Radiative corrections in neutrino-deuterium scattering

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(Received 22 January 1998)

The radiative corrections for neutrino-deuterium scattering are computed for the charged-current (CC) reaction, $\nu_e + d \to p + p + e^-$, and for the neutral-current (NC) reaction, $\nu_x + d \to p + n + \nu_x$. Nonrelativistic kinematics are used for the hadrons, which considerably simplifies the calculations. The impact radiative corrections have on the observables to be detected in the Sudbury Neutrino Observatory (SNO) are discussed and for most observables is found to be negligible. Only in the case where the internal bremsstrahlung photons emitted in the reaction $\nu_e + d \to p + p + e^- + \gamma$ are detected, is the expectation for the ratio of the number of CC to the number of NC events seen in SNO shifted by about one standard deviation. [S0556-2813(98)00508-1]

PACS number(s): 25.30.Pt, 13.15.+g, 26.65.+t, 96.60.Jw

I. INTRODUCTION

The Sudbury Neutrino Observatory (SNO) [1] will utilize the interaction of ^{8}B solar neutrinos with deuterium in heavy water to measure the total neutral-current (NC) cross section in the reaction

$$\nu_x + d \to p + n + \nu_x . \tag{1}$$

Also to be measured is the charged-current (CC) cross section in the reaction

$$\nu_e + d \to p + p + e^- . \tag{2}$$

Since the NC reaction is independent of neutrino flavor, the ratio of the number of charged-current to neutral-current events, $N_{\rm CC}/N_{\rm NC}$, will be a powerful indicator of the presence of neutrino oscillations. A measurement of this ratio is a primary goal for the SNO experiment. A secondary experiment that could provide an independent check is to measure the shape of the recoil spectrum in the CC reaction. A distortion in the electron spectrum from that expected in standard weak-interaction theory could also suggest neutrino oscillations. The likely measured signals are the first moment, $\langle T_e \rangle$, and possibly the second moment, $\langle T_e^2 \rangle$, of the recoil electron's kinetic energy, where the averages are taken over the electron spectrum for a detection threshold set at $T_{\rm min} = 5$ MeV.

Bahcall, Krastev, and Lisi [2,3] have given values for the SNO observables, $\langle T_e \rangle$, $\langle T_e^2 \rangle$, and $N_{\rm CC}/N_{\rm NC}$ for the standard solar model with no neutrino oscillations, and for a number of cases representing various neutrino-oscillation scenarios. The shift in SNO observables with each scenario is a measure of the discriminatory ability of the SNO detector to uncover new physics.

In the analysis so far the role that radiative corrections might have on the cross sections of Eqs. (1) and (2) has not been considered. However, one might expect it to be reasonably small. In weak-interaction studies, the correction to the total decay rate is typically at the few percent level. Indeed, in considering the energy dependence of the electron spectrum and the SNO observables, we find radiative corrections have a negligible impact on the shape test and an almost negligible 0.5% effect on the ratio $N_{\rm CC}/N_{\rm NC}$. This analysis, however, assumes the internal bremsstrahlung photon emitted in the charge-current reaction, Eq. (2), has not been detected. In principle, the SNO detector can measure these bremsstrahlung photons. If we make a very schematic assumption that the efficiency of photon detection equals the efficiency of electron detection, then the analysis simplifies significantly and we find that the impact of radiative corrections on the $N_{\rm CC}/N_{\rm NC}$ observable is increased to a 3.7% effect. If, for example, the error that has been assigned to the $N_{\rm CC}/N_{\rm NC}$ observable from such causes as counting statistics, uncertainties in the neutrino spectrum, and the energy resolution and absolute energy calibration of the SNO detector represents one standard deviation, then the change in this observable caused by the inclusion of radiative corrections is found to be about one standard deviation. This is not sufficient to spoil the discriminatory test that could, for example, distinguish between neutrino oscillations and no neutrino oscillations in the $N_{\rm CC}/N_{\rm NC}$ observable. However, it is sufficient that if an analysis were to be inverted, namely, the SNO observable was used to determine the parameters of a neutrino-oscillation scenario, then the value of the parameters will depend on whether radiative corrections have been considered or not. So photon detection with the SNO detector may become an interesting issue.

