Medium modification of nucleon properties in the Skyrme model

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A Skyrme-type Lagrangian for a skyrmion imbedded in nuclear matter is proposed. The dependence of the static nucleon properties and the nucleon-nucleon tensor interaction on nuclear density is investigated. [S0556-2813(98)04009-6]

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I. INTRODUCTION

Properties of a single nucleon in free space are understood in classical terms by means of a solitonlike solution of nonlinear Lagrangians such as that of Skyrme [1]. Models have been constructed to consider the pion-nucleon and the nucleon-nucleon (NN) interactions [2–4] and even to deform the nucleons [5]. An important outstanding problem is the study of these solitonlike structures in a many-body nuclear system. In collisions the properties of hadron are to be studied in a hot (nonzero temperature) and dense (a density much larger than ρ_0 is the density of normal nuclear matter) nuclear matter. There are good reasons to believe that the properties of hadrons such as mass, radii, and coupling to external currents change in a hot and dense nuclear medium. Furthermore, the modifications of the NN interaction in such a medium have to be understood. The medium is expected to play an important role in changing the overall strength of the interaction and the relative strengths of the central, tensor, and spin-orbit components of the interaction.

Quantum chromodynamics (QCD), the fundamental theory of the strong interactions, has to be replaced with an effective theory to consider the nuclear interactions in a medium in any consistent manner. In arriving at an effective theory the important symmetry constraints of QCD, chiral symmetry, and scale invariance have to be retained. Recent considerations of Brown and Rho [6] are an attempt to find and elucidate such a theory. In constructing an effective Lagrangian $\mathcal{L}_{\rm eff}$ the in-medium modification is reflected by a change in the vacuum expectation value of a dilatation field. The resulting Lagrangian, which obeys the trace anomaly of QCD almost coincides with the Schechter's Lagrangian [7] in form but includes some modified parameters. However, this in-medium $\mathcal{L}_{\rm eff}$ does not take into account the possible modification of the chiral field, since it is considered here as a massless Goldstone boson. On the other hand it is quite natural to assume that, $\mathcal{L}_{\rm eff}$ has to include the direct distortions of the chiral fields. In fact, in a linear approach, the Skyrme Lagrangian describes the free pion field and its in-medium modified version must be relevant to the pion fields in nuclear matter.

The aim of this paper is to consider a nucleon placed in nuclear matter and try to describe this nucleon in the framework of the Skyrme model, taking into account the influence of the medium (see also Refs. [8,9]). It is not our goal to describe the whole nuclear system. Instead we shall concentrate on the changes of nucleon properties embedded in nuclear matter taking into account the influence of baryon rich environment as an external parameter. The basic idea is that, in the linear approach the $\mathcal{L}_{\rm eff}$ should give the well known [10] equation for the pion field $\partial^\mu\partial_\mu\vec{\pi}+(m_\pi^2+\hat{\Pi})\vec{\pi}=0$, where $\hat{\Pi}$ is the polarization function or the self-energy of the pion field in the medium. The extension to finite nuclear systems requires modifications due to the presence of the surface.

The paper is organized as follows. In Sec. II, we propose a modified Skyrme Lagrangian $\mathcal{L}_{\rm eff}$, including the distortion of chiral field in the medium. The solitonlike solutions of this Lagrangian represent a Skyrmion embedded in the nuclear medium; the Lagrangian is applied to calculate the static properties of the nucleon in Sec. III. In Sec. IV, we consider the possible modifications of nucleon-nucleon tensor interaction due to the presence of the medium. We summarize and discuss the results in Sec. V.

II. THE IN-MEDIUM SKYRME LAGRANGIAN

The Skyrme model is a theory of nonlinear meson fields where baryons can emerge as soliton solutions. The Skyrme Lagrangian may be written as [1]

$$\begin{aligned}\mathcal{L}_{\rm sk}&=\mathcal{L}_2+\mathcal{L}_{4a}+\mathcal{L}_{\chi sb},\\ \mathcal{L}_2&=-\frac{F_\pi^2}{16}\,{\rm Tr}(\vec{\nabla}U)\cdot(\vec{\nabla}U^+),\\ \mathcal{L}_{4{\rm a}}&=\frac{1}{32e^2}\,{\rm Tr}[U^+\partial_i U U^+\partial_j U]^2,\\ \mathcal{L}_{\chi{\rm sb}}&=-\frac{F_\pi^2}{16}\,{\rm Tr}[(U^+-1)m_\pi^2(U-1)],\end{aligned}\tag{2.1}$$

