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Cumulant moments in hadron-nucleus collisions and stochastic processes

N. Suzuki
Matsusho Gakuen Junior College, Matsumoto 390-12, Japan

M. Biyajima
Department of Physics, Shinshu University, Matsumoto 390, Japan

G. Wilk
The Andrej Sołtan Institute for Nuclear Studies, Warsaw, Poland

Z. Wlodarczyk
Institute of Physics, Pedagogical University, Kielce, Poland

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Cumulant moments of negatively charged particles observed in hadron-nulceus collisions are analyzed by a leading particle cascade model. A modified negative binomial distribution (MNBD) or a negative binomial distribution (NBD) is used for multiplicity distribution from each participant hadron. If multiplicity distributions are truncated, both calculated results with the MNBD and the NBD can explain the oscillation of cumulant moments obtained from the data. [S0556-2813(98)01409-5]

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I. INTRODUCTION

From a reduction of a generating functional based on the QCD, it has been pointed out that the cumulant moment of the multiplicity distribution oscillates irregularly as the rank of the moment increases [1]. Analyses of the cumulant moments in hadron-hadron (hh) and e^+e^- collisions show that the jth rank normalized cumulant moment of observed negatively charged multiplicity distributions oscillates irregularly around the zero with increasing the rank j [2]. The minimum points are around $j \simeq 5$. However the gross feature of the oscillation of the cumulant obtained from the experimental data are not described by the QCD at present.

It is known that multiplicity distributions in hh and e^+e^- collisions are well described by the solutions of stochastic processes [3], for example, by the negative binomial distribution (NBD) or by the modified negative binomial distribution (MNBD). The NBD is derived from the birth process with immigration under the condition that no particles exist at the initial stage [4]. The MNBD is derived from the pure birth process under the initial condition of the binomial distribution [4,5].

The NBD and the MNBD differ distinctively in the following sense: for the untruncated multiplicity distributions, the jth rank normalized cumulant moment of the NBD is positive and decrease with increasing the rank j, whereas that of the MNBD can oscillate according to the choice of parameters. Therefore the behavior of the cumulant moments obtained from the experimental data puts a new constraint on the model of multiplicity distributions.

The cumulant moments of negatively charged particles in e^+e^- collisions are analyzed by the NBD and the MNBD in Ref. [6]. The cumulant moment calculated from the truncated MNBD oscillates, and can explain the gross feature of the oscillatory behavior of the data in e^+e^- collisions. However, the oscillation of the cumulant moment calculated from NBD is much weaker than that from the MNBD, and cannot explain the behavior of the data. On the other hand, in hh collisions, calculated cumulant moments both from truncated MNBD and NBD well describe the behavior of the data [7]. These results indicate that truncation of multiplicity distributions are important to describe the observed behavior of the experimental data [6–8].

In the present paper, we would like to extend previous analyses to the case of hadron-nucleus (hA) collisions. It is known that the cumulant moment obtained from the experimental data in hA collisions also oscillates, and the magnitude of it is comparable to that observed in hh collisions [9]. In hA collisions, there will be an additional condition compared with those in hh collisions that an incident hadron can collide inelastically with nucleons more than once inside a target nucleus. In order to estimate the effect of multiple collisions of the incident hadron, cumulant moments in hA collisions are investigated with a one-dimensional leading particle cascade model. The details of the model are explained in the next section.

In general, the generating function $\Pi(z)$ of the multiplicity distribution $P(n)$ is defined by

$$\Pi(z) = \sum_{n=0}^{\infty} P(n) z^n. \tag{1}$$

Then, the multiplicity distribution is given from Eq. (1) as

$$P(0) = \Pi(0),$$

$$P(n) = \frac{1}{n!} \frac{\partial^n \Pi(z)}{\partial z^n}\bigg|_{z=0}, \quad n = 1,2,\ldots\ . \tag{2}$$

The jth rank factorial moment f_j of the multiplicity distribution is given by

$$f_j=\langle n(n-1)\cdots(n-j+1)\rangle=\left.\frac{\partial^j\Pi(z)}{\partial z^j}\right|_{z=1}, \quad j=1,2,\ldots. \tag{3}$$

The cumulant moment is given by

$$\kappa_j=\left.\frac{\partial^j H(z)}{\partial z^j}\right|_{z=1}, \tag{4}$$

where

$$H(z)=\ln \Pi(z). \tag{5}$$

From Eqs. (3)–(5), we have the relation between the normalized cumulant moment K_j and the normalized factorial moment F_j:

$$K_1=F_1=1,$$

$$K_j=F_j-\sum_{m=1}^{j-1} {}_{j-1}C_{m-1}\, F_{j-m}\, K_m, \quad j=2,3,\ldots, \tag{6}$$

where

$$K_j=\frac{\kappa_j}{\langle n\rangle^j}, \quad F_j=\frac{f_j}{\langle n\rangle^j}. \tag{7}$$

The H_j moment is defined as

$$H_j=K_j/F_j, \tag{8}$$

which should be used in the analysis of the experimental data.