II. BASIC REACTION RATES

We start by evaluating the cross-section for the CC reaction, Eq. (2). In the lab frame of reference the following four-vectors are defined:[1]

$p_d = (0, iM_d) =$ initial four-momentum of deuteron,

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[1] We use the Pauli metric defined in De Wit and Smith [4] in which a four-vector is written as $A_\mu = (\mathbf{A}, iA_0)$ with imaginary fourth component, and a scalar product as $A \cdot B = A_\mu B_\mu = \mathbf{A} \cdot \mathbf{B} - A_0 B_0$.

$p_\nu=(\mathbf{p}_\nu, iE_\nu)$ = initial four-momentum of neutrino,

$p_1=(\mathbf{p}_1, iE_1)$ = final four-momentum of one of the protons,

$p_2=(\mathbf{p}_2, iE_2)$ = final four-momentum of the other proton,

$p_e=(\mathbf{p}_e, iE_e)$ = final four-momentum of the electron.

The differential cross-section is

$$d\sigma_{\rm CC}=\frac{1}{(2\pi)^5}\int\frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{p}_e}{32M_d E_\nu E_1 E_2 E_e}T \times\delta^4(p_d+p_\nu-p_1-p_2-p_e), \tag{3}$$

where T is the square of the T-matrix element, averaged over initial spins and summed over final spins

$$T=32G^2V_{ud}^2 g_A^2 M_d E_\nu E_1 E_2 E_e |I|^2(1+ay),$$

$$I=\int u_{2p}^*(r)u_d(r)dr. \tag{4}$$

Here $u_d(r)/r$ is the radial wave function for an S-state deuteron, $u_{2p}(r)/r$ is the radial function of relative motion for the two emerging protons, a is the electron-neutrino angular correlation coefficient ($a=-1/3$ for a pure Gamow-Teller transition), y is the cosine of the angle between the electron and neutrino directions, and G the weak interaction coupling constant determined from muon decay. It is assumed that in the initial state all the deuterium is in the ^{3}S state and then the only allowed transition is to the ^{1}S diproton state and the transition, being $1^+\rightarrow 0^+$, is pure Gamow-Teller. Thus the expression for the cross section explicitly displays the axial-vector coupling constant ($g_A=1.26$); V_{ud} is the Kobayashi-Maskawa mixing matrix element ($V_{ud}=0.975$). It is convenient to introduce relative and center-of-mass coordinates: $P=p_1+p_2$, $p=\frac{1}{2}(p_1-p_2)$ and use the three-momentum part of the delta function to integrate over $d^3\mathbf{P}$:

$$d\sigma_{\rm CC}=G^2V_{ud}^2 g_A^2\frac{1}{(2\pi)^5}\int d^3\mathbf{p}d^3\mathbf{p}_e|I|^2(1+ay) \times\delta(M_d+E_\nu-E_1-E_2-E_e). \tag{5}$$

Since only neutrino energies less than 15 MeV are considered, it is adequate to treat the nucleons nonrelativistically and write the protons' energies as $E_1=M_p+\mathbf{p}_1^2/(2M_p)$, $E_2=M_p+\mathbf{p}_2^2/(2M_p)$, where M_p is the proton mass. Then E_1+E_2 becomes $2M_p+(\mathbf{p}^2+\frac{1}{4}\mathbf{P}^2)/M_p$. We will assume that the center of mass of the diproton is essentially at rest, $\mathbf{P}=0$, the no-recoil approximation frequently used in beta-decay studies. Choosing the quantization axis along the direction of the incident neutrino and integrating over the angles of the protons and the polar angles of the electron gives

$$d\sigma_{\rm CC}=G^2V_{ud}^2 g_A^2\frac{1}{(2\pi)^3}\int p\,d(p^2)p_eE_e dE_e dy|I|^2 \times(1+ay)\,\delta(\Delta+E_\nu-E_e-p^2/M_p), \tag{6}$$

where $p=|\mathbf{p}|$, and Δ is the mass difference, $\Delta=M_d-2M_p$. The energy delta function is now used to integrate over $d(p^2)$, and if no attempt is made to measure the electron's angular distribution then integrating over dy yields the required expression for the differential cross section

$$\frac{d\sigma_{\rm CC}}{dE_e}=\frac{G^2}{4\pi^3}V_{ud}^2 g_A^2 M_p[M_p(\Delta+E_\nu-E_e)]^{1/2}p_eE_e|I|^2. \tag{7}$$

It should be stressed that the use of nonrelativistic approximations for the nucleons and the ability to use the energy delta function to effect the $d(p^2)$ integration constitute a great simplification. Normally in three-body final-state kinematics, the energy delta function imposes awkward limits on the other integration variables that are cumbersome for subsequent algebraic work. Naturally, we will use these simplifications in deriving expressions for the radiative corrections.

