Continuum quasiparticle random-phase approximation description of isovector $E1$ giant resonances

S. Kamerdzhiev,[1,2] R. J. Liotta,[1] E. Litvinova,[3] and V. Tselyaev[1,4]
[1]*Royal Institute of Technology, S-10405 Stockholm, Sweden*
[2]*Institute of Physics and Power Engineering, 249020 Obninsk, Russia*
[3]*Institute of Nuclear Power Engineering, 249020 Obninsk, Russia*
[4]*Institute of Physics, St. Petersburg State University, 198904 St. Petersburg, Russia*
(Received 14 October 1997)

Isovector $E1$ giant resonances in nonmagic nuclei are calculated by using a method which takes into account the single-particle continuum in the framework of a continuum quasiparticle random-phase approximation. A special procedure is formulated which gives the spurious dipole state at zero energy. The calculations are performed in a stable nonmagic tin isotope as well as unstable magic and nonmagic tin isotopes. The role of the single-particle continuum and the spurious dipole state is discussed. It turned out that the first ingredient has a noticeable influence on the resonance form. [S0556-2813(98)00607-4]

PACS number(s): 24.30.Cz, 21.60.Jz, 27.60.+j

I. INTRODUCTION

Giant resonances are excitations embedded in the continuum part of nuclear spectra and, therefore, one would expect that the continuum has to be taken into account explicitly in their description. However, the gross properties of these resonances can be studied suitably by means of bound (e.g., harmonic oscillator) representations [1]. This can be understood, since the particle in the particle-hole excitations moves in high angular momentum orbits. Therefore, the corresponding centrifugal and, for protons, Coulomb barriers tend to trap the system within the nuclear volume, hindering the particle decay of the giant resonance. The effective potential thus ''felt'' by the particle is well approximated by a harmonic-oscillator potential [2]. But bound representations are not suited to describe quantities, like decay widths, which are closely linked to the continuum. In cases like this, one has to include in the formalism the degrees of freedom that cause the particle decay of the giant resonance. The inclusion of the continuum in particle-hole (ph) excitations is usually achieved by using the continuum random-phase approximation (CRPA) [3]. This formalism was recently used to evaluate the poles and residues of the S matrix, that is, the position and total decay width as well as the partial decay widths corresponding to giant resonances [4]. More to the point of the present paper, in Refs. [5,6] the CRPA ph excitations were used as a basis to describe more complex states, namely, the $1p1h\otimes$phonon excitations that induce the spreading width of the giant resonance.

These investigations established the importance of treating the continuum properly in order to evaluate quantities related to the decay of the resonance. Thus, in Ref. [5], it was found that, for $E1$ giant resonances, the contribution of the continuum to the total width, including the spreading width, is large. For instance, the continuum contributes with 14% of the total width in ^{40}Ca, with 28% in ^{48}Ca and 7% in ^{208}Pb.

All these calculations were performed in normal nuclei. The importance of the continuum in such nuclei indicates that a similar feature may be present in superfluid nuclei and, therefore, a proper calculation, including both BCS and continuum degress of freedom is necessary. That is, one has to include from the outset a continuum quasiparticle RPA (QRPA) treatment of the ph degrees of freedom.

In Ref. [7], such an approach was developed based on the projection operator method using a separable interaction. A more elaborated approach, but for charge-exchange excitations, was presented in Ref. [8]. It uses the self-consistent density-functional method, and takes into account the RPA continuum exactly in both particle-hole and particle-particle channels. To overcome difficulties related to basis truncations, which is necessary in superfluid nuclei, a method similar to the so-called (r,λ) representation [9] was used in Ref. [8].

In this paper we present a continuum QRPA approach to ph excitations, based on the Green-function method, for neutral excitations. Our approach is similar to that of Ref. [8] in the sense that we take into account the RPA continuum exactly, and use the (r,λ) representation in the ''pairing'' part of our propagator. We use the standard Migdal theory of finite Fermi systems (TFFS's) [10], which in fact corresponds to the QRPA with a Landau-Migdal-type interaction. We apply a special procedure to obtain the spurious dipole state at zero energy. The calculations are performed for the isovector $E1$ resonances in stable and unstable isotopes of Sn.

In Sec. II, we present the outline of our approach. The description of our ''forced consistency'' procedure is given in Sec. III. In Sec. IV, the most important details of the calculations, e.g., the choice and fitting of single-particle levels in non-normal nuclei and the parameters of the Landau-Migdal interaction, are discussed. The results of the calculations are in Sec. V, and the conclusions in Sec. VI.

