Photoproduction of a Λ on ^{12}C

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The photoproduction of a Λ on ^{12}C is investigated by using the recently developed Saclay-Lyon amplitudes for the $\gamma p \rightarrow K^+\Lambda$ reaction and the single-particle wave functions from a relativistic mean-field model of nuclei and Λ hypernuclei. With the nuclear transition matrix elements taken from a shell-model calculation, the predicted bound-Λ production cross sections are close to the $^{12}\mathrm{C}(\gamma,K^+)^{12}_{\Lambda}\mathrm{B}$ reaction data. The dependence of the predictions on the model of $\gamma p \rightarrow K^+\Lambda$ amplitudes has been investigated. The cross sections of quasifree processes leading to an unbound Λ are also calculated in a simple three-body model. The predicted cross sections of the inclusive $^{12}\mathrm{C}(\gamma,K^+)$ reaction reproduce the energy dependence of the data up to 1.1 GeV, but overestimate the magnitude by a factor of about 2.2. We discuss the extent to which this overestimation can be understood in terms of medium effects on the propagation of the outgoing K^+ and Λ. [S0556-2813(98)05109-7]

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I. INTRODUCTION

It has been well recognized [1] that electromagnetic probes are complementary to hadronic probes in investigating the structure of hypernuclei. With the recent developments at several GeV electron facilities, data on the photoproduction and electroproduction of hypernuclei will soon be very extensive. To make progress, it is necessary to understand the reaction mechanisms of these electromagnetic processes. In this work, we make an attempt in this direction with a theoretical interpretation of recent $^{12}\mathrm{C}(\gamma,K^+)$ reaction data [2].

Most of the previous theoretical investigations [3–6] of (γ,K) reactions on nuclei were carried out using an approach similar to that developed in the study of (γ,π) reactions [7,8]. The transition amplitude of the reaction is calculated from a kaon photoproduction operator on the nucleon and the wave functions of the initial nuclear and the final hypernuclear systems. The outgoing kaon wave functions are calculated by using either an optical potential or the eikonal approximation. No data for 1p-shell and heavier nuclei were available for testing these earlier theoretical predictions.

Motivated by the recent $^{12}\mathrm{C}(\gamma,K^+)$ reaction data [2], we test the validity of the theoretical scheme developed in Refs. [3–6] by taking the advantage of two recent developments. First, a model of kaon photoproduction and electroproduction amplitudes has been developed recently by a Saclay-Lyon Collaboration [9]. It is parametrized in terms of low-order Feynman amplitudes involving all identified resonances in the s, u and t channels. The parameters are determined by a global fit to all existing data of kaon photoproduction and electroproduction on the nucleon. It was found that only 2 nucleonic, 4 hyperonic, and 2 kaonic resonances out of a total of 25 resonances contribute significantly to the reaction mechanism. Second, a relativistic mean-field model of hypernuclei has been developed [10] to reproduce the binding energies of hypernuclei throughout the Periodic Table. The main objective of this work is to see the extent to which data of Ref. [2] can be understood by using these two theoretical inputs.

The data [2] on the $^{12}\mathrm{C}(\gamma,K^+)$ reaction has two components. The first one is due to the bound-Λ production leading to bound $^{12}_{\Lambda}\mathrm{B}$ states. The second component is due to the production of a K^+ associated with an unbound hyperon. In this work, we will focus on the predictions for the bound-Λ production mainly because the needed nuclear transition matrix elements are available from a shell-model calculation [11,12]. Furthermore, the distortion effects on the outgoing kaons in the 1p-shell region are found [4] mainly to reduce the magnitudes by about 30% but not to modify significantly the shapes of the angular distributions for all of the strongly excited states. For the still rather qualitative data we are considering here, it is therefore sufficient to perform calculations without including kaon distortions. In comparing with the data, we can simply scale the predictions by about 30%.

On the contrary, a calculation for the production of an unbound Λ is more difficult. For investigating the inclusive data of Ref. [2], one can follow either the response function formulation [13] that is well developed in (e,e') studies or the distorted-wave approach of Ref. [14]. In either approach, one needs to know not only the kaon-nucleus potential but also the Λ-nucleus potential. While the former one has been studied to some extent [15], our knowledge of the Λ-nucleus potential is still very limited. We therefore will only carry out a quasifree calculation based on a simple three-body model in which the distortion effects on the outgoing K^+ and Λ are neglected. The differences between our predictions and the data will indicate the importance of medium effects on the propagation of outgoing hadrons.

In Sec. II, we define the photoproduction operator in terms of Saclay-Lyon amplitudes and present the formula for calculating cross sections for the $^{12}\mathrm{C}(\gamma,K^+)^{12}_{\Lambda}\mathrm{B}$ reaction. The three-body model for calculating the production of an unbound Λ in the inclusive $^{12}\mathrm{C}(\gamma,K^+)$ reaction will be given in Sec. III. The results are presented and discussed in Sec. IV.