Equation (7) has been derived by Kelly and Überall [5], and by Ellis and Bahcall [6], who with the effective-range theory to evaluate the radial integrals, were the first to estimate the cross section for the absorption of solar ^{8}B neutrinos in deuterium. The expression, however, is not exact. First, trivially, the expression should be multiplied by the Fermi function, $F(2,E_e)$, to account for the Coulomb interaction between the two protons and the electron in the final state. Second, the restriction to S-state deuteron wave functions should be relaxed and more realistic wave functions used. Third, the allowed approximation of beta decay should be extended to include higher multipoles; and fourth, meson-exchange currents should be included. All these improvements have been implemented by Kubodera and co-workers [7,8], and by Ying, Haxton, and Henley [9,10] as summarized by Kubodera and Nozawa [11]. For our final numerical work we will use the computer code of Bahcall and Lisi [2] for the calculation of the neutrino-deuterium cross sections.

Following a similar analysis, the differential cross-section for the neutral-current reaction, Eq. (1), is

$$\frac{d\sigma_{\rm NC}}{dE'_\nu}=\frac{1}{4}\frac{G^2}{4\pi^3}g_A^2 M_p[M_p(\Delta'+E_\nu-E'_\nu)]^{1/2}E_\nu'^2|I'|^2, \tag{8}$$

where E'_ν is the energy of the final-state neutrino (in the lab system), I' the radial integral for the overlap of the relative proton-neutron ^{1}S scattering function with a ^{3}S-state deuteron function, and $\Delta'=M_d-M_p-M_n$, with M_n the neutron mass.

III. RADIATIVE CORRECTIONS

Radiative corrections to the electron spectrum from allowed beta decay have been considered in a number of papers [12–18]. In obtaining the corrections, integrations are carried out over the allowed neutrino and photon energies and the results exhibited as a differential spectrum in the electron energy. Essentially the same analysis follows for neutrino absorption reactions. The contributions to the radiative corrections have two components: the emission of real photons (internal bremsstrahlung) and virtual radiative corrections due, for example, to the exchange of photons between charged particles. We discuss each in turn. However, the results cannot be evaluated separately. This is because

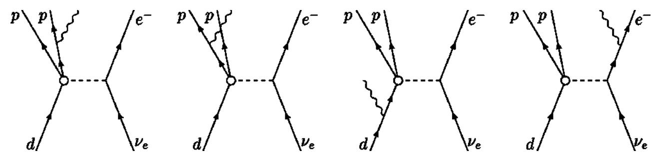

FIG. 1. Feynman diagrams for the emission of bremsstrahlung photons from charged particles present in the initial or final state. As explained in the text, only bremsstrahlung from electrons will be computed.

the bremsstrahlung graphs yield an infrared divergence that is exactly canceled by the virtual graphs.

A. Bremsstrahlung graphs in the CC reaction

Each of the external charged particles can emit bremsstrahlung photons, as shown in Fig. 1, and it is the sum of these graphs that make up a gauge-invariant set. However, the contributions from the nucleon bremsstrahlung graphs are much smaller than the contribution from electron bremsstrahlung. This is because the magnitude of the graph is largely determined by the energy denominator in the propagator for the internal fermion line. For nucleon bremsstrahlung this denominator is $(p-k)^2+M_p^2=-2p\cdot k=-2\mathbf{p}\cdot\mathbf{Q}+2E_pE_k\simeq 2M_pE_k$, in the nonrelativistic approximation for a nucleon in which $\mathbf{p}\simeq 0$. Here the four-vector for the photon momentum is written $k=(\mathbf{Q},iE_k)$. For electron bremsstrahlung, the denominator is $-2p_e\cdot k=2E_eE_k(1-\beta x)$, where $\beta=p_e/E_e$ and x the cosine of the angle between the electron and photon directions. Thus the ratio of the nucleon and electron bremsstrahlung graphs is of the order E_e/M_p. Since the average electron energy in the present work is around 7 MeV compared to a nucleon mass of 1 GeV, the nucleon bremsstrahlung graphs can clearly be neglected.