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where e is the Skyrme parameter and $F_\pi = 2f_\pi$ with f_π being the pion decay constant. The expansion around the vacuum value ($U \approx 1$)

$$U = \exp[2i(\vec{\tau}\vec{\pi})/F_\pi] \approx 1 + \frac{2i}{F_\pi}(\vec{\tau}\vec{\pi}) + \cdots \qquad (2.2)$$

in Eq. (2.1) gives a Lagrangian for the free pion field

$$\mathcal{L}_{sk} \approx \mathcal{L}_\pi = -\frac{1}{2}(\vec{\nabla}\vec{\pi})^2 - \frac{1}{2}m_\pi^2\vec{\pi}^2. \qquad (2.3)$$

Let us consider a Skyrmion inserted in a nucleus. It is well known that pions in nuclei are described [10] by the Lagrangian

$$\mathcal{L}_\pi^\star = \mathcal{L}_\pi - \frac{1}{2}\,\vec{\pi}\hat{\Pi}\,\vec{\pi} \qquad (2.4)$$

(the asterisk indicates the medium) where $\hat{\Pi}$ is the self-energy or the polarization operator, which characterizes the modification of the pion propagator in the medium. Bearing in mind an expansion such as (2.2) we may generalize Eq. (2.1) as

$$\mathcal{L}_{sk}^* = \mathcal{L}_2 + \mathcal{L}_{4a} + \mathcal{L}_{\chi sb}^*,$$
$$\mathcal{L}_{\chi sb}^* = -\frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}[(U^+ - 1)(1 + \hat{\Pi}/m_\pi^2)(U - 1)], \qquad (2.5)$$

where the only modified term $\mathcal{L}_{\chi sb}^*$ describes the distortion of the pion field in the medium.

The calculation of the pion self-energy in the coordinate space within the Skyrme model in a self-consistent way is a special problem. It is not our goal to calculate it in the present paper, since we are not describing the whole system of nucleons in the framework of the Skyrme model. Instead, in the coordinate space we use a simple relation between $\hat{\Pi}$ and the pion-nuclear optical potential $\hat{U}_{\rm opt}$: $\hat{\Pi} \approx 2\omega_\pi \hat{U}_{\rm opt}$ [10].

In general, the operator $\hat{\Pi}$ acts on the coordinate R, of the center of mass of the soliton as well as on its internal collective coordinate $\vec{r}$, i.e., $\hat{\Pi} = \hat{\Pi}(\vec{R}, \vec{r} - \vec{R})$. For heavy nuclei the R dependence is weak and for the homogeneous nuclear matter it may be neglected entirely. Letting $\hat{\Pi} = 2\omega_\pi \hat{U}_{\rm opt}(\vec{r})$ in Eq. (2.5) we may choose an optical potential that is used widely. It is clear that when $\hat{U}_{\rm opt}$ is local, as the ''Laplacian potential'' is [10], the modification of the Lagrangian is trivial and mainly consists in changing the pion mass into an effective mass $m_\pi^* = m_\pi\sqrt{1 + 2\hat{U}_{\rm opt}/m_\pi}$ in the medium.

Clearly, the most interesting case is to use the nonlocal Kisslinger potential, used both in describing the pionic atoms and the pion nuclear scattering. At threshold, when $\omega_\pi \approx m_\pi$, it may be represented in a schematic way [10] as

$$\hat{\Pi} = \chi_s(r) + \vec{\nabla}\cdot\chi_p(r)\vec{\nabla}, \qquad (2.6)$$

where χ_s and χ_p are functionals of S-wave and P-wave pion nucleon scattering lengths and the nuclear density $\rho(r)$. Using this form for polarization operator and integrating by parts we obtain the following Lagrangian:

$$\mathcal{L}_{sk}^* = \mathcal{L}_2^* + \mathcal{L}_{4a} + \mathcal{L}_{\chi sb}^*,$$
$$\mathcal{L}_2^* = -\frac{F_\pi^2}{16}\alpha_p(r)\mathrm{Tr}(\vec{\nabla}U)\cdot(\vec{\nabla}U^+),$$
$$\mathcal{L}_{4a} = \frac{1}{32e^2}\,\mathrm{Tr}[U^+\partial_i U U^+ \partial_j U]^2, \qquad (2.7)$$
$$\mathcal{L}_{\chi sb}^* = \frac{F_\pi^2 m_\pi^2}{16}\alpha_s(r)\mathrm{Tr}(U + U^+ - 2),$$

where $\alpha_p(r) = 1 - \chi_p(r)$, $\chi_p(r)$ is the pion dipole susceptibility of the medium, and $\alpha_s(r) = 1 + \chi_s(r)/m_\pi^2$.

Thus, the nonlocal Kisslinger potential modifies not only the pion mass term but also the kinetic term $\mathcal{L}_2$. Note that in our model the fourth order derivative term $\mathcal{L}_{4a}$ remains unchanged. This is not surprising, since this term corresponds to the infinite mass limit of the ρ meson [11], whose self-energy operator is not considered here. Thus our basic Lagrangian is given in Eq. (2.7) and will be used to investigate modifications of the nucleon properties in the medium.