II. FORMULATION FOR ^{12}C$(\gamma,K^+)^{12}_{\Lambda}$B REACTION

Following the previous investigations [3–6], we assume that the (γ,K) reaction can be described in terms of the elementary $\gamma N \to K\Lambda$ amplitudes. As is well known [16–19], there are some ambiguities in implementing this impulse approximation into practical calculations within the multiple-scattering theory [20]. In this work, we are guided by the formulation developed in the (γ,π) study of Ref. [8]. In order to indicate clearly the dynamical content of our calculation and also to define notations for later discussion, we will present in this section the explicit formula used in this work.

The formulation presented below is based on the factorization approximation that neglects the dependence on the Λ momentum (apart from the overall δ function for three-momentum conservation) and uses the final $K^+\Lambda$ energy to evaluate the $\gamma N \to K\Lambda$ amplitude in nuclei. This simplification (and also the other possible forms of the factorization approximation discussed in the literature [16–19,8]) allows the separation of the transition amplitude into a structure part and a reaction part. The transition amplitude for the (γ,K) reaction in the γ-nucleus center-of-mass frame (A-CM) is then determined by the following production operator:

$$A(\vec{k},\vec{q},\hat{\varepsilon}) = \sum_{i=1}^{A} O(\vec{k},\vec{q},\hat{\varepsilon},\vec{\sigma}_i) e^{i(\vec{q}-\vec{k})\cdot \vec{r}_i}, \tag{1}$$

where $\vec{q}$ and $\vec{k}$ are, respectively, the momenta of the initial photon and the final kaon, $\hat{\varepsilon}$ is the photon polarization vector, $\vec{\sigma}_i$ is the Pauli operator, and r_i is the position vector of the ith nucleon. The interaction dynamics is contained in

$$O(\vec{k},\vec{q},\hat{\varepsilon},\vec{\sigma}) = \Gamma(\vec{k},\vec{k}_c,\vec{q},\vec{q}_c) \sum_{i=1}^{4} F_i(W_c,\theta_c) O_i(\vec{q}_c,\vec{k}_c,\hat{\varepsilon},\vec{\sigma}), \tag{2}$$

where $F_i(W_c,\theta_c)$ are the Chew-Goldberger-Low-Nambu (CGLN) amplitudes defined in the γN and $K\Lambda$ center-of-mass frame (2-CM), and

$$O_1(\vec{q}_c,\vec{k}_c,\hat{\varepsilon},\vec{\sigma}) = \vec{\sigma}\cdot\hat{\varepsilon},$$

$$O_2(\vec{q}_c,\vec{k}_c,\hat{\varepsilon},\vec{\sigma}) = i(\vec{\sigma}\times\hat{q}_c\cdot\hat{\varepsilon}),$$

$$O_3(\vec{q}_c,\vec{k}_c,\hat{\varepsilon},\vec{\sigma}) = \vec{\sigma}\cdot\hat{q}_c\hat{k}_c\cdot\hat{\varepsilon},$$

$$O_4(\vec{q}_c,\vec{k}_c,\hat{\varepsilon},\vec{\sigma}) = \vec{\sigma}\cdot\hat{k}_c\hat{k}_c\cdot\hat{\varepsilon}. \tag{3}$$

In the above expressions, we have used the simplicity of the chosen factorization approximation that the Λ is frozen in the nuclear center-of-mass frame. Thus we have in the A-CM frame the following expressions of the relative momenta:

$$\vec{k}_c = \frac{E_\Lambda(\vec{p}_\Lambda)\vec{k} - E_K(\vec{k})\vec{p}_\Lambda}{E_K(\vec{k}) + E_\Lambda(\vec{p}_\Lambda)},$$

$$\vec{q}_c = \frac{E_N(\vec{p}_N)\vec{q} - q\vec{p}_N}{E_N(\vec{p}_N) + q}, \tag{4}$$

where

$$\vec{p}_\Lambda = -\frac{\vec{k}}{A},$$

$$\vec{p}_N = \vec{k} + \vec{p}_\Lambda - \vec{q}, \tag{5}$$

$$= \frac{(A-1)}{A}\vec{k} - \vec{q}.$$

The factor Γ in Eq. (2) is due to the transformation of the elementary amplitude from the 2-CM to the A-CM frame and is of the following form:

$$\Gamma(\vec{k},\vec{k}_c,\vec{q},\vec{q}_c) = \left[\frac{E_K(\vec{k}_c)E_\Lambda(\vec{k}_c)E_N(\vec{q}_c)q_c}{E_K(\vec{k})E_\Lambda(\vec{p}_\Lambda)E_N(\vec{p}_N)q}\right]^{1/2}. \tag{6}$$

The invariant mass W_c and the scattering angle θ_c in the 2-CM frame are defined by the final $K\Lambda$ subsystem

$$W_c = E_K(\vec{k}_c) + E_\Lambda(\vec{k}_c),$$

$$\cos\,\theta_c = \hat{k}_c\cdot\hat{q}_c. \tag{7}$$

By using Eqs. (2)–(6), the Saclay-Lyon amplitudes can be used directly in our calculations. These equations define one of the possible off-shell extrapolations [16–19] which are needed in any multiple-scattering calculation of intermediate-energy nuclear calculations. Here we are guided by the formulation developed in the (γ,π) study of Ref. [8].