The differential cross section for the reaction $\nu_e+d\rightarrow p+p+e^-+\gamma$, is

$$d\sigma^{\gamma}_{\rm CC}=\frac{1}{(2\pi)^8}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{p}_ed^3\mathbf{Q}}{64M_dE_\nu E_1E_2E_eE_k}\times T^{\gamma}\delta^4(p_d+p_\nu-p_1-p_2-p_e-k), \tag{9}$$

where T^{γ} is the square of the T-matrix element, averaged over initial spins and summed over final spins. As before, we introduce relative and center-of-mass momenta for the two protons, p and P, and we use the momentum delta function to integrate over $d^3\mathbf{P}$ and the energy delta function to integrate over $d(p^2)$. No restrictions have been imposed on any of the angular integrations and so these can all be done trivially except for dx, where x is the cosine of the angle between the electron and photon directions. The result is

$$d\sigma^{\gamma}_{\rm CC}=\frac{1}{16\pi^5}\int\frac{p_eE_edE_eQ^2dQdx}{64M_dE_\nu E_1E_2E_eE_k}M_p\times[M_p(\Delta+E_\nu-E_e-E_k)]^{1/2}T^{\gamma}, \tag{10}$$

where

$$T^{\gamma}=32G^2e^2V_{ud}^2g_A^2M_dE_\nu E_1E_2E_e|I|^2\times\left\{\frac{E_k}{E_e^2(E_k-\beta Qx)}+\left(\frac{E_e+E_k}{E_e}\right)\frac{\beta^2(1-Q^2x^2/E_k^2)}{(E_k-\beta Qx)^2}\right\} \tag{11}$$

with $\beta=p_e/E_e$. This simple result is possible because of the nonrelativistic approximation used for the nucleon kinematics and of the ability to use the energy delta function to integrate over $d(p^2)$. Contrast this with, for example, the very complicated result [19–23] obtained for the bremsstrahlung correction in neutrino-electron scattering. We define the radiative correction to be

$$\frac{d\sigma_{\rm CC}}{dE_e}+\frac{d\sigma^{\gamma}_{\rm CC}}{dE_e}=\frac{d\sigma_{\rm CC}}{dE_e}\left[1+\frac{\alpha}{\pi}g_b(E_e,E_\nu)\right] \tag{12}$$

and

$$g_b(E_e,E_\nu)=\int_{-1}^{+1}dx\int_0^{Q_{\rm max}}Q^2dQ\left[\frac{\Delta+E_\nu-E_e-E_k}{\Delta+E_\nu-E_e}\right]^{1/2}\frac{1}{E_k}\times\left\{\frac{E_k}{E_e^2(E_k-\beta Qx)}+\left(\frac{E_e+E_k}{E_e}\right)\frac{\beta^2(1-Q^2x^2/E_k^2)}{(E_k-\beta Qx)^2}\right\}, \tag{13}$$

where α is the fine-structure constant, $\alpha=e^2/(4\pi)$. The photon energy is written as $E_k=[Q^2+\lambda^2]^{1/2}$, where λ is a small nonzero photon mass introduced to regulate the infrared divergence. The integrations over Q and x are very delicate: the logarithmic pole in λ has to be extracted to cancel with the λ dependence coming from the virtual corrections. The range of integration for Q runs from 0 to $Q_{\rm max}$. In Sec. III D we will discuss the possibility that the bremsstrahlung photons are detected by SNO. If they are, then $Q_{\rm max}$ equals ω, the threshold energy for photon detection. If they are not, then $Q_{\rm max}=\Delta+E_\nu-E_e\equiv y$.