III. THE IN-MEDIUM NUCLEON PROPERTIES

In finite nuclei there may arise some difficulties concerned with surface effects and localization of the Skyrmion in the nuclear medium. Therefore for simplicity, only medium modifications in homogeneous nuclear matter are considered. In this case $\chi_p(r)$ and $\chi_s(r)$ in the Lagrangian are clearly constants [$\chi_p(r) \equiv \chi_p, \chi_s(r) \equiv \chi_s$] and the Skyrmion may be assumed to have spherical symmetry.

For the spherically symmetric static Skyrme ansatz $U(r) = U_0 = \exp[i\vec{\tau}\hat{r}\Theta(r)]$, $\hat{r} = \vec{r}/|r|$, the mass functional for the dimensionless variable $x = eF_\pi r$ has the form[1]

$$M_H^* = \frac{4\pi F_\pi}{e}\int_0^\infty dx(\tilde{M}_2^* + \tilde{M}_{4a} + \tilde{M}_{\chi sb}^*),$$
$$\tilde{M}_2^* = (\Theta'^2 x^2/2 + s^2)(1 - \chi_p)/4,$$
$$\tilde{M}_{4\mathrm{a}} = s^2(d/2 + \Theta'^2), \qquad (3.1)$$
$$\tilde{M}_{\chi sb}^* = (1 - c)x^2\beta^2(1 + \chi_s/m_\pi^2)/4,$$

where $c \equiv \cos(\Theta)$, $s \equiv \sin(\Theta)$, $d = (s/x)^2$, and $\beta = m_\pi/(F_\pi e)$. Since the nuclear dipole susceptibility χ_p is nearly proportional to the nuclear density ρ, for large densities the $\tilde{M}_2^*$ term becomes negative and a Skyrmion may

[1]Here the Skyrmion is assumed to be placed right at the center of mass of the nucleus.

disappear. Let us discuss this point in detail. The Euler-Lagrange equation for the shape function $\Theta(x)$ is given as

$$\Theta''[x^2\alpha_p+8s^2]+2\Theta' x\alpha_p+4\Theta'^2 s_2 -[s_2\alpha_p+4ds_2+x^2\beta^2 s\alpha_s]=0, \tag{3.2}$$

where $s_2\equiv\sin(2\Theta)$ and the prime corresponds to a derivative with respect to x. As we are not interested in describing the nuclear system as a whole, solutions with $\Theta(0)=\pi$ corresponding to the baryon number $B=1$ are used. The asymptotic behavior of $\Theta(x)$ at large distances is similar to that for the free case

$$\lim_{x\to\infty}\Theta(x)=\gamma\frac{(1+\beta^* x)\exp(-\beta^* x)}{x^2},\qquad \beta^*=\beta\sqrt{\frac{1+\chi_s/m_\pi^2}{1-\chi_p}}. \tag{3.3}$$

It is well known [10] that for finite nuclei the pion susceptibility is always less than unity, $\chi_p<1$. However, for infinite nuclear matter with a constant density $\rho=\lambda\rho_0(\rho_0=0.5m_\pi^3)$ there is some critical value of λ when the expression under the square root sign becomes negative which leads to an exponentional dissipation of the soliton solutions. Thus the condition for survival of a Skyrmion in dense matter is equivalent to comparing the dipole susceptibility with unity as in the usual pion nuclear physics [10]. This result may be compared with the model proposed in Ref. [12], where there are no Skyrmion solutions even for real nuclei.

In order to carry out numerical calculations the following expressions for χ_s and χ_p [10] are adopted

$$\chi_s=-4\pi\eta b_0\rho,\quad \chi_p=\frac{\kappa}{1+g_0'\kappa},\quad \kappa=4\pi c_0\rho/\eta, \tag{3.4}$$

where $\eta=1+m_\pi/M_N$ is a kinematical factor and M_N is the mass of the nucleon. The parameters b_0 and c_0 are effective pion-nucleon S and P wave scattering lengths, respectively, and g_0' is the Lorentz-Lorenz or correlation parameter.

The set of empirical parameters $b_0=-0.024m_\pi^{-1}$, $c_0=0.21m_\pi^{-3}$ [13] are used in this calculation. Parameters $F\pi$ and e have the values $F\pi=108$ MeV and $e=4.84$ [1], so for the free nucleon and isobar masses we have $M_N=939$ MeV and $M_\Delta=1232$ MeV. Using these values in Eqs. (3.3) and (3.4) the critical density of nuclear matter $\rho_{\rm crit}$ may be estimated, when a stable Skyrmion solution does not exist as $\rho_{\rm crit}\geqslant 1.3\rho_0$ and $\rho_{\rm crit}\geqslant 3\rho_0$ for $g_0'=1/3$ and 0.7, respectively. Clearly for real nuclei where $\rho\leqslant\rho_0$ this model is valid. The standard canonical quantization method [1] gives the familiar expressions for the mass of the nucleon and Δ isobar

$$M_N^*=M_H^*+3/8\lambda_M^\star,\qquad M_\Delta^*=M_H^*+15/8\lambda_M^\star, \tag{3.5}$$

where M_H^* is the soliton mass (3.1) and $\lambda_M^\star$ is the moment of inertia of the rotating Skyrmion

$$\lambda_M^\star=\frac{8\pi}{3e^3F_\pi}\int_0^\infty dx x^2 s^2[1/4+\Theta'^2+d]. \tag{3.6}$$