To account for the shell structure of the initial nuclei and final hypernuclei, it is more convenient to cast Eq. (1) into the following second quantization form:

$$O = \sum_{\alpha\beta} F^{LM}_{\alpha\beta}(\vec{k},\vec{q},\hat{\varepsilon})[b^\dagger_{l_\alpha j_\alpha}(\Lambda)h^\dagger_{l_\beta j_\beta}(N)]^{LM}, \tag{8}$$

where $b^\dagger$ and $h^\dagger$ are, respectively, the creation operators for Λ-particle and nucleon-hole states, and

$$F^{LM}_{\alpha\beta}(\vec{k},\vec{q},\hat{\varepsilon}) = \sum_{n=0,1}\sum_{l} \Gamma(\vec{k},\vec{k}_c,\vec{q},\vec{q}_c)\left[\frac{4\pi}{2l+1}\right]\sqrt{\frac{2j_\alpha+1}{2L+1}}(-1)^{2j_\beta}N_{L,-M}(n,l,\hat{q}_c,\hat{k}_c,\hat{\varepsilon})$$
$$\times\left\langle l_\alpha \frac{1}{2} j_\alpha \middle| |\mathbf{T}_L(\xi_n,\mathbf{Y}_l(\hat{r}))| \middle| l_\beta \frac{1}{2} j_\beta \right\rangle \int_0^\infty r^2 dr R^*_{l_\alpha j_\alpha}(r) j_l(|\vec{q}-\vec{k}|r) R_{l_\beta j_\beta}(r), \tag{9}$$

where $\boldsymbol{\xi}_0 = 1$, $\boldsymbol{\xi}_1 = \vec{\sigma}$, and $R_{lj}(r)$ is the radial wave function. All angle dependence of the reaction is absorbed in

$$N_{LM}(n=1,l,\hat{q}_c,\hat{k}_c,\hat{\varepsilon})=F_1(W_c,\theta_c)T_{LM}(\hat{\varepsilon},\mathbf{Y}_l(\hat{t}))-F_2(W_c,\theta_c)T_{LM}(\hat{k}_c\times(\hat{q}_c\times\hat{\varepsilon}),\mathbf{Y}_l(\hat{t}))+F_3(W_c,\theta_c)(\hat{k}_c\cdot\hat{\varepsilon})T_{LM}(\hat{q}_c,\mathbf{Y}_l(\hat{t}))+F_4(W_c,\theta_c)(\hat{k}_c\cdot\hat{\varepsilon})T_{LM}(\hat{k}_c,\mathbf{Y}_l(\hat{t})), \tag{10}$$

and

$$N_{LM}(n=0,l,\hat{q}_c,\hat{k}_c,\hat{\varepsilon})=F_2(W_c,\theta_c)[i\hat{k}_c\cdot(\hat{q}_c\times\hat{\varepsilon})]Y_{LM}(\hat{t})\,\delta_{L,l}, \tag{11}$$

where $\vec{t}=\vec{q}-\vec{k}$ and the angular tensor is defined, in the convention of Ref. [21], by

$$T_{LM}(\mathbf{V},\mathbf{Y}_l(\hat{t}))=\sum_{m_v,m_l}\langle LM|1lm_vm_l\rangle V_{1m_v}Y_{lm_l}(\hat{t}).$$

By using the above definitions (8)–(11), the differential cross section of the bound-Λ production that leads to a bound state of $^{12}_{\Lambda}$B with spin J can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_J=\frac{(2\pi)^4}{E^2}\frac{E_A(\vec{q})kE_K(\vec{k})E_{A-1,\Lambda}(\vec{k})}{2}\sum_{\lambda=\pm1}\sum_M\left|\sum_{\alpha,\beta}\langle{}^{12}_{\Lambda}\mathrm{B}(J)\|[b^\dagger_{l_\alpha j_\alpha}(\Lambda)h^\dagger_{l_\beta j_\beta}(N)]^J\|{}^{12}\mathrm{C(g.s.)}\rangle F^{JM}_{\alpha\beta}(\vec{k},\vec{q},\hat{\varepsilon}_\lambda)\right|^2, \tag{12}$$

where $E_A(\vec{q})$ and $E_{A-1,\Lambda}(\vec{k})$ are, respectively, the energies of the initial ^{12}C and the final $^{12}_{\Lambda}$B states, and $E=q+E_A(\vec{q})$. If the kaon distortion is included, Eq. (9) needs to be modified and the calculations become more involved. In this work, we are guided by Ref. [4] and will use the above expressions in our calculations. In comparing with the data, we need to scale the predicted magnitudes by about 30%.