II. A LEADING PARTICLE CASCADE MODEL

For particle production processes in hA collisions, the leading particle cascade model is taken. Main assumptions of the model are the following:

(i) The incident hadron can repeat inelastic collisions with nucleons inside the nucleus with constant inelastic cross section $\sigma_{\rm in}$. The nucleons which participate inelastic collisions and the incident hadron is called participant hadrons. If the incident hadron collides inelastically ν times inside the nucleus, the number of participant hadrons are $\nu+1$.

(ii) The secondary particles are emitted only from participant hadrons, and do not interact inelastically (or do not hadronize) inside the nucleus.

(iii) The multiplicity distribution from each participant hadron is assumed to be the same and the generating function of it is written as $G_0(z)$.

From the assumption (i), the number distribution of inelastic collisions of the incident particle inside the nucleus at impact parameter $\mathbf{b}$ is given by the binomial distribution. Then the number distribution of inelastic collisions of the incident particle is given by

$$p(k)=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}\ {}_AC_k\ \delta_b^k(1-\delta_b)^{A-k}, \quad k=1,2,\ldots,A,$$

$$\sigma_{\rm abs}=\int d^2\mathbf{b}[1-(1-\delta_b)^A],$$

$$\delta_b=\sigma_{\rm in}T(\mathbf{b}), \tag{9}$$

where A is the atomic mass number of the target nucleus, $\sigma_{\rm abs}$ is the absorption cross section of hA collisions. In Eq. (9), $T(\mathbf{b})$ is a nuclear thickness at impact parameter $\mathbf{b}$, and it is normalized as

$$\int d^2\mathbf{b}\ T(\mathbf{b})=1.$$

The generating function of Eq. (9) is given by

$$\pi(u)=\sum_{k=1}^{A} p(k)\ u^k=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}[\{1+\delta_b(u-1)\}^A -(1-\delta_b)^A]. \tag{10}$$

The kth moment of inelastic collisions of the incident particle inside the nucleus is given from Eq. (10) as

$$\langle \nu(\nu-1)\cdots(\nu-k+1)\rangle =\left.\frac{\partial^k\pi(u)}{\partial u^k}\right|_{u=1}=\frac{A(A-1)\cdots(A-k+1)}{\sigma_{\rm abs}}\int d^2\mathbf{b}\ [\delta_b]^k.$$

Then the average number $\langle\nu\rangle$ of inelastic collisions of the incident hadron is written as

$$\langle\nu\rangle=\frac{A\ \sigma_{\rm in}}{\sigma_{\rm abs}}.$$

As the generating function of the secondary particles emitted from each participant hadron is given by $G_0(z)$, the generating function $\Pi(z)$ of the multiplicity distribution in the final states is given by the following equation:

$$\Pi(z)=\pi[G_0(z)]\ G_0(z)=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}(\{1+\delta_b[G_0(z)-1]\}^A -(1-\delta_b)^A)G_0(z).$$

It is convenient to introduce the generating function $\Pi_b(z)$ at impact parameter $\mathbf{b}$,

$$\Pi_b(z)=(\{1+\delta_b[G_0(z)-1]\}^A-(1-\delta_b)^A)G_0(z),$$

$$\Pi(z)=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}\ \Pi_b(z). \tag{11}$$

Then we have

$$[(1-\delta_b)G_0(z)+\delta_b G(z)]\frac{\partial^k \Pi_b(z)}{\partial z^k}$$

$$=\frac{A\delta_b}{2}\sum_{j=1}^{k}\frac{1}{k}\left(j-\frac{2k-3j}{A}\right)\ {}_kC_j\frac{d^jG(z)}{dz^j}\frac{d^{k-j}\Pi_b(z)}{dz^{k-j}}$$

$$+(1-\delta_b)\sum_{j=1}^{k}\frac{2j-k}{k}\ {}_kC_j\frac{d^jG_0(z)}{dz^j}\frac{d^{k-j}\Pi_b(z)}{dz^{k-j}}$$