The mass M_H^* may be interpreted as the mass of a soliton of the nonlinear pion field affected by the medium. Note that the moment of inertia $\lambda_M^\star$ does not include the nuclear density ρ explicitly, since the nonstatic parts of the self-energy operator are not included. Similarly, the isoscalar and isovector mean square radii, defined by zero components of the baryon and vector currents, have the same formal expressions as in the free case:

$$\langle r^2\rangle_{I=0}^*=-\frac{2}{e^2F_\pi^2\pi}\int_0^\infty x^2\Theta' s^2 dx,\qquad \langle r^2\rangle_{I=1}^*=\frac{1}{e^2F_\pi^2}\frac{\int_0^\infty x^4 s^2[1+4(\Theta'^2+d)]dx}{\int_0^\infty x^2 s^2[1+4(\Theta'^2+d)]dx}. \tag{3.7}$$

Changes in the moment of inertia and size of the nucleon are not crucial, since they are caused only by a modification of the profile function Θ. In contrast, the expression for the isovector magnetic moments defined by the space component of the vector current

$$\mu_{I=1}=\frac{1}{2}\int d\vec{r}\vec{r}\times\vec{V}_3 \tag{3.8}$$

includes medium characteristics explicitly, which arise from the contributions of the kinetic term $\mathcal{L}_2^*$ to the vector current

$$\vec{V}_k=-i\frac{F_\pi^2}{16}(1-\chi_p)\mathrm{Tr}\ \vec{\tau}(L_k+R_k)+\frac{i}{16e^2}\mathrm{Tr}\ \vec{\tau}\{[L_\nu[L_k,L_\nu]]+[R_\nu[R_k,R_\nu]]\}, \tag{3.9}$$

where $L_\mu=U^+\partial_\mu U$ and $R_\mu=U\partial_\mu U^+$. Hence for the nucleon in nuclear matter simple relations between magnetic moments and momentum of inertia such as $\mu_{I=1}^p=\lambda_M/3$ shown in Ref. [1] do not work. Table I illustrates modifications of the static properties of the nucleon in infinite nuclear matter.

Early arguments [14] about changes of the nucleon size in the medium were, in part, based on the expectation that $r^*/r=M_N/M_N^*$, where r^* and M_N^* are the nucleon radius and mass in the nuclear medium, respectively, and r, M_N are the same two quantities for a free nucleon. In the present model the renormalization of the nucleon mass is much larger than the renormalization of the nucleon radius. The renormalization of the nucleon radii in Eq. (3.7) has been caused only by a modification of the profile function $\Theta(r)$ (see Fig. 1) in nuclear matter, while the modification of M_N is caused by the additional factor $(1-\chi_p)$ in Eq. (3.1).

There are no direct experimental values of the static properties of a nucleon bound in nuclei or nuclear matter. In contrast many theoretical approaches have been proposed to estimate them. Many of them deal with an explanation of the EMC effect. For example, in the nuclear binding model [15],

TABLE I. Ratio of the static properties of the nucleon in the medium (denoted by asterisks) to that of the free nucleon for various values of the nuclear density $\rho=\lambda 0.5 m_\pi^3$ ($g_0'=1/3$).

λ	$\frac{M_N^*}{M_N}$	$\frac{g_A^*}{g_A}$	$\sqrt{\frac{\langle r^2\rangle^*_{M,I=0}}{\langle r^2\rangle_{M,I=0}}}$	$\sqrt{\frac{\langle r^2\rangle^*_{I=0}}{\langle r^2\rangle_{I=0}}}$	$\sqrt{\frac{\langle r^2\rangle^*_{I=1}}{\langle r^2\rangle_{I=1}}}$	$\sqrt{\frac{\langle r^2\rangle^*_{p}}{\langle r^2\rangle_{p}}}$	$\sqrt{\frac{\langle r^2\rangle^*_{n}}{\langle r^2\rangle_{n}}}$
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.788	0.845	1.121	1.170	1.060	1.095	0.969
0.75	0.696	0.743	1.177	1.255	1.086	1.140	0.939
1.00	0.609	0.619	1.230	1.344	1.113	1.188	0.900