III. UNBOUND-Λ PRODUCTION IN ^{12}C(γ,K^+)X REACTION

We assume that the production of an unbound Λ is due to a quasifree mechanism in which the final state is a three-body system with two plane-wave states for K and Λ and a one proton-hole state of ^{12}C. Explicitly, we define

$$|\Psi_f\rangle=a^\dagger_{\vec{k}}b^\dagger_{\vec{p}_\Lambda m_{s_\Lambda}}(\Lambda)|\Psi_{A-1}\rangle_\alpha, \tag{13}$$

where $a^\dagger$ is the creation operator for kaons, and $|\Psi_{A-1}\rangle_\alpha=h^\dagger_\alpha(p)|^{12}\mathrm{C}\rangle_{\mathrm{g.s.}}$. The differential cross section for quasifree production can then be written as

$$\frac{d\sigma}{d\Omega}=\frac{(2\pi)^4E_A(q)}{E}\int_0^{k_{max}}k^2dk\times\int d\Omega_\Lambda\frac{p_\Lambda E_\Lambda(\vec{p}_\Lambda)E_{A-1}(\vec{q}-\vec{k}-\vec{p}_\Lambda)}{[E_{A-1}(\vec{q}-\vec{k}-\vec{p}_\Lambda)+E_\Lambda(\vec{p}_\Lambda)(1-2\hat{\mathbf{p}}_\Lambda\cdot(\vec{q}-\vec{k})/p_\Lambda)]}\frac{1}{2}\sum_{\lambda=\pm1}\sum_{j_\alpha m_{j_\alpha}}\sum_{m_{s_\Lambda}}|T^\lambda_{j_\alpha m_{j_\alpha},m_{s_\Lambda}}(\vec{k},\vec{q})|^2, \tag{14}$$

with

$$T^\lambda_{j_\alpha m_{j_\alpha},m_{s_\Lambda}}(\vec{k},\vec{q})=\sum_{l_\alpha m_{l_\alpha}}\sum_{m_{s_N}}\left\langle j_\alpha m_{j_\alpha}\middle|l_\alpha\frac{1}{2}m_{l_\alpha}m_{s_N}\right\rangle Y_{l_\alpha m_{l_\alpha}}(\hat{p}_N)R_{l_\alpha j_\alpha}(p_N)(-1)^{j_\alpha-m_{j_\alpha}}\langle\vec{k}\vec{p}_\Lambda m_{s_\Lambda}|t(W)|\vec{q}\lambda\vec{p}_Nm_{s_N}\rangle, \tag{15}$$

where k_{max} and p_Λ are restricted by

$$q+M_A=E_K(\vec{k})+E_\Lambda(\vec{p}_\Lambda)+E_{A-1}(\vec{q}-\vec{k}-\vec{p}_\Lambda), \tag{16}$$

and

$$\vec{p}_N=\vec{p}_\Lambda+\vec{k}-\vec{q},$$

$$W=E_K(\vec{k})+E_\Lambda(\vec{p}_\Lambda). \tag{17}$$

In Eq. (15), $R_{l_\alpha j_\alpha}(p_N)$ is the nucleon single-particle wave function in momentum space. We evaluate the $\gamma N\to K\Lambda$ amplitude $\langle\vec{k}\vec{p}_\Lambda m_{s_\Lambda}|t(W)|\vec{q}\vec{p}_Nm_{s_N}\rangle$ exactly from the Saclay-Lyon amplitude by using the A-CM to 2-CM transformation defined in Ref. [8]. No frozen Λ approximation, such as that defined by Eq. (5), is assumed. This is important since the outgoing Λ in the quasifree production is unbound.

IV. RESULTS AND DISCUSSIONS

The most essential input to our calculations is the set of $\gamma p\to K^+\Lambda$ amplitudes defined by Eqs. (2) and (3). In this work, we use the amplitudes developed by the Saclay-Lyon group [9]. The accuracy of this model in the energy region

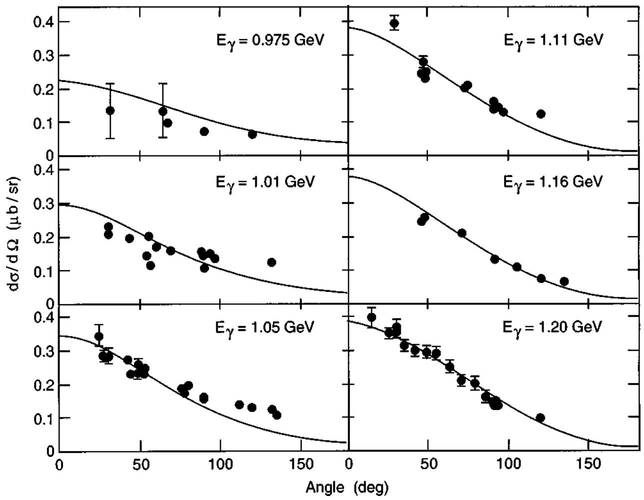

FIG. 1. The $\gamma p \rightarrow K^+\Lambda$ cross sections predicted by using the Saclay-Lyon amplitudes [9] are compared with the data.