$$+A\delta_b(1-\delta_b)^A\sum_{j=1}^{k}\frac{j}{k}\ {}_kC_j\frac{d^jG_0(z)}{dz^j}\frac{d^{k-j}G(z)}{dz^{k-j}},$$

$$k=1,2,\ldots, \quad (12)$$

where

$$G(z)=[G_0(z)]^2.$$

It should be noted that as $G_0(z)$ denotes the generating function for one participant, $G(z)$ is that for two participants, and corresponds to that in hadron-nucleon (hN) collisions. The jth rank factorial moment $f_j^{(0)}$ of the multiplicity distribution from one participant hadron is given from $G_0(z)$ as

$$f_j^{(0)}=\langle n_0(n_0-1)\cdots(n_0-j+1)\rangle=\left.\frac{\partial^jG_0(z)}{\partial z^j}\right|_{z=1}.$$

The jth rank factorial moment $f_j^{(1)}$ is given from $G(z)$ as

$$f_j^{(1)}=\langle n_1(n_1-1)\cdots(n_1-j+1)\rangle=\left.\frac{\partial^j\ G(z)}{\partial z^j}\right|_{z=1}.$$

The normalized factorial moments are defined by

$$F_j^{(0)}=\frac{f_j^{(0)}}{\langle n_0\rangle^j},\quad F_j^{(1)}=\frac{f_j^{(1)}}{\langle n_1\rangle^j}.$$

From the relation $G(z)=G_0(z)^2$, we have

$$\langle n_1\rangle=2\langle n_0\rangle,\quad F_2^{(1)}=\frac{1}{2}(F_2^{(0)}+1).$$

From Eqs. (3), (7), (11), and (12), the following relations are obtained:

$$\langle n\rangle=\langle n_0\rangle(\langle\nu\rangle+1)=\frac{\langle n_1\rangle}{2}(\langle\nu\rangle+1),$$

$$F_2=\frac{\langle\nu(\nu-1)\rangle+2\langle\nu\rangle}{(\langle\nu\rangle+1)^2}+\frac{1}{\langle\nu\rangle+1}F_2^{(0)}, \quad (13)$$

where $\langle n\rangle$ is the average multiplicity in the final states. The first equation in Eq. (12) roughly satisfy the relation between $\langle\nu\rangle$ and $\langle n\rangle$ obtained from the experiments [10,12]. In addition, The normalized factorial moment F_k satisfies the following relation:

$$F_k=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}\ F_k(\mathbf{b}),$$

$$F_0(\mathbf{b})=1-(1-\delta_b)^A,$$

$$F_k(\mathbf{b})=\frac{A\delta_b}{2}\sum_{j=1}^{k}\frac{1}{k}\left(j-\frac{2k-3j}{A}\right)\ {}_kC_j\left(\frac{2}{\langle\nu\rangle+1}\right)^jF_j^{(1)}F_{k-j}(\mathbf{b})$$

$$+(1-\delta_b)\sum_{j=1}^{k}\frac{2j-k}{k}\ {}_kC_j\left(\frac{1}{\langle\nu\rangle+1}\right)^jF_j^{(0)}F_{k-j}(\mathbf{b})$$

$$+A\delta_b(1-\delta_b)^A\left(\frac{2}{\langle\nu\rangle+1}\right)^k$$

$$\times\sum_{j=1}^{k}\frac{j}{2^jk}\ {}_kC_jF_j^{(0)}F_{k-j}^{(1)},\quad k=1,2,\ldots. \quad (14)$$

The multiplicity distribution $p_0(n)$ from each participant hadron is given by

$$p_0(0)=G_0(0),$$

$$p_0(n)=\frac{1}{n!}\left.\frac{\partial^nG_0(z)}{\partial z^n}\right|_{z=0},\quad n=1,2,\ldots.$$

The multiplicity distribution $p_1(n)$ is defined from $G(z)$ as

$$p_1(0)=G(0),$$

$$p_1(n)=\frac{1}{n!}\left.\frac{\partial^nG(z)}{\partial z^n}\right|_{z=0},\quad n=1,2,\ldots.$$