$M_N^*=700$ MeV (appropriate for Fe) and $M_N^*=600$ MeV (appropriate for Au) have been found. On the other hand a calculation of the nucleon effective mass M_N^* is an important problem in quantum hadrodynamics (QHD). The recent results obtained by including π,ρ,ω meson fields explicitly in the Lagrangian of QHD give $M_N^*\approx 620$ MeV at zero temperature for $\rho=\rho_0$ [16]. In comparison with these results, our model gives $M_N^*=572$ MeV for nuclear matter for $g_0'=1/3$. Note that, our results are very sensitive to the value of g_0'. For example, using a different value, $g_0'=0.4$, we get $M_N^*=596$ MeV. This fact is illustrated in Fig. 2(a), where the dependence of M_N^* on g_0' is plotted for $\rho=0$, $\rho=0.5\rho_0$, and $\rho=\rho_0$. The present approach is similar in some respects to the soliton model of Ref. [17] where the mean field approximation for Friedberg-Lee approach is used. A swelling of the nucleon size $\sim 30\%$ predicted there, is in good agreement with our result (see Table I). On the other hand, in the pion excess model [18], which explains the swelling by a distortion of the pion cloud in the medium where a very large modification of the nucleon size, nearly doubling of the free value of the rms radius, was obtained only for the isovector radius, whereas here the swelling affects both the isovector and the isoscalar radii.

In nuclei the axial coupling constant g_A, governing the Gamow-Teller transitions, is modified significantly from its free-space value of ≈ 1.25. It is shown that g_A is systematically renormalized downward in finite nuclei [19]. A most remarkable observation [20] based on a model-independent analysis of β decay and magnetic moments data of the mirror nuclei ($5\leqslant A\leqslant 39$) is that the g_A in nuclei equals unity to a very good accuracy: $g_A^*=1.00\pm 0.02$, that is, $g_A^*/g_A=0.8$ for nuclear matter.

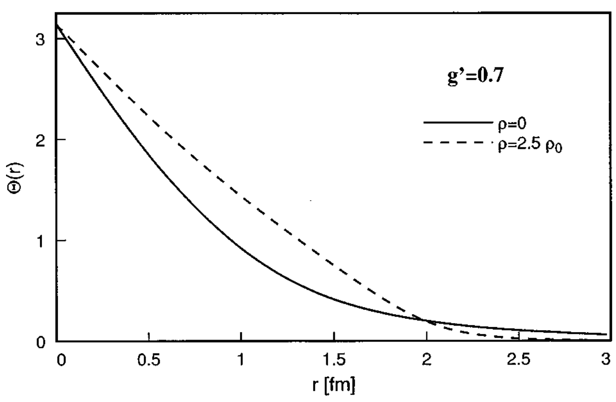

FIG. 1. The profile function $\Theta(r)$ of a free skyrmion (solid curve) and that of a skyrmion in the nuclear matter $\rho=2.5\rho_0$ (dashed curve). Here $g_0'=0.7$.

Although the Skyrme model, especially in its original version, gives an underestimate for the value of g_A ($g_A=0.65$ for the free case) we may still try to investigate the quenching phenomenon within the present approach. It is easy to show that the expression for g_A is the same as in the free case but there is an additional factor arising from the kinetic term

$$g_A{}^*=-\frac{\pi}{3e^2}\int_0^\infty dx x^2[g_2^*(x)+g_4(x)],$$

$$g_2^*(x)=(1-\chi_p)\cdot(\Theta'+s_2/x), \tag{3.10}$$

$$g_4(x)=4[s_2(\Theta'^2+d)/x+2\Theta' d].$$

In nuclear matter ($\chi_p<1$) g_A decreases due to the factor $(1-\chi_p)$ in $g_2^*(x)$. The decrease of g_A approaches 38% for nuclear matter ($\rho=\rho_0$ with $g_0'=1/3$) (Table I). This is consistent with the estimates carried out in the Δ-hole coupling model using the random phase approximation: $g_A{}^*/g_A\approx 0.67$ for $\rho/\rho_0=1$ and $g_A{}^*/g_A\approx 0.8$ for $\rho/\rho_0=1/2$. The g_0' dependence of $g_A{}^*/g_A$ is shown in Fig. 2(b). This dependence is in qualitative agreement with the formula $g_A{}^*/g_A=[1-4g_0'L(0)/9]^{-1}$ presented by Rho [19].