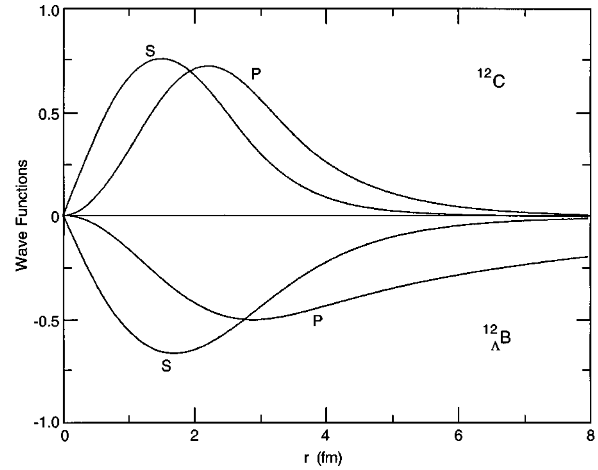

FIG. 2. The single-particle wave functions for the $0s_{1/2}$ (S) and $0p_{3/2}$ (P) states in ^{12}C and $^{12}_{\Lambda}$B calculated from the relativistic mean-field model of Ref. [10].

considered here is illustrated in Fig. 1. We see that the $\gamma p \rightarrow K^+\Lambda$ data can be described very well.

A. Exclusive $^{12}\mathrm{C}(\gamma,K^+)^{12}_{\Lambda}\mathrm{B}$ cross sections

The $^{12}\mathrm{C}(\gamma,K^+)$ data considered in this work were obtained from an experiment limited to measuring cross sections in the kinematic range where the outgoing kaons are within $10° \leqslant \theta_L \leqslant 40°$ with respect to the incident photons. By investigating the dependence of the averaged cross sections on the missing mass, the bound $^{12}_{\Lambda}$B states with total energies in the range of $11.2 \leqslant M_x \leqslant 11.4$ GeV were identified. However the data are not accurate enough for identifying individual states. The total bound-Λ production cross section of these unresolved bound hypernuclear states was estimated to be $\overline{\sigma_B} = 0.21 \pm 0.05$ μb/sr in the $E_\gamma = 1.0$–1.1 GeV energy region. We will first investigate whether the calculation based on Eqs. (9)–(12) can explain this data.

To proceed, we need the reduced matrix elements, $\langle ^{12}_{\Lambda}\mathrm{B}(J) \| [b^+_\alpha h^+_\beta]^J \| ^{12}\mathrm{C}(\mathrm{g.s.}) \rangle$ in Eq. (12), for nuclear transitions. Fortunately, this information can be obtained from the shell-model calculations of Refs. [11,12]. The calculations were performed within a model space spanned by the configurations that involve active nucleons in $(0p_{3/2}, 0p_{1/2})$ and a Λ in $(0s_{1/2}, 0p_{3/2}, 0p_{1/2})$ orbitals. For comparing with the reaction data, we follow the suggestion of Ref. [12] to normalize the predicted energies of negative-parity states to the experimental value of the Λ binding energy, $\mathrm{B}_\Lambda = 11.37$ MeV, for the 1^- ground state identified from emulsion data [22]. The energies of the positive-parity states are normalized to the first 2^+ state with an energy 9.86 MeV above the ground state (in ^{12}C), as suggested in Ref. [23].

For single-particle wave functions, we use those of the mean-field calculation of Ref. [10]. The predicted Λ single-particle energies are -10.8 MeV and -0.036 MeV for the $0s_{1/2}$ and $0p_{3/2}$ states, respectively. These values are in good agreement with experimental data, as discussed in Ref. [10]. The corresponding single-particle wave functions are shown in Fig. 2. We see that $0p_{3/2}$ Λ is barely bound. These wave functions are not changed much if we adjust the parameters of the mean-field calculation to give a slightly larger binding energy for the $0p_{3/2}$, as suggested by Ref. [12]. We neglect such possible corrections and use the wave functions displayed in Fig. 2. The small differences between the $0p_{1/2}$ and $0p_{3/2}$ wave functions are also neglected. These simplifications should be reasonable for investigating the still very qualitative data of Ref. [2]. For the same reason, we also do not consider other possible phenomenological methods, such as adjusting the Wood-Saxon potential to reproduce the empirical single-particle energies, in generating the single-particle wave functions.

In Fig. 3, we present the predicted cross sections at $E_\gamma = 1.1$ GeV and scattering angle $\theta = 10°$ in the center-of-mass frame. As expected, this reaction, which involves a large momentum transfer ($|\vec{k} - \vec{q}| \sim 400$ MeV), gives the largest cross sections for the stretched (highest spin) $(2^-)_1$ and $(3^+)_1$ states formed from converting a $0p_{3/2}$ proton to a Λ in either the $0s_{1/2}$ or the $0p_{3/2}$ state. To test our predictions in