Then from Eqs. (2), (11), and (12), the multiplicity distribution in the final states is given by

$$P(n)=\frac{1}{\sigma_{\rm abs}}\int d^2\mathbf{b}\ P_b(n),$$

$$P_b(0)=(\{1-\delta_b[1-p_1(0)]\}^A-(1-\delta_b)^A)\ p_0(0),$$

$$P_b(k)=\frac{A\delta_b}{2}\sum_{j=1}^{k}\frac{1}{k}\left(j-\frac{2k-3j}{A}\right)$$

$$\times\ {}_kC_j\frac{p_1(j)\ P_b(k-j)}{(1-\delta_b)p_0(0)+\delta_bp_1(0)}$$

$$+(1-\delta_b)\sum_{j=1}^{k}\frac{2j-k}{k}$$

$$\times\ {}_kC_j\frac{p_0(j)\ P_b(k-j)}{(1-\delta_b)p_0(0)+\delta_bp_1(0)}+A\delta_b(1-\delta_b)^A$$

$$\times\sum_{j=1}^{k}\frac{j}{k}\ {}_kC_j\frac{p_0(j)\ p_1(k-j)}{(1-\delta_b)p_0(0)+\delta_b\ p_1(0)},$$

$$k=1,2,\ldots, \quad (15)$$

where the multiplicity distribution at impact parameter $\mathbf{b}$ is defined by

$$P_b(n)=\frac{1}{n!}\left.\frac{\partial^n \Pi_b(z)}{\partial z^n}\right|_{z=0}.$$

III. MULTIPLICITY DISTRIBUTION FROM ONE PARTICIPANT

For the multiplicity distribution from each participant hadron, two cases are considered: one is that the distribution is given by the modified negative binomial distribution (MNBD), and the second is that it is given by the negative binomial distribution (NBD).

A. The MNBD

The MNBD is derived from the pure birth process with the binomial distribution as the initial condition [4]. The generating function of the MNBD is written as

$$G_0(z)=\left(\frac{1-r_1(z-1)}{1-r_2(z-1)}\right)^N, \tag{16}$$

where N is an integer, r_1 is real and $r_2>0$ [4,5].

The MNBD is obtained from $G_0(z)$ as

$$p_0(0)=G_0(0)=\left(\frac{1+r_1}{1+r_2}\right)^N,$$

$$\begin{aligned} p_0(n)&=\frac{1}{n!}\left.\frac{\partial^n G_0(z)}{\partial z^n}\right|_{z=0} \\ &=N\left(\frac{r_1}{1+r_2}\right)^n\sum_{j=1}^{\min(n,N)}\frac{(N+n-j-1)!}{j!\,(n-j)!\,(N-j)!} \\ &\quad\times\left(\frac{-r_1}{r_2}\right)^j\left(\frac{1+r_1}{1+r_2}\right)^{N-j}, \quad n=1,2,\ldots. \end{aligned} \tag{17}$$

From Eq. (16), we have the kth rank factorial moment

$$\begin{aligned} f_k^{(0)}&=\left.\frac{\partial^k G_0(z)}{\partial z^k}\right|_{z=1} \\ &=N{r_2}^k\sum_{j=1}^{\min(k,N)}{}_kC_j\frac{(N+k-j-1)!}{(N-j)!}\left(\frac{-r_1}{r_2}\right)^j. \end{aligned} \tag{18}$$

The first and the second factorial moments are given from Eq. (18) by

$$f_1^{(0)}=\langle n_0\rangle=N(r_2-r_1),$$

$$f_2^{(0)}=\langle n_0(n_0-1)\rangle=\langle n_0\rangle^2+\langle n_0\rangle(r_1+r_2).$$

Then, the parameters r_1 and r_2 are expressed as

$$r_1=\frac{1}{2}\left(F_2^{(0)}-1-\frac{1}{N}\right)\langle n_0\rangle,$$

$$r_2=\frac{1}{2}\left(F_2^{(0)}-1+\frac{1}{N}\right)\langle n_0\rangle. \tag{19}$$

The generating function $G(z)$ is given as

$$G(z)=G_0(z)^2=\left(\frac{1-r_1(z-1)}{1-r_2(z-1)}\right)^{2N}.$$

The factorial moment $f_k^{(1)}$ and the multiplicity distribution $p_1(n)$ are given from Eqs. (17) and (18), if N is replaced by $2N$.

B. The NBD

The NBD is given from the birth process with the immigration with no particles in the initial stage [4]. The generating function of the NBD is given by

$$G_0(z)=\left[1-\frac{\langle n_0\rangle}{\lambda}(z-1)\right]^{-\lambda}. \tag{20}$$

The NBD is obtained from Eq. (20) as

$$p_0(0)=\left[1+\frac{\langle n_0\rangle}{\lambda}\right]^{-\lambda},$$

$$p_0(n)=\frac{\Gamma(\lambda+n)}{\Gamma(\lambda)\Gamma(n)}\left(\frac{\langle n_0\rangle}{\lambda}\right)^n\left(1+\frac{\langle n_0\rangle}{\lambda}\right)^{-n-\lambda}. \tag{21}$$

