mathcal{J}_{\rm rig}$ from Eqs. (9) and (10) and using $\mathcal{J}_{\rm rig}\approx(2/5)AMR_0^2\approx 0.014A^{5/3}\hbar^2/{\rm MeV}$, we find

$$B(M1;0_1^+\rightarrow 1_{\rm sc}^+)=0.66\,\frac{\delta^3}{1+(3\delta)^2}A^{4/3}g_{\rm eff}^2\mu_N^2. \tag{13}$$

This expression has the same mass dependence as the semi-empirical formula derived in Eq. (9) of Ref. [4], if the $A^{-1/3}$ mass dependence of the excitation energy of the scissors mode is taken into account there. We note that the δ^2 law follows from that semiempirical formula only through the (correct) assumption of a constancy of the scissors mode energy. We use, furthermore, for the g factor $g_{\rm eff}=c_g g_p=c_g 2g_R$, where $g_R=Z/A$ is the rigid body value, obtaining

$$B(M1;0_1^+\rightarrow 1_{\rm sc}^+)=2.6c_g^2\,\frac{\delta^3}{1+(3\delta)^2}\,\frac{Z^2}{A^{2/3}}\,\mu_N^2. \tag{14}$$

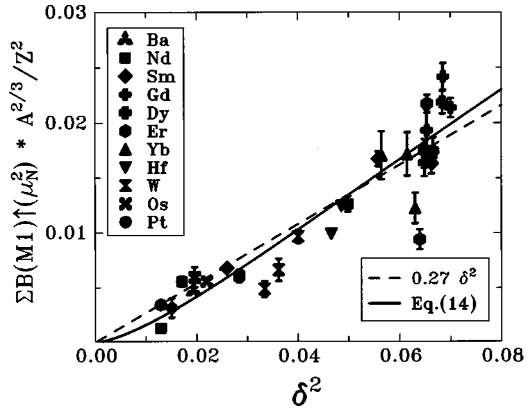

FIG. 5. Check of the consistency of the schematic approach with the δ^2 law. The expression from Eq. (14) is plotted with the solid line versus the square of the deformation parameter δ. For the effective g factor a scaling factor $c_g=0.8$ has been used. The dashed line indicates a pure δ^2 law with a scaling factor 0.27. Superimposed are the experimental data from Fig. 1.

The deformation dependence of the expressions Eqs. (13) and (14) is practically indistinguishable from a δ^2 dependence. This fact is shown in Fig. 5, where we compare the expression from Eq. (14) with a pure δ^2 dependence and with the data on the total $M1$ strengths from Fig. 1. In order to describe on an absolute scale the $B(M1)$ values found experimentally, we must use the scaling factor $c_g=0.8$ in the effective g factor. The difference between the pure δ^2 law and Eq. (14) is of the order of the experimental error bars and it is smaller than the overall scatter of the data points.

Consequently, using the approach of a schematic QRPA for the description of a scissors mode, one can derive both main features for the strong low-lying $M1$ excitations in deformed nuclei: a nearly δ^2 dependence of the $M1$ strength and an approximate constancy of its center of gravity, close to 3 MeV.

V. SUMMARY

From the photon scattering data on nuclei in the mass region $130<A<200$ we have extracted and tabulated mean values for the excitation energy of the scissors mode. The mean excitation energy is calculated as the centers of gravity of the observed low-lying $M1$ strength distributions. The mass dependence of the scissors mode energy is in rough agreement with an $A^{-1/3}$-mass dependence. For a wide range of deformation parameters $0.12<\delta<0.3$, for which experimental data are available, a deformation dependence of the scissors mode energy cannot be observed. The inclusion of the deformation dependence of pairing effects in a formula, derived earlier by Bes and Broglia within a schematic QRPA, improves their estimate of the scissors mode energy for moderately deformed nuclei. The schematic approach, which results in the nearly constancy of the scissors mode energy is consistent with the δ^2 law for the $M1$ strength.

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