II. THEORY

In this section we give the main formulas of the TFFS's [10], as well as a generalization of this formalism which allows one to include the single-particle continuum in superfluid nuclei. We will consider neutral excitations only, but

the generalization to non-neutral channels is straightforward.

The cross section for electric multipole photoabsorption is given by [1]

$$\sigma(E)=\frac{8\pi^3(L+1)e^2}{L[(2L+1)!!]^2}\left(\frac{E}{\hbar c}\right)^{2L-1}S(E). \tag{2.1}$$

Under the influence of an external field carrying energy ω, the density matrix ρ changes by $\delta\rho$. The corresponding strength function S is

$$S(E)=-\frac{2L+1}{\pi}\mathrm{Im}\sum_{S,\tau}\delta_{S0}e_\tau\int_0^\infty dr\ r^{J+2}\delta\rho_{S,\tau}(r,\omega), \tag{2.2}$$

where $\omega=E+\imath\eta$ with $\eta\to+0$, L is the angular momentum of the electric excitation, τ and S are the isotopic ($\tau=n,p$) and spin ($S=0,1$) indexes, e_τ is the local dimensionless nucleon charge, and $J=L$ for $L\geqslant 1$.

The corresponding system of TFFS equations can be solved conveniently in coordinate representation. After the separation of angular variables, this system has the form

$$\begin{aligned}\delta\rho_{S,\tau}(r_1,\omega)=&\,\delta\rho^{(0)}_{S,\tau}(r_1,\omega)\\&-\sum_{S',\tau'}\int_0^\infty dr_2 r_2^2 A^{(\tau)L}_{LS,LS'}(r_1,r_2;\omega)\\&\times\mathcal{F}^{\tau\tau'}_{S'}(r_2)\delta\rho_{S',\tau'}(r_2,\omega)\end{aligned} \tag{2.3}$$

where

$$\delta\rho^{(0)}_{S,\tau}(r_1,\omega)=-e_\tau\int_0^\infty dr_2 r_2^{J+2}A^{(\tau)L}_{LS,L0}(r_1,r_2;\omega), \tag{2.4}$$

and $\mathcal{F}$ is the amplitude of the effective particle-hole interaction parametrized according to the TFFS, i.e.,

$$\begin{aligned}\mathcal{F}^{\tau\tau'}_S(r)=&\,\delta_{\tau\tau'}C_0[\delta_{S,0}(f+f')+\delta_{S,1}(g+g')]+(1-\delta_{\tau\tau'})\\&\times C_0[\delta_{S,0}(f-f')+\delta_{S,1}(g-g')],\end{aligned} \tag{2.5}$$

where the functions $f(r)$ and $f'(r)$ are determined by interpolation. For this the parameters f_{ex}, f_{in}, f'_{ex}, and f'_{in} are introduced such that, e.g., for $f(r)$, one obtains

$$f(r)=f_{\mathrm{ex}}+(f_{\mathrm{in}}-f_{\mathrm{ex}})\frac{\rho_0(r)}{\rho_0(0)}, \tag{2.6}$$

where $\rho_0(r)$ is the nuclear density in the ground state.

The propagator $A(r_1,r_2;\omega)$ in Eq. (2.3) contains both the single-particle continuum and the pairing contribution. In Ref. [3], difficulties related to truncations of the basis as well as the problem of treating the continuum properly in normal nuclei were overcome by using the Green-function method. However, this is not the case for superfluid nuclei where there is a contribution of the particle-particle channel from static pairing fields. To take this into account, in Ref. [9] the so-called method of mixed (r,λ) representation was proposed and used for the first 2^+ levels in nonmagic nuclei. We will apply this method to giant resonances. In addition, we will also consider the spin component of the density matrix. The main idea is to single out the levels near the Fermi level, where the gap is not equal to zero, which contribute to the propagators. This part of the propagator is then evaluated in coordinate representation using standard techniques. The remaining part of the propagator, which contains the continuum, is calculated using the method of Refs. [3]. Thus, according to these ideas, the propagator $A^{(\tau)L}_{LS,LS'}$ has the form

$$A^{(\tau)L}_{LS,LS'}(r,r';\omega)=A^{\mathrm{cont}(\tau)L}_{LS,LS'}(r,r';\omega)+A^{\mathrm{disc}(\tau)L}_{LS,LS'}(r,r';\omega), \tag{2.7}$$