The kth rank factorial moment is given by

$$f_k^{(0)}=\frac{\Gamma(\lambda+k)}{\Gamma(\lambda)}\left(\frac{\langle n_0\rangle}{\lambda}\right)^k. \tag{22}$$

The normalized second factorial moments are given from Eq. (22) by

$$F_2^{(0)}=1+\frac{1}{\lambda}. \tag{23}$$

The generating function $G(z)$ is given as

$$G(z)=G_0(z)^2=\left[1-\frac{\langle n_1\rangle}{2\lambda}(z-1)\right]^{-2\lambda}.$$

The factorial moment $f_k^{(1)}$ and the multiplicity distribution $p_1(n)$ are given from Eqs. (21) and (22), if $\langle n_0\rangle$ is replaced by $\langle n_1\rangle$ and λ by 2λ.

IV. ANALYSIS OF EXPERIMENTAL DATA

The cumulant moments obtained from multiplicity distributions of observed negatively charged particles in hA collisions [11,12] are analyzed by the leading particle cascade model in this section. Observed values of $\langle n\rangle$ and F_2 are shown in Table I.

TABLE I. The observed values of $\langle n\rangle$, F_2, and $n_{\max}$ in hA collisions [11,12].

	$E_{\rm in}$ GeV/c	$\langle n\rangle$	F_2	$n_{\max}$
K^+Al	250	5.00	1.12	16
K^+Au	250	6.68	1.22	22
p Ar	200	5.39	1.21	17
p Xe	200	6.84	1.27	24

In order to calculate the H_j moments by the cascade

TABLE II. The parameters used in the calculations of the cumulant moments in hA collisions. $N=4$ is used for the MNBD, and $\langle n_0\rangle$ and $F_2^{(0)}$ are common with the MNBD and the NBD.

	A	$\langle n_0\rangle$	$F_2^{(0)}$
K^+Al	27	1.95	1.079
K^+Au	197	1.93	1.251
p Ar	40	1.62	1.297
p Xe	131	1.55	1.429

model, two parameters, the inelastic hN cross section $\sigma_{\rm in}$ and the nuclear thickness $T(\mathbf{b})$ should be fixed, in addition to the parameters in the multiplicity distribution from one participant hadron. For the inelastic cross section, we use $\sigma_{\rm in}=18.0$ mb for kaon-nucleon collisions, and $\sigma_{\rm in}=32.0$ mb for proton-nucleon collisions.

In our calculations, a somewhat simplified picture of a nucleus is adopted: It has a uniform density and is approximated by a sphere with radius R. Then the nuclear density is given by

$$\rho=\frac{3}{4\pi R^3},$$

and the nuclear thickness at impact parameter $\mathbf{b}$ is

$$T(\mathbf{b})=2\rho\sqrt{R^2-\mathbf{b}^2}.$$

In order to integrate over impact parameter, variable is changed as

$$x=\sqrt{1-(\mathbf{b}/R)^2},$$

and the Gaussian integral formula with eight sample points is used.

The nuclear radius R is parametrized as

$$R=1.23A^{1/3}+1.23A^{-1/3},$$

to reproduce the absorption cross section $\sigma_{\rm abs}$ in hA collisions [10]. The mass numbers of the target nuclei used in the calculations are shown in Table II.

At first, the H_j moments are calculated from the factorial moments derived from the generating function without truncation of multiplicities, in other words from Eqs. (6), (8), and (14). In Fig. 1, the calculated H_j moments are shown for p Ar and p Xe collisions. Parameters used in our calculations are adjusted to reproduce the observed values of $\langle n\rangle$ and F_2 listed in Table I; $\langle n_0\rangle=1.596$ and $F_2^{(0)}=1.185$ for p Ar collisions and $\langle n_0\rangle=1.535$ and $F_2^{(0)}=1.380$ for p Xe collisions. The values of $\langle n_0\rangle$ and $F_2^{(0)}$ are common for the MNBD and the NBD. In addition $N=4$ is used for the MNBD.

Results for p Ar collisions, obtained from both the MNBD and the NBD, oscillate with almost the same strength. However, those oscillations are smaller by almost two orders of magnitude from those seen in data (cf. Fig. 3). For example, relative minimum by the NBD within $0\leqslant j\leqslant 15$ is $H_8=-3.83\times 10^{-4}$.

In the case of p Xe collisions, the H_j moments calculated by the MNBD and the NMD almost coincide, therefore only the result by the NBD is shown in Fig. 1. The oscillation becomes much weaker than that in p Ar collisions. In this case the relative minimum is now equal to $H_8=7.66\times 10^{-5}$.