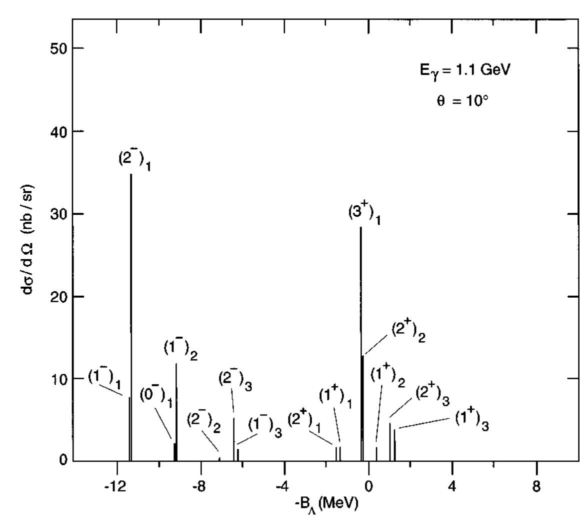

FIG. 3. The calculated differential cross sections for the $^{12}\mathrm{C}(\gamma,K^+)^{12}_{\Lambda}\mathrm{B}$ reaction at photon energy $E_\gamma = 1.1$ GeV and scattering angle $\theta = 10°$ in the center-of-mass frame.

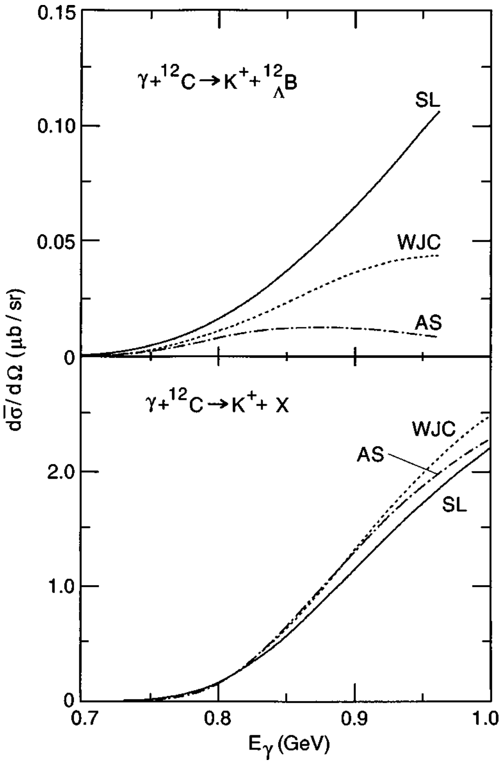

FIG. 4. The cross sections, averaged over a range of scattering angles $10° \leqslant \theta_L \leqslant 40°$, predicted by three models for the $\gamma p \to K^+\Lambda$ amplitudes are compared. The model SL is from Ref. [9], WJC is from Ref. [23], and AS is from Ref. [24]. The upper half is for the exclusive process $^{12}C(\gamma,K^+)^{12}_{\Lambda}B$, and the lower half is for the inclusive process $^{12}C(\gamma,K^+)X$. See text for the explanation.

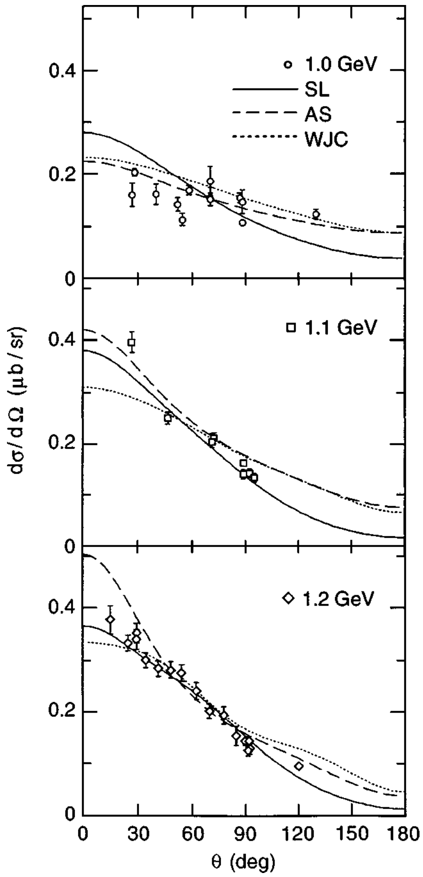

FIG. 5. The differential cross sections of $\gamma p \to K^+\Lambda$ predicted by the models of SL [9], AS [24], and WJC [23] are compared at three energies $E_\gamma = 1.0$, 1.1, and 1.2 GeV.

detail, a precision measurement with high energy resolution is clearly needed. The data from Ref. [2] can only be compared with the sum of contributions from all bound $^{12}_{\Lambda}B$ states. Furthermore, an average over scattering angle $10° \leqslant \theta_L \leqslant 40°$ should also be taken. Below we will present only these averaged cross sections.

Before we compare our predictions with the data, it is interesting to examine the dependence of our predictions on the input $\gamma p \to K^+\Lambda$ amplitudes. In addition to the Saclay-Lyon (SL) model, we also consider the models in Ref. [24] (WJC) and Ref. [25] (AS). The predicted $^{12}C(\gamma,K^+)^{12}_{\Lambda}B$ cross sections are compared in the upper part of Fig. 4. We see that the differences between the three models are very large, while their predictions on the proton target are comparable, as shown in Fig. 5. This is not surprising, since the contribution from each CGLN amplitude, F_i's in Eq. (2), to the transition amplitude is weighted by different nuclear matrix elements. This can be seen easily in Eqs. (9) and (10). Thus, the large differences seen in the upper half of Fig. 4 are due to the fact that the F's predicted by the three considered models are very different. This is illustrated in Fig. 6.