where A^{cont} is the part of the propagator containing the transitions from the ''paired'' levels to the continuum, i.e.,

$$\begin{aligned}&A^{\mathrm{cont}(\tau)L}_{LS,LS'}(r,r';\omega)\\&=-\sum_1\delta_{\tau\tau_1}v_1^2R_1(r)R_1(r')\sum_{l_2j_2}T^{LSS'}_{12}\\&\quad\times[G^\tau_{l_2j_2}(r,r';\mu_\tau-E_1+\omega)+(-1)^{S+S'}\\&\quad\times G^\tau_{l_2j_2}(r,r';\mu_\tau-E_1-\omega)],\end{aligned} \tag{2.8}$$

where v_1^2 is the quasiparticle occupation number, $R_1(r)$ the corresponding radial single-particle wave function, E_1 the quasiparticle energy, and μ_τ the chemical potential. The partial components G^τ_{lj} of the Green function without pairing are expressed in closed form in terms of the regular and irregular solutions of the single-particle Schrödinger equation [3]. Therefore, we take into account exactly the contribution of the single-particle continuum within this formalism.

In Eq. (2.7), A^{disc} is the ''pairing'' part of the propagator which includes the sum $\Sigma^{\mathrm{disc}}_{12}$ over discrete single-particle levels near the Fermi surface, where the gap is not zero, i.e.,

$$\begin{aligned}&A^{\mathrm{disc}(\tau)L}_{LS,LS'}(r,r';\omega)\\&=\sum_{12}^{\mathrm{disc}}\delta_{\tau\tau_1}\delta_{\tau\tau_2}R_1(r)R_2(r)R_1(r')R_2(r')T^{LSS'}_{21}\\&\quad\times\left[\frac{v_1^2(1-v_2^2)}{\omega+E_1+E_2}-\frac{(1-v_1^2)v_2^2}{\omega-E_1-E_2}\right.\\&\quad+(-1)^S\frac{\Delta_1\Delta_2}{4E_1E_2}\left(\frac{1}{\omega+E_1+E_2}-\frac{1}{\omega-E_1-E_2}\right)\\&\quad\left.+\frac{v_2^2}{\omega+\mu_\tau-\epsilon_1-E_2}-\frac{v_1^2}{\omega-\mu_\tau+\epsilon_2+E_1}\right].\end{aligned} \tag{2.9}$$

Here ϵ_1 are single-particle energies, the index $1\equiv(n_1,l_1,j_1)$ is the set of single-particle quantum numbers, and $T^{LSS'}_{12}$ is the product of the reduced matrix elements of the spherical tensor operator:

$$T^{LSS'}_{12}=\frac{1}{2L+1}\langle j_1l_1\|T_{LLS}\|j_2l_2\rangle\langle j_1l_1\|T_{LLS'}\|j_2l_2\rangle. \tag{2.10}$$

III. ''FORCED CONSISTENCY''

Our approach is not self-consistent in the sense that there is no connection between the mean field and the particle-hole interaction used. The parameters of this interaction in the TFFS have been determined reliably, except for the parameter f_{ex} which is usually fitted so as to obtain the spurious dipole state at zero energy. This phenomenological procedure is used very often, particularly when dealing with separable interactions. In this case it is the strength of the interaction which is fitted to obtain the spurious isoscalar dipolar state at zero energy [4]. However, one may doubt the reliability of this procedure, particularly regarding the exact depletion of the Thomas-Reiche-Kuhn sum rule. It is therefore important to investigate the possibility of obtaining the spurious dipole state at zero energy without requiring any fit of parameters. Here we will perform such a task following a rather simple procedure. That is, the condition that the spurious dipole state should be at zero energy means that the density matrix $\delta\rho_{S,\tau}(r,\omega)$ [Eq. (2.3)], must have a pole at that energy for $L=1$, i.e.,

$$\delta\rho_{S,\tau}(r,\omega)=\frac{\zeta_{S,\tau}(r)}{\omega-\omega_0}+\delta\rho^{\rm reg}_{S,\tau}(r,\omega), \tag{3.1}$$

where $\omega_0\to 0$, and the function $\delta\rho^{\rm reg}_{S,\tau}(r,\omega)$ is regular at $\omega=\omega_0$. This condition can be satisfied if one adds an additional term to the particle-hole effective interaction in Eq. (2.3). Thus, instead of the amplitude $\mathcal{F}_S^{\tau\tau'}$ in Eq. (2.3) one obtains the more general amplitude

$$\tilde{\mathcal{F}}_{S\tau,S'\tau'}(r,r')=\delta_{S,S'}\frac{\delta(r-r')}{rr'}\mathcal{F}_S^{\tau\tau'}(r)+\mathcal{F}^{\rm rest}_{S\tau,S'\tau'}(r,r'), \tag{3.2}$$

where $\mathcal{F}^{\rm rest}$ is a restoring amplitude which is undefined so far.