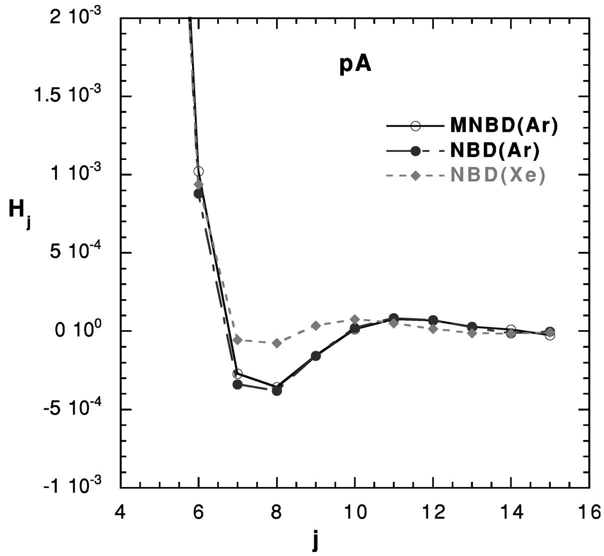

FIG. 1. The H_j moments calclulated from Eqs. (6), (8), and (14) for p Ar and p Xe collisions without truncation of multiplicities. Open circles are calculated results by the MNBD for p Ar collisions. Full circles and full diamonds are by the NBD for p Ar and p Xe collisions, respectively.

One can see from our calculations that as the average collision number $\langle\nu\rangle$ increases from 2.38 for p Ar collisions to 3.46 for p Xe collisions, the oscillation of the H_j moments become much weaker.

The calculated H_j moments for K^+Al ($\langle\nu\rangle=1.57$) and K^+Au ($\langle\nu\rangle=2.50$) collisions show similar behavior as those for p Ar and p Xe collisions. However, the difference between two results in KA collisions is not so significant.

It should be noted that although the H_j moment calculated directly from the NBD does not show oscillatory behavior without truncation of multiplicity, that from our leading particle cascade model does show. This effect will come from the finiteness of the collision number of the incident hadron.

Second, the H_j moments are calculated by the use of truncated multiplicity distributions. The multiplicity distributions used in the analysis are normalized as

$$\sum_{n=0}^{n_{\rm max}} p_0(n)=1,\quad \sum_{n=0}^{n_{\rm max}} p_1(n)=1,\quad \sum_{n=0}^{n_{\rm max}} P(n)=1,$$

where $n_{\rm max}$ is the maximum multiplicity of observed negatively charged particles. The factorial moments of negatively charged particles in the final states are calculated by the use of Eq. (15) as

$$f_j=\sum_{n=1}^{n_{\rm max}} n(n-1)\cdots(n-j+1)\ P(n),\quad j=1,2,\ldots. \tag{24}$$

Then the H_j moments of negatively charged particles are calculated from Eqs. (6), (7), and (8).