To compare with the bound-Λ production data, $\overline{\sigma_B} = 0.21 \pm 0.05$ μb/sr, of Ref. [2], we calculate the sum of the cross sections for all bound $^{12}_{\Lambda}B$ states and take an average of the results over the energy range $1.0 \leqslant E_\gamma \leqslant 1.1$ GeV and the angle range $10° \leqslant \theta_L \leqslant 40°$. Our result is 0.19 μb/sr for the Saclay-Lyon model. Taking into account a $\sim$30% reduction due to the kaon distortion [4], our prediction is $\sim$0.13 μb/sr. This result can be increased by a few percent if we use a slightly more bound Λ wave function for the $0p$ orbitals, as suggested by Ref. [12]. Thus our prediction of the bound-Λ production is close to the experimental value 0.21

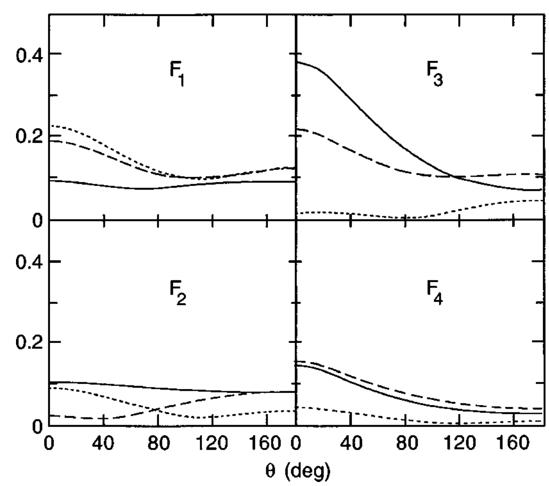

FIG. 6. The absolute magnitudes of the CGLN amplitudes of Eq. (2) calculated at $E_\gamma = 1$ GeV from the models of SL [9] (solid curve), AS [24] (dashed curve), and WJC [23] (dotted curve) are compared. Differences in phases are also significant but are not shown here.

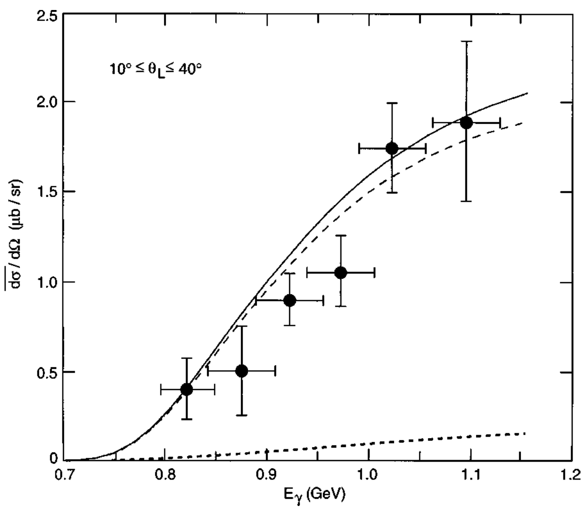

FIG. 7. The calculated cross sections, averaged over $10° \leqslant \theta_L \leqslant 40°$, are compared with the data [2]. The short-dashed curve is the calculated cross section of the exclusive process $^{12}C(\gamma,K^+)^{12}_{\Lambda}B$ reduced by 30% according to Ref. [4]. The long-dashed curve is obtained by dividing the contribution from the quasifree $^{12}C(\gamma,K^+)X$ process by a factor of 2.2. The solid curve is the sum of these two contributions.

± 0.05 μb/sr of Ref. [2]. From the upper half of Fig. 4, it is clear that the predictions from the WJC and AS models are well below the experimental value.

B. Inclusive $^{12}C(\gamma,K^+)X$ cross sections

To compare our predictions with the total inclusive cross section data in the entire energy region up to $E_\gamma = 1.1$ GeV, we need to include the cross section for producing an unbound Λ. This part of cross section (called quasifree cross section) is calculated by using the expressions in Eqs. (14)–(17). In the lower half of Fig. 4, the quasifree cross sections predicted by the considered three models of $\gamma p \rightarrow K^+\Lambda$ amplitudes are compared. It is seen that their differences are much smaller than for the exclusive $^{12}C(\gamma,K^+)^{12}_{\Lambda}B$ reaction displayed in the upper half of Fig. 4. This can be understood since Eq. (14) can be cast qualitatively into the following form:

$$\frac{d\sigma}{d\Omega} \sim \int d\vec{p}_N \rho(\vec{p}_N) \left[\frac{\overline{d\sigma}}{d\Omega}\right]_{\gamma+p\rightarrow K^+\Lambda} (\vec{p}_N), \tag{18}$$

where $(\overline{d\sigma}/d\Omega)(\vec{p}_N)$ is the spin-averaged elementary cross section evaluated at a momentum $\vec{p}_N$, and $\rho(\vec{p}_N)$ is the momentum distribution of protons in ^{12}C. The results seen in the lower half of Fig. 4 simply reflect the fact that the considered three models are comparable in reproducing the elementary cross sections (see Fig. 5). The large differences in the elementary amplitudes F's illustrated in Fig. 6, which are crucial in understanding the results in the upper part of Fig. 4, do not play a role in Eq. (18).