For nuclear matter the effective Lagrangian, Eq. (2.7), with the polarization operator, Eqs. (2.6) and (3.4), has only two parameters depending on the pion nucleon scattering. Here the effective pion-nucleon scattering lengths have been used. However, in nuclear matter the pion field is localized very close to the nucleons in contrast to the case of the pionic atoms. One may ask if the present model is able to make predictions about the constants b_0 and c_0 in nuclear matter? To do this we have to compare our results with the experimental data. The ratio $g_A{}^*/g_A$ is well established to be 0.8, while the pionic data analysis yields a value of 0.62 (see the last line of Table I). In the nuclear matter c_0 is reduced by a factor of 2 almost independent of the value of g_0' to get the correct quenching (Table II). For this optimal case the effective nucleon mass is also close to the common value of 700 MeV. In addition $g_A{}^*/g_A$ is not sensitive to the S-wave scattering length b_0. A reduction of the effective P-wave scattering length in nuclei may be clearly understood by the fact that quenching of g_A is equivalent to a reduction of the pion-nucleon coupling constant $g_{\pi NN}$ and hence the pion-nucleon amplitude.

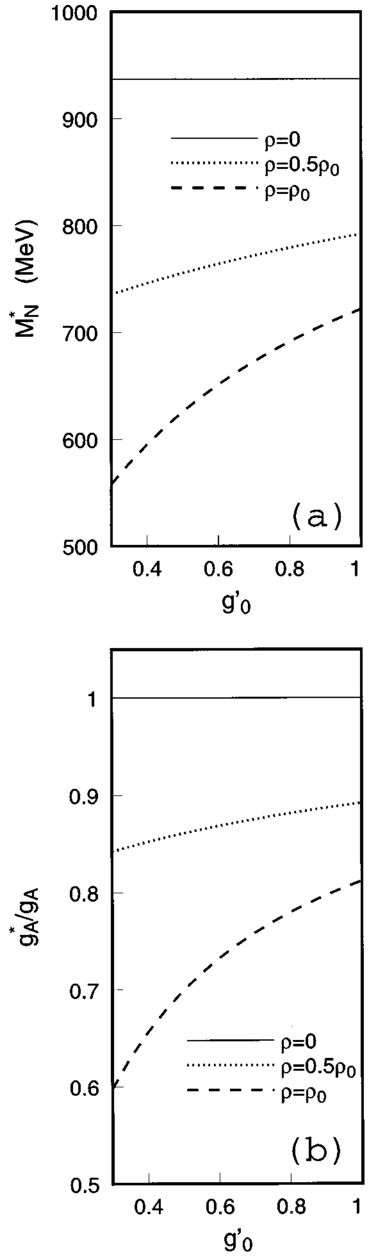

FIG. 2. (a) The dependence of the effective nucleon mass on the Lorentz-Lorenz parameter g_0'. Solid, dotted, and dashed curves are for $\rho=0$, $\rho=0.5\rho_0$, and $\rho=\rho_0$, respectively. (b) The same as in (a), but for the ratio g_A^*/g_A.

IV. IN-MEDIUM *NN* TENSOR INTERACTION

Not only the static properties of hadrons but also the dynamical ones are modified by the presence of the medium. The in-medium NN interaction differs from the corresponding one in the free space due to Pauli blocking (which is not considered here) and due to the modification of propagators of exchanged mesons [21]. We investigate the nucleon-nucleon interaction potential by using the product approximation

$$U(\vec{x};\vec{r}_1,\hat{A}_1;\vec{r}_2,\hat{A}_2)=\hat{A}_1U_0(\vec{x}-\vec{r}_1)\hat{A}_1^+\hat{A}_2U_0(\vec{x}-\vec{r}_2)\hat{A}_2^+ \equiv U_1U_2, \tag{4.1}$$

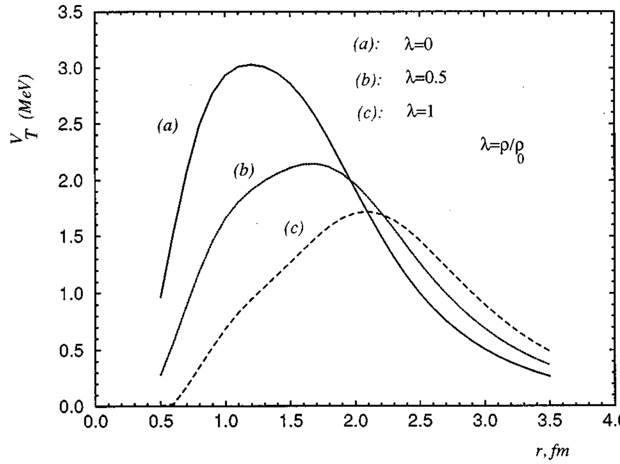

FIG. 3. The tensor part of the NN potential-$V_T(r)$. Solid, dotted, and dashed curves are for $\rho=0$, $\rho=0.5\rho_0$, and $\rho=\rho_0$, respectively. Here $g_0'=0.6$, $b_0=-0.024m_\pi^{-1}$, and $c_0=0.15m_\pi^{-3}$.