Substituting Eqs. (3.1) and (3.2) into Eq. (2.3), one finds that there should be a solution to the following homogeneous equation:

$$\zeta_{S\tau}(r_1)=-\sum_{S',S'',\tau'}\int_0^\infty dr_2 r_2^2\int_0^\infty dr_3 r_3^2 \times A^{(\tau)1}_{1S,1S''}(r_1,r_2;\omega_0)\tilde{\mathcal{F}}_{S''\tau,S'\tau'}(r_2,r_3)\zeta_{S'\tau'}(r_3). \tag{3.3}$$

There are several methods to satisfy the condition of existence of a solution of this equation using the different forms of the amplitude $\mathcal{F}^{\rm rest}$ in Eq. (3.2). An approximate method suitable for the QRPA was used in Ref. [11]. We choose the quantity $\mathcal{F}^{\rm rest}$ as follows:

$$\mathcal{F}^{\rm rest}_{S\tau,S'\tau'}(r,r')=\sum_{k=1}^{2}\kappa_k f_k^{S\tau}(r)f_k^{S'\tau'}(r'), \tag{3.4}$$

where the functions $f_2^{S\tau}(r)$ and $f_1^{S\tau}(r)$ are connected to each other by the expressions

$$f_2^{S\tau}(r)=\sum_{\tau'}\mathcal{F}_S^{\tau\tau'}(r)\xi_{S\tau'}(r), \tag{3.5}$$

$$\xi_{S\tau}(r)=\sum_{S'}\int_0^\infty dr' r'^2 A^{(\tau)1}_{1S,1S'}(r,r';\omega_0)f_1^{S'\tau}(r'), \tag{3.6}$$

and the constants κ_1 and κ_2 are determined by

$$\kappa_k=-\left(\sum_{S\tau}\int_0^\infty dr\ r^2 f_k^{S\tau}(r)\xi_{S\tau}(r)\right)^{-1}. \tag{3.7}$$

After substituting Eqs. (3.2) and (3.4)–(3.7) in Eq. (3.3), one readily finds that the function $\xi_{S\tau}(r)$ [Eq. (3.6)] satisfies Eq. (3.3). Moreover, this equation is satisfied for any arbitrary function $f_1^{S\tau}(r)$, any amplitude $\mathcal{F}_S^{\tau\tau'}(r)$, and any mean-field potential $U^\tau(r)$. These conditions are as general as those provided by a self-consistent treatment of the problem and, therefore, the alternative to self-consistency presented here is more realistic than the fitting of parameters mentioned above.

In our model, the amplitude $\mathcal{F}_S^{\tau\tau'}$ and the potential U have been already fixed. But we still have the function $f_1^{S\tau}(r)$, which so far has not been defined. We determine this function by requiring that

$$f_1^{S\tau}(r)=\delta_{S0}\frac{dU^\tau(r)}{dr}. \tag{3.8}$$

Thus we have defined the quantity $\mathcal{F}^{\rm rest}$ in such a way as to obtain the ''ghost'' energy to be exactly equal to zero without the usual procedure of fitting force parameters.

The choice of form (3.8) is very natural because if one fulfills the usual self-consistency condition, i.e., [10,12],

$$\frac{dU^\tau(r)}{dr}=-\sum_{\tau'}\mathcal{F}_0^{\tau\tau'}(r)\int_0^\infty dr' r'^2 A^{(\tau')1}_{10,10}(r,r';\omega_0)\frac{dU^{\tau'}(r')}{dr'} \tag{3.9}$$

then one obtains, from Eqs. (3.5)–(3.8),

$$f_2^{S\tau}(r)=-f_1^{S\tau}(r),\quad \kappa_1=-\kappa_2 \tag{3.10}$$

and then $\mathcal{F}^{\rm rest}_{S\tau,S'\tau'}(r,r')\equiv 0$.