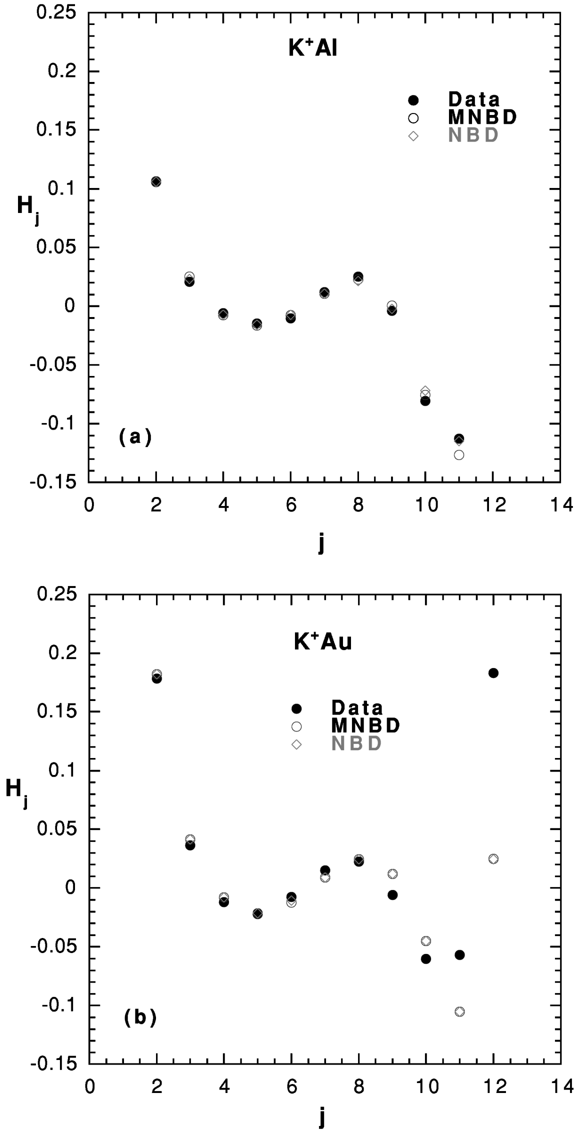

FIG. 2. The H_j moments of negatively charged particles in (a) K^+Al collisions and (b) K^+Au collisions. Full circles are calculated from the data in Ref. [11]. Open circles and open diamonds are our calculations by the MNBD and by the NBD, respectively. Calculations are done from Eqs. (15) and (24) with truncated multiplicity distributions.

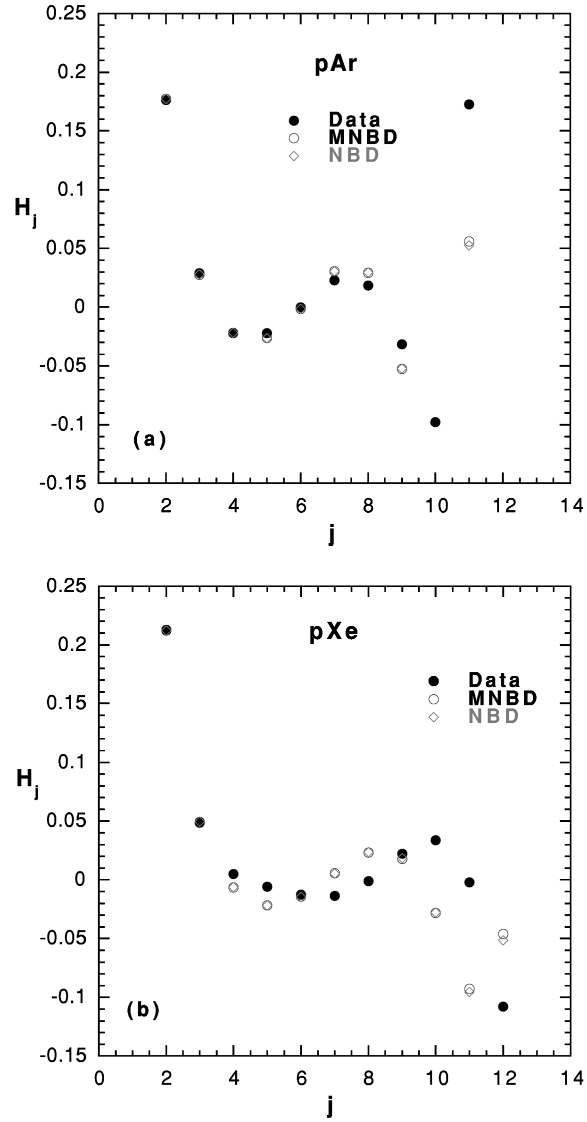

FIG. 3. The H_j moments of negatively charged particles in (a) p Ar collisions and (b) p Xe collisions. Full circles are calculated from the data in Ref. [12]. Open circles and open diamonds are our calculations by the MNBD and by the NBD, respectively. Calculations are done from Eqs. (15) and (24) with truncated multiplicity distributions.

The observed values of $\langle n \rangle$, F_2, and $n_{\max}$ in the experiments are listed in Table I, and parameters used in our calculations are shown in Table II. The values of $\langle n_0 \rangle$ and $F_2^{(0)}$ are common with the MNBD and the NBD, and in the case of the MNBD we use $N=4$. By these input parameters, the values of $\langle n \rangle$ and F_2 listed in Table I are well reproduced.

The H_j moments calculated from the MNBD and the NBD are compared with those of observed negatively charged particles for K^+Al and K^+Au collisions [11], respectively, in Figs. 2(a) and 2(b). Those for p Ar and p Xe collisions are presented in Figs. 3(a) and 3(b).

As can be seen from the Figs. 2 and 3, both calculated results almost coincide with each other, and well reproduce the gross feature of the oscillations obtained from the experimental data. If $N=3$ or $N=5$ and same values of $\langle n_0 \rangle$ and $F_2^{(0)}$ with those in Table I are used for the MNBD, almost the same results with $N=4$ are obtained.