where $U_0(\vec{x}-\vec{r}_i)$ for $i=1,2$ is the hedgehog solution $(U_0(\vec{r})=\exp[i\vec{\tau}\hat{r}\Theta(r)],\ \hat{r}=\vec{r}/|r|)$ located at $\vec{r}_i$, and A_i is the collective coordinate to describe the rotation. The in-medium NN interaction may be defined by

$$V_{NN}(\vec{r})=-\int d\vec{x}[\mathcal{L}_{\rm sk}^*(U_1U_2)-\mathcal{L}_{\rm sk}^*(U_1)-\mathcal{L}_{\rm sk}^*(U_2)], \tag{4.2}$$

where $\vec{r}$ is the relative coordinate between the two Skyrmions $(\vec{r}=\vec{r}_1-\vec{r}_2)$. The static NN potential may be obtained by using a standard technique [2] which gives the following general representation:

$$V_{NN}(\vec{r})=V_C(r)+(\vec{\tau}_1\vec{\tau}_2)(\vec{\sigma}_1\vec{\sigma}_2)V_{\sigma\tau}(r)+(\vec{\tau}_1\vec{\tau}_2)S_{12}V_T(r). \tag{4.3}$$

Unfortunately, the original Skyrme model for the free case cannot describe the intermediate range attraction in the central potential within this approximation [2]. This is accomplished by the inclusion of a scalar σ meson in the Lagrangian [3,4], which is not taken into account in the present calculations. Here, it is more interesting to consider the tensor part $V_T(r)$ of V_{NN} due to the exchange of pions, modified in the medium. This part of V_{NN} plays an important role in the spin-isospin excitations and the pionlike excited states in nuclei.

Actually for finite nuclei the product ansatz (4.1) should be modified to take into account the nonspherical effects. But for homogenous nuclear matter the present approximation of the product of hedgehog solutions is valid as in the case of free space. In fact, formally in this approach, the main difference between the in-medium case $(\rho\neq 0)$ and the free one $(\rho=0)$ is that the contribution to the potential arising from $\mathcal{L}_2$ and $\mathcal{L}_{\chi sb}$ should be multiplied by factors of $(1-\chi_p)$ and $(1+\chi_s)$, respectively. The resulting $V_T(r)$ is presented in Fig. 3 for nuclear matter densities of $\rho=0$, $\rho=0.5\rho_0$, and $\rho=\rho_0$. The parameters of the optical potential are chosen so as to reproduce the relation $g_A{}^*/g_A=0.8$ for nuclear matter (II line of Table II). The nucleon-nucleon tensor interaction in a nucleus appears to be weaker than it is in free space

($\rho=0$). This suppression of $V_T(r)$ in nuclear matter was shown earlier [22] using renormalized exchange meson masses in accordance with the scaling model [6]. For finite nuclei it was shown [23] by analyzing the energy difference of $T=1$ and $T=0$, $J=0$ states in ^{16}O.

V. DISCUSSION AND SUMMARY

We have proposed a modified Lagrangian $\mathcal{L}^*_{\rm sk}$ for a Skyrmion placed in a nuclear medium. In constructing the linear approximation to the Lagrangian it is required that the well-known equation for the pion field $\partial^\mu\partial_\mu\vec{\pi}+(m_\pi^2+\Pi)\vec{\pi}=0$ is realized. The inclusion of the pion self-energy Π in to the free space Skyrme Lagrangian, determines the explicit coordinate dependence of Π. Actually, for a moving Skyrmion with $U=U(\vec{r}-\vec{R})$ a similar dependence $\Pi=\Pi(\vec{r}-\vec{R})$ could be obtained. Otherwise this equation would not be consistent with the medium modified Lagrangian.

A much more general choice such as $\Pi=\Pi(\vec{R},\vec{r}-\vec{R})$, which is essential for finite nuclei, would give information about energy levels of the bound Skyrmion as well as about the in-medium modifications of its internal parameters (mass, size, etc.). The latter is an exciting topic in the relativistic heavy-ion experiments.

As an input data, apart from F_π and e, the present approach uses the nuclear density and effective pion-nucleon scattering lengths. Using the modified Skyrme model it is easy to study the in-medium nucleon-nucleon interaction using the standard product approximation.

Let us recall here our main results.

(i) The critical nuclear density $\rho_{\rm crit}$ where a Skyrmion in nuclear matter remains stable is $\rho_{\rm crit}\leqslant 1.3\rho_0$ and $\rho_{\rm crit}\leqslant 3\rho_0$ for $g_0'=1/3$ and 0.7, respectively. This fact shows the strong dependence of $\rho_{\rm crit}$ on the Landau parameter g_0'.

(ii) The in-medium effects such as the swelling of a nucleon and a decrease of its mass are not as large as predicted by the pion excess models. The change of nucleon mass is mainly due to the modification of the second derivative term $\mathcal{L}_2$ and depends on the size of the isoscalar P-wave pion nucleon scattering volume (C_0).