Further, we should take care of the condition of having the ''ghost'' excitation probability be equal to zero. The equality $B_0(E1)=0$ is a consequence of general equations (see, for example, Ref. [19]), and must be fulfilled in the self-consistent model. In our calculations we made this by renormalizing the well depth parameters of the potentials $U^\tau(r)$ in Eq. (3.8) in order to obtain the spurious state probability $B_0(E1)$ equal to zero. This can be done easily because

$$B_0(E1)=3\sum_\tau e_\tau\int_0^\infty dr\ r^3\zeta_{0\tau}(r), \tag{3.11}$$

where $e_p=eN/A$ and $e_n=-eZ/A$. Then, choosing the potential-well depth parameters in Eq. (3.8) which ensure the equality

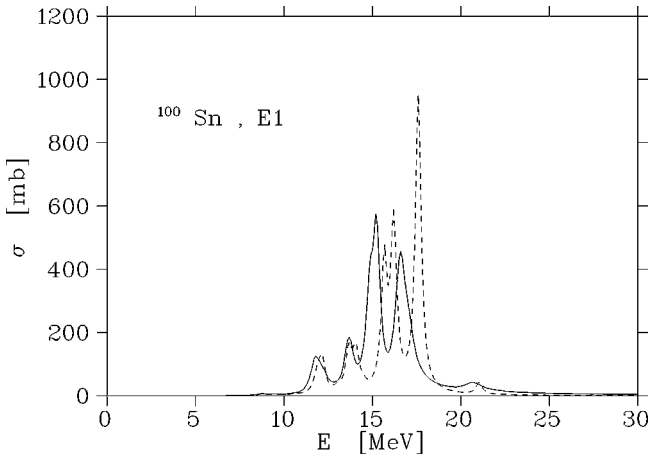

FIG. 1. The $E1$ photoabsorption cross section for the unstable ^{100}Sn nucleus obtained in the RPA with the single-particle continuum (solid line) and without the continuum (dashed line). The smearing parameter is equal to 200 keV.

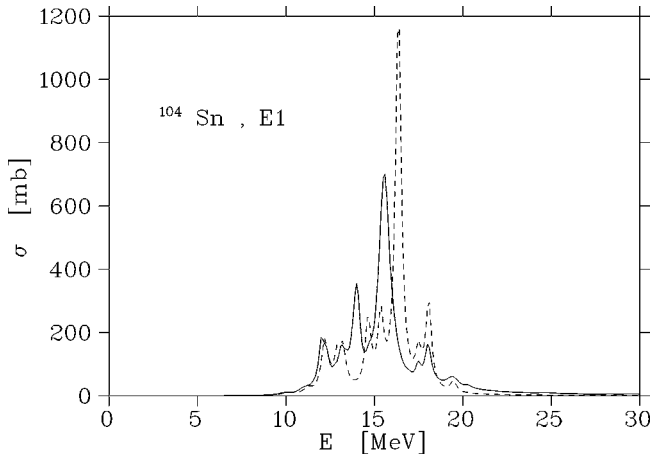

FIG. 2. Same as in Fig. 1, but for the unstable ^{104}Sn nucleus obtained in the QRPA with the continuum (solid line) and without it (dashed line).

$$Z\int_0^\infty dr\ r^3\xi_{0n}(r)=N\int_0^\infty dr\ r^3\xi_{0p}(r), \qquad (3.12)$$

we obtain $B_0(E1)=0$. Our method of ''forced consistency'' is suited for any propagator A, e.g., when more complex excitations than the QRPA are included since, by definition, the spurious dipole state will in any case exactly appears at zero energy.

IV. NUMERICAL DETAILS

In the applications to be presented in this section we will use the Landau-Migdal interaction with parameters as in Refs. [5,6,13], i.e., [see Eqs. (2.5) and (2.6)],

$$f_{\rm in}=-0.002,\quad f'_{\rm ex}=2.30,\quad f'_{\rm in}=0.76,\quad g=0.05,$$

$$g'=0.96,\quad C_0=300\ \text{MeV fm}^3. \qquad (4.1)$$

For the parameter $f_{\rm ex}$, we choose the value $f_{\rm ex}=-1.73$, taken from a previous calculation [13]. We calculate the ground-state nuclear density $\rho_0(r)$ in Eq. (2.6) by using our Woods-Saxon single-particle wave functions. This procedure of evaluating the nuclear density, instead of using the standard Fermi distribution form [5,6], is more consistent with our model.