With the Saclay-Lyon $\gamma p \rightarrow K^+\Lambda$ amplitude, the sum of the bound Λ production and the quasifree Λ-production cross sections is found to be about a factor of 2.2 larger than the data. As seen in Fig. 7, our results divided by a reduction factor $R=2.2$ are consistent with the energy dependence of the data. As expected, the quasifree Λ production (long-dashed curve) is much larger than the bound Λ production (short-dashed curve).

The reduction factor $R \sim 2.2$ must be mainly due to the medium effects on the incoming photons and outgoing hadrons. We can estimate the reduction factor due to the distortion effects on photons and kaons by using the experimental cross sections $\sigma^{tot}_{\gamma N}$ and σ^{tot}_{KN} and the eikonal approximation. This was done in Ref. [2] with $R = Z/Z_{eff} \sim 1.6$, where $Z = 6$ is the proton number in ^{12}C and Z_{eff} is the effective proton number. Their value is much smaller than the value 2.2 needed here to reproduce the data. We therefore reexamine the eikonal approximation estimate by using the refined formula developed in Ref. [26]. In this approximation, the effective proton number is

$$Z_{eff} = \int d\vec{r} \rho_p(r) |\chi^{(+)}_\gamma(\vec{r})|^2 |\chi^{(-)}_{K^+}(\vec{r})|^2, \tag{19}$$

where

$$\chi^{(+)}_\gamma(\vec{r}) = \exp\left[i\vec{q}\cdot\vec{r} - \int_{-\infty}^{z} \frac{\sigma^{tot}_{\gamma N}}{2} \rho(\vec{r}) d\vec{r} \right], \tag{20}$$

$$\chi^{(-)}_{K^+}(\vec{r}) = \exp\left[-i\vec{k}\cdot\vec{r} - \int_{z}^{\infty} \frac{\sigma^{tot}_{K^+N}}{2} \rho(\vec{r}) d\vec{r} \right]. \tag{21}$$

In above equations, $\rho_p(\vec{r})$ is the proton density normalized to $Z=6$, and $\rho(\vec{r})$ is the total density normalized to $A=N+Z=12$. By using the procedure of Ref. [26] to evaluate Eq. (19), we find that the effective proton number for a ^{12}C density defined in terms of harmonic oscillator wave functions can be calculated analytically

$$Z_{eff} = \frac{\pi}{2} \int dx T(x) \exp\left[-\frac{\sigma^{tot}_{\gamma N} + \sigma^{tot}_{K^+N}}{2} T(x) \right], \tag{22}$$

with

$$T(x) = \frac{4}{\pi b^2} e^{-x/b^2} \left(\frac{5}{3} + \frac{4}{3}\frac{x}{b^2} \right). \tag{23}$$

Here, the oscillator length $b = 1.64$ fm is chosen to reproduce the charge mean-square-radius extracted from elastic electron scattering from ^{12}C. With the values $\sigma^{tot}_{\gamma N} \sim 0.2$ mb and $\sigma^{tot}_{K^+N} \sim 12.0$ mb, Eq. (21) yields $Z_{eff} \sim 4.10$. Thus our value of $R = Z/Z_{eff}$ is only ~ 1.46 which is close to the value ~ 1.6 of the eikonal calculation of Ref. [2]. It is much smaller than the value ~ 2.2 needed here to reproduce the data. This suggests that the medium effect on K^+ propagation is significant, but is not the whole mechanism for understanding the reduction factor $R \sim 2.2$. One obvious possibility is to also consider the medium effect on the outgoing Λ. A full distorted-wave approach similar to that developed in the $(e,e'\pi N)$ study of Ref. [14] is perhaps needed to make progress in this direction. This can be pursued only when a good Λ optical potential is well developed. The recent

Bruckner-Hartree-Fock calculation [27] of mean fields of hyperons in nuclear matter is certainly an important step toward this direction.

In conclusion, we have investigated the $^{12}\mathrm{C}(\gamma,K^{+})$ reaction by using the recently developed Saclay-Lyon amplitudes [9] of $\gamma p \rightarrow K^{+}\Lambda$ reaction and wave functions from the relativistic mean-field model of nuclei and hypernuclei developed in Ref. [10]. With the nuclear transition matrix elements taken from a shell-model calculation [11,12], the predicted bound-Λ production cross section is close to the data [2]. In a quasifree calculation based on a simple three-body model, the predicted unbound-Λ production is about a factor of 2.2 larger than the data. This factor cannot be fully understood in terms of the medium effects on the incoming photons and outgoing kaons. Our results suggest that the medium effects on the Λ propagation need to be included in future investigations.

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