(iii) The quenching of the axial coupling constant g_A in nuclear matter shows that the effective c_0 is much smaller than that predicted by the pionic atom analysis.

TABLE II. Nucleon effective mass M_N^* and modification of the nucleon size in normal nuclear matter ($\rho=\rho_0=0.5m_\pi^3$). The effective pion-nucleon scattering length b_0 and scattering volume c_0 are chosen so that $g_A^*/g_A=0.8$.

g_0'	b_0 (m_π^{-1})	c_0 (m_π^{-3})	M_N^* (MeV)	$\sqrt{\frac{\langle r^2\rangle_p^*}{\langle r^2\rangle_p}}$
1/3	−0.024	0.125	719	1.089
0.6	−0.024	0.150	714	1.092
0.6	0.0	0.140	680	1.10

(iv) The medium modifies the NN interaction, in particular the tensor part of the interaction in nuclei appears to be weaker than in free space.

Modification of the nucleon properties found in the present paper are understood in terms of the medium effects on the chiral nonlinear field and consequently on the shape and mass of the soliton.

Another explanation of the in-medium modifications, based on scale invariant arguments, has been proposed recently by Brown and Rho [6]. The (broken) scale invariance of QCD in the Skyrme model was implemented and this suggested that changes in the hadron properties may arise from a universal scaling related to the scaling anomaly of QCD. However, further analyses [24] have shown that these changes must be small due to the large mass of the dilaton, associated here with a glueball.

At a more fundamental level the origin of these changes is hidden in a partial restoration of the chiral symmetry, i.e., in a decrease of the quark condensate in nuclear matter [6,24]. Unfortunately, there are no quark degrees of freedom in the Skyrme model. So, in the framework of this model it is natural to believe that the modification of nucleon properties in the medium are caused by the influence of the latter on the nonlinear pion fields.

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[1] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983); G. S. Adkins and C. R. Nappi, *ibid.* **B233**, 109 (1984); I. Zahed and G. E. Brown, Phys. Rep. **142**, 1 (1986).

[2] T. Otofuji, S. Saito, M. Yasuno, T. Kurihara, and H. Kanada, Phys. Rev. C **34**, 1559 (1986); E. M. Nyman and D. O. Riska, Phys. Scr. **34**, 533 (1986); A. De Pace, H. Müther, and A. Faessler, Z. Phys. A **325**, 229 (1986); H. Yabu and K. Ando, Prog. Theor. Phys. **74**, 750 (1985).

[3] M. M. Musakhanov and A. Rakhimov, Mod. Phys. Lett. A **10**, 2297 (1995).

[4] H. Yabu, B. Schwesinger, and G. Holzwarth, Phys. Lett. B **224**, 25 (1989).

[5] A. Rakhimov, T. Okazaki, M. M. Musakhanov, and F. C. Khanna, Phys. Lett. B **378**, 12 (1996).

[6] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991); Phys. Rep. **269**, 333 (1996).

[7] H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **33**, 3476 (1986).

[8] Ulf-G. Meissner, Nucl. Phys. **A503**, 801 (1989).

[9] G. Kalbermann, Nucl. Phys. **A612**, 359 (1997).

[10] T. Ericson and W. Weise, *Pions and Nuclei* (Clarendon, Oxford, 1988); J. M. Eisenberg and D. S. Koltun, *Theory of Meson Interactions with Nuclei* (Wiley, New York, 1980).

[11] Rajat K. Bhaduri, *Models of Nucleon from Quarks to Solitons*

(Addison-Wiley, New York, 1988).
[12] E. Mishustin, Sov. Phys. JETP **71**, 21 (1990).
[13] L. Tausher, *Physics of Exotic Atoms* (Frascati INFN, Erice, 1977).
[14] K. S. Celenza, A. Rozental, and C. M. Shakin, Phys. Rev. Lett. **53**, 892 (1984).
[15] J. V. Nobel, Nucl. Phys. **A329**, 354 (1979).
[16] Song Gao, Yi-Jun Zhang, and Ru-Keng Su, Nucl. Phys. **A593**, 362 (1995).
[17] M. Jandel and G. Peters, Phys. Rev. D **30**, 1117 (1984).
[18] M. Ericson, Prog. Theor. Phys. Suppl. **91**, 244 (1987).
[19] M. Rho, Annu. Rev. Nucl. Sci. **34**, 531 (1984).
[20] B. Buck and S. M. Perez, Phys. Rev. Lett. **50**, 1975 (1983).
[21] G. Q. Li and R. Machleidt, Phys. Rev. C **48**, 1702 (1993).
[22] A. Hosaka and H. Toki, Nucl. Phys. **A529**, 429 (1991).
[23] D. C. Zheng, L. Zamick, and H. Muther, Ann. Phys. (N.Y.) **230**, 118 (1994).
[24] M. Birse, J. Phys. G **20**, 1537 (1994).