Since here we are interested only in the isovector $E1$ resonance, the results of the calculations should not be very sensitive to the value of the pairing gap. For Δ, we therefore use for Δ the standard odd-even mass formula. This is quite reasonable, especially considering that the isotopes with nucleon numbers which differ by two nucleons from the normal magic core are not considered.

We take the single-particle energies and wave functions as given by a realistic Woods-Saxon potential [14]. However, we renormalize the levels that lie near the Fermi surface by changing the well depth U_{jl} of the potential so as to obtain agreement with the experimental single-particle energies. For nonmagic nuclei, these energies are given by

$$\epsilon_1^{\rm expt}=\mu\pm\sqrt{(\epsilon_1-\mu)^2+\Delta^2} \qquad (4.2)$$

where the sign $+$ $(-)$ is taken for the $\epsilon_1^{\rm expt}$ determined from the spectra of the $N+1$ $(N-1)$ odd nucleus, and ϵ_1 is the single-particle energy calculated with our Woods-Saxon potential. The fitting procedure consists of changing U_{jl} and μ in each iteration just to obtain the experimental value given by Eq. (4.2). We also impose the usual particle conservation constraint. The experimental single-particle levels corresponding to ^{120}Sn are from Ref. [15], those in ^{100}Sn are from Ref. [16], and those in ^{132}Sn are from Ref. [17].

In the evaluation of the term $A^{\rm disc}$ in Eq. (2.7), we use a substraction procedure to avoid a double counting in the propagator. The form of this equation is very useful to study the influence of the single-particle continuum, since one can readily compare the results obtained with and without including the continuum in the framework of the same calculation scheme. Thus the results without the continuum are obtained by evaluating the propagator A in Eq. (2.9) without the last two terms in the square brackets. The summation was performed over four shells near the Fermi surface.

To perform the calculations without continuum, where single-particle levels with positive energy may be necessary, we used the standard procedure of immersing the system in a box, with the boundary condition that on the walls of the box the wave functions vanish. This procedure is even necessary in the continuum QRPA in order to obtain a reasonable single-particle basis in the ''pairing'' part of the propagator. Notice that in the continuum part of the propagator [the first term in Eq. (2.7)], all single-particle wave functions and Green-function partial components are evaluated by using the more ''natural'' boundary condition of the continuum RPA, i.e., by using the scattering single-particle Green function given by G^{τ}_{lj} in Eq. (2.8). This difference treatment of the continuum in the two terms of Eq. (2.7) may induce inaccuracies in the calculation procedure. We therefore corrected the term $A^{\rm cont}$ in this equation by choosing the box boundary condition for states with the same j and l as those with positive energies entering $A^{\rm disc}$. For example, in ^{120}Sn this correction was made for the $i_{13/2}$ and $f_{5/2}$ neutron components. It is worthwhile to point out once again that these corrections are necessary because the subtraction of two slightly different pole terms may have very unpleasant numerical consequences.

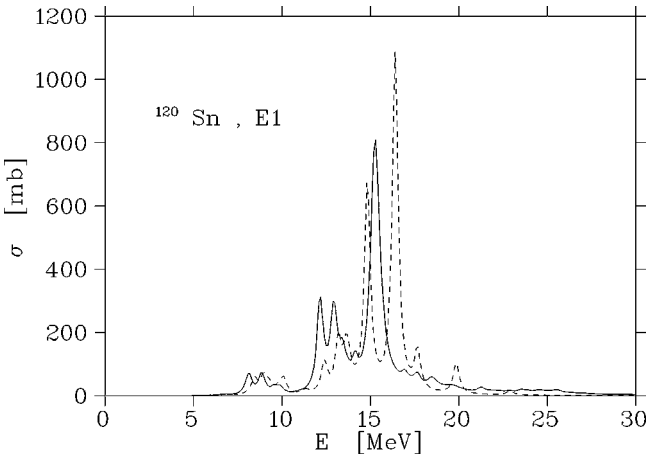

FIG. 3. Same as in Fig. 2, but for ^{120}Sn.