From the analysis of K^+Al and K^+Au collisions, the relation between $\langle \nu \rangle$ and the strength of oscillation in H_j moments is not clear. However, from the analysis of pA collisions, as the average collision number $\langle \nu \rangle$ increases, the oscillations of H_j moments obtained both from the data and our calculations become weaker.

To clarify this effect, we calculate the H_j moments for KA and pA collisions using $\langle n_0 \rangle = 5.00$ and $F_2^{(0)} = 1.20$. The maximum multiplicity is determined by the formula $n_{\max} = 4\langle n_0 \rangle (\langle \nu \rangle + 1)$. The calculated results by the NBD are almost the same with those by the MNBD with $N=4$ for any process. The calculated results show that the oscillation of H_j moments for KA collisions weakly decreases as the average collision number $\langle \nu \rangle$ increases from 1.57 for K^+Al collisions to 2.50 for K^+ Au collisions.

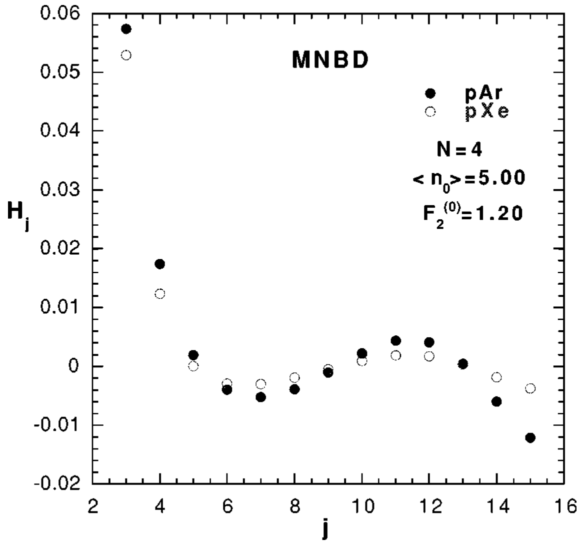

FIG. 4. The H_j moments of negatively charged particles calculated by the truncated MNBD. Full circles are for p Ar collisions, and open circles are for p Xe collisions.

For p Ar and p Xe collisions, the calculated results for the MNBD with $N=4$ are shown in Fig. 4. The corresponding results for the NBD are almost the same with those for the MNBD, and therefore those are not plotted. It can be seen as the mass number of target nucleus increases, the oscillation of H_j moment becomes weaker. For comparison, we calculate H_j moments for p Be collisions corresponding to $A=9$ $(\langle \nu \rangle = 1.54)$ with $\langle n_0 \rangle = 5.00$ and $F_2^{(0)} = 1.20$. The results are almost the same with those for p Ar collisions.

V. SUMMARY AND DISCUSSIONS

The H_j moments of multiplicity distributions of observed negatively charged particles in hA collisions are analyzed by the leading particle cascade model. The incident particle is assumed to collide with a nucleon inside the nucleus with the constant inelastic cross section, and the number distribution of inelastic collisions at fixed impact parameter is given by the binomial distribution. We use the MNBD or the NBD for multiplicity distribution from each participant hadron. Our model roughly reproduces the relation between the collision number $\langle \nu \rangle$ and $\langle n \rangle$ observed by experiments [10,12].

In order to compare our calculations with the experimental data, parameters are adjusted to reproduce the observed values of $\langle n \rangle$ and F_2. Calculated results without truncation of multiplicity for both the MNBD and the NBD oscillate. However, the magnitude of oscillation is much weaker than that of the experimental data. Moreover, as the mass number A of the target nucleus, or the average collision number $\langle \nu \rangle$ of the incident hadron increases, the magnitude of oscillations become weaker.

If multiplicity distributions are truncated, both calculated H_j moments by the MNBD and by the NBD well describe the experimental data. For pA collisions, the oscillation of H_j moments decrease clearly as $\langle \nu \rangle$ increases.

We also calculate the H_j moment with $\langle n_0 \rangle = 5$ and $F_2^{(0)} = 1.20$ for K^+Al, K^+Au, p Ar, and p Xe collisions. The oscillation of H_j moment becomes weaker as the mass number of target nucleus increases. This effect appears much clearer for $\langle \nu \rangle \geqslant 2$. Therefore from our analysis, it can be said that the oscillation of the H_j moment in hA collisions decreases as the collision number $\langle \nu \rangle$ increases.

If we directly apply the MNBD or the NBD to the analysis of the H_j moments in hA collisions, calculated moments with truncated multiplicity distribution, the MNBD or the NBD, well describe the behaviors of H_j moments obtained from the data, as in hh collisions.

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