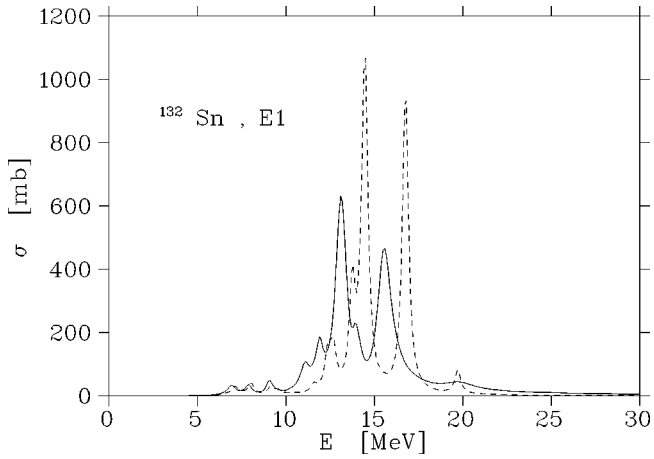

FIG. 4. Same as in Fig. 1, but for the unstable ^{132}Sn nucleus.

In all calculations the smearing parameter $\eta=200$ keV has been used. This value allows one to see the role of the single-particle continuum clearly.

The mean energies of the $E1$ resonances were calculated using the expression

$$\overline{E}=(m_1/m_{-1})^{1/2}, \quad m_k=\int dE\ S(E)E^k. \tag{4.3}$$

We give integral cross sections, i.e.,

$$\sigma_{\rm int}=\int dE\ \sigma(E), \tag{4.4}$$

as a percentage of the Thomas-Reiche-Kuhn cross section [1] $\sigma_{\rm int}=60(NZ/A)$ MeV mb. In the calculations below, the cross sections were calculated by perfoming the integral using a contour in the complex energy plane [18], which encloses the energy interval 0–200 MeV.

V. RESULTS AND DISCUSSION

In the applications to be presented below, we used the Landau-Migdal interaction with parameters as in Refs. [5,6,13], i.e., without any free parameter. The introduction of the pairing interaction in the case of superfluid systems requires the quantization of the continuum for some partial waves. As mentioned above, we have carried this out by inmersing the nucleus in a box and using the boundary condition of vanishing wave function on the walls of the box.

The calculated photoabsortion cross sections are presented in Figs. 1–4. Since we did not include configurations beyond the RPA, our results do not describe the width of giant resonances. For this, at least $1p1h\otimes$phonon excitations has also to be included, as was done for normal systems in Refs. [5,6]. In superfluid nuclei the treatment of these excitations is more complicated. The formalism presented here is a first step in this direction.

The mean energies are presented in Table I. In this table we also give the percentage of the total cross section [Eq. (4.4)] relative to the corresponding Thomas-Reiche-Kuhn value which is the depletion of the sum rule corresponding to the giant resonance.

As can be seen from the comparison of the results for the giant resonance envelope with and without the continuum, which is given in Figs. 1–4, the inclusion of the continuum is rather noticeable both for magic and nonmagic nuclei. It is clear that this effect will become even more appreciable if we take the smearing parameter $\eta<200$ keV. This means that it is necessary to take the single-particle continuum into account in QRPA calculations at least if one uses the smearing parameter $\eta\leqslant 200$ keV.

However, the influence of the continuum on the integral characteristics under investigation is not considerable; see Table I. Due to the inclusion of the continuum, we have obtained a small decrease of $\overline{E}$ and an increase of $\sigma_{\rm int}$ by 4% to 6%. The reason for the latter is the influence of the tail at higher energies. The tail gives contributions at energies $E>40$ MeV. We have therefore integrated the curves with the continuum up to 200 MeV, and obtained an increase of $\sigma_{\rm int}$ by about 2%.

We have also calculated the mean energies using another definition for $\overline{E}$, namely, $\overline{E}=(m_3/m_1)^{1/2}$ [cf. Eq. (4.3)], which emphasizes the role of the continuum and the resonance high-energy tail. This definition of $\overline{E}$ gives the values 18.3, 18.0, 17.2, and 17.0 MeV for ^{100}Sn, ^{104}Sn, ^{120}Sn, and ^{132}Sn, respectively, and is decreased by about 2 MeV due to exclusion of the continuum. Thus we see a noticeable influence of the continuum on these mean energies.

TABLE I. Mean energy [Eq. (4.3)] and percentage of the integrated cross section [Eq. (4.4)] with respect to the corresponding Thomas-Reiche-Kuhn value.

Isotopes	$\overline{E}$, MeV		$\sigma_{int}/60\frac{NZ}{A}$,%	
	QRPA+cont	QRPA−cont	QRPA+cont	QRPA−cont
^{100}Sn	15.6	15.7	105.3	100.0
^{104}Sn	15.3	15.3	108.3	102.2
^{120}Sn	14.0	14.3	98.7	94.7
^{132}Sn	13.8	14.2	101.5	97.2

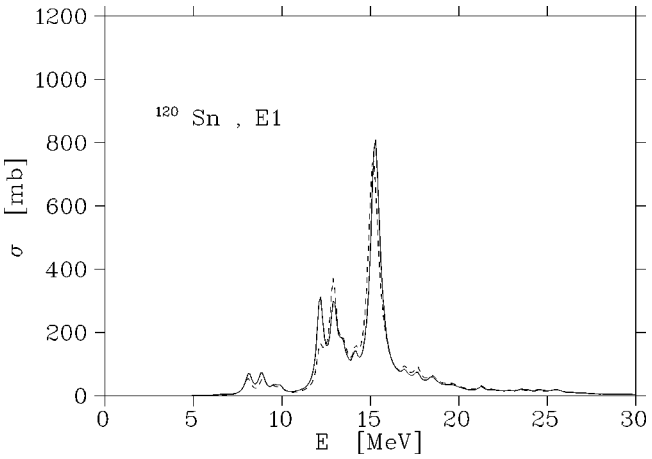

FIG. 5. The $E1$ photoabsorption cross section for ^{120}Sn calculated within the continuum QRPA with (solid line) and without (dashed line) the ''forced self-consistent'' procedure.

It is of interest to investigate the role of the ''forced consistency'' procedure. We have repeated the calculations with the continuum without this procedure. It turned out that there is a redistribution of the strength, especially for the nuclei ^{120}Sn and ^{132}Sn. As an example, in Fig. 5 we present the comparison of these results for ^{120}Sn. One can see an influence of the spurious dipole state on the form of the curve. This influence is negligible in ^{100}Sn. The difference between these two nuclei is probably due to the larger value of $N-Z$ in ^{120}Sn, which tends to mix the $T=0$ and 1 isospin values.

The similarity of the results with and without the ''forced consistency'' procedure means that the exclusion of the ''ghost'' is satisfactory or, in other words, the force parametrizations of the TFFS's have been chosen reasonably enough, and in this sense the theory gives results which are similar to the ones provided by the self-consistent theory.

Finally, it is worthwhile to mention that the depletion of the sum rule shown in Table I exceeds 100%. In our calculations, this is connected to the influence of pairing correlations (which are ground-state correlations), although it might also have to do with the omission of the particle-particle channel [20] or with the fitting of single-particle energies, which introduces a certain dependence of the single-particle potential on the operator l^2. These effects can change the calculated sum rule.

VI. CONCLUSION

In this paper we have investigated the influence of the continuum on dipole giant resonances by using the continuum RPA in the normal nuclei ^{100}Sn and ^{132}Sn. We have also applied a continuum QRPA, which we have developed for giant resonances, to the superfluid nuclei ^{104}Sn and ^{120}Sn. We have introduced a method to avoid spuriosities associated with the isoscalar dipole state which appears as a result of the breaking of the translational symmetry, and which, within the RPA, appears at zero energy if a self-consistent calculation is performed. Our method does not use any free parameter (like fitting the strength of a separable interaction to obtain the zero energy [4]), and is general; i.e., it is independent of the particle-hole interaction that one uses. The resulting formalism allows one to make the calculation with or without the continuum in a clear and straightforward fashion.

The comparison of the $E1$ resonance envelope calculated within the same numerical scheme with and without the single-particle continuum showed that the inclusion of the continuum gives a noticeable change of the envelope both for magic and nonmagic nuclei. Moreover, the influence of the continuum is noticeable for the mean energies if these are defined as $\overline{E}=(m_3/m_1)^{1/2}$. Thus the continuum at least redistributes the strength and, therefore, photoabsortion cross sections, like those presented in Figs. 1–4, can provide important information about the influence of the continuum in nuclei.

Our ''forced consistency'' procedure to isolate the spurious state is general, independent of the interaction or the method used to study particle-hole excitations, and sets the spurious state at exactly zero energy in all cases. The fact that the inclusion of this procedure does not change the results strongly confirms that the parameters used in the TFFS are reasonable. The approach developed in this paper may be useful to treat cases in which more complex configurations are included in the formalism.

ACKNOWLEDGMENTS

We are very grateful to J. Blomqvist for useful discussions. This work was supported by the Royal Swedish Academy of Sciences and by the Russian Foundation for Fundamental Research.

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