The first term in Eq. (13) is free of the logarithmic singularity and can be trivially evaluated in the limit $E_k\rightarrow Q$ and $\lambda^2\rightarrow 0$. For the second term, the singularity is isolated by writing the integrand as

$$\frac{Q^2}{E_k}\left[\left(1-\frac{E_k}{y}\right)^{1/2}\left(1+\frac{E_k}{E_e}\right)-1\right]\frac{\beta^2(1-Q^2x2/E_k^2)}{(E_k-\beta Qx)^2}+\frac{Q^2}{E_k}\frac{\beta^2(1-Q^2x^2/E_k^2)}{(E_k-\beta Qx)^2}. \tag{14}$$

The first part is now free of the singularity and can be evaluated in the limit $E_k\rightarrow Q$ and $\lambda^2\rightarrow 0$. The second part contains the singularity and is exactly the integral evaluated by Kinoshita and Sirlin [12]. The result is

$$g_b(E_e,E_\nu)=I_1+I_2+I_{KS},$$

$$I_1 = -\frac{1}{E_e^2}\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)\frac{y^2}{15}$$
$$\times\left[\left(5-3\left(1-\frac{Q_{\rm max}}{y}\right)\right]\left(1-\frac{Q_{\rm max}}{y}\right)^{3/2}-2\right],$$
$$I_2 = 2\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right]$$
$$\times\int_0^{Q_{\rm max}}\frac{dQ}{Q}\left[\left(1-\frac{Q}{y}\right)^{1/2}\left(1+\frac{Q}{E_e}\right)-1\right],$$
$$I_{KS} = 2\ln\left(\frac{Q_{\rm max}}{\lambda}\right)\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right]+\mathcal{C}, \tag{15}$$

with

$$\mathcal{C} = 2\ln 2\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right]+1+\frac{1}{4\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)$$
$$\times\left[2+\ln\left(\frac{1-\beta^2}{4}\right)\right]+\frac{1}{\beta}[L(\beta)-L(-\beta)]$$
$$+\frac{1}{2\beta}\left[L\left(\frac{1-\beta}{2}\right)-L\left(\frac{1+\beta}{2}\right)\right], \tag{16}$$

where $L(z)$ is a Spence function

$$L(z) = \int_0^z \frac{dt}{t}\ln(|1-t|). \tag{17}$$

The integral in I_2 has to be done carefully because of the apparent pole at $Q=0$. In fact, there is no pole. For small Q, the integrand is expanded binomially and the $1/Q$ factor canceled.

B. Virtual radiative corrections

The virtual radiative corrections are those in which a photon is exchanged between the electron and the hadronic charged particles. A loop integration over the photon momentum, k, is involved. Because of the need to isolate the infrared singularity it is convenient to evaluate the correction in two photon-energy regimes, $k \leqslant M_p$ and $k \gg M_p$. In the first case the approximation is made that the W-boson mass is large, $m_W \to \infty$, and the interaction reduces to the four-fermion contact interaction of the prestandard model days. In this situation the loop integral is dominated by the $k^2 \to 0$ regime, so the photon coupling at the hadronic vertex is simply Dirac-like, the anomalous terms can be neglected. The loop integration, however, is ultraviolet divergent and some form of cutoff at an energy of the order of the proton mass has to be imposed. The virtual correction in this limit has been evaluated by Berman and Sirlin [13], Sirlin [14], and by Yahoo *et al.* [15] and we quote from the latter. If we write the T-matrix for the bare process as $T_{fi}^{\rm bare}$ and the T-matrix for the virtual correction as $T_{fi}^{\rm virtual}$, then the virtual radiative correction is written

$$T_{fi}^{\rm bare}+T_{fi}^{\rm virtual} = T_{fi}^{\rm bare}\left(1+\frac{\alpha}{2\pi}g_v(E_e)\right), \tag{18}$$

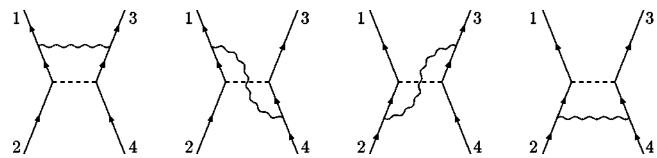

FIG. 2. Feynman diagrams for the exchange of virtual photons (or Z-bosons) between quarks and leptons. Note there are no virtual photon (or Z-boson) exchanges between particles 1 and 2, or between 3 and 4. These exchanges (identically zero in the Landau gauge) are part of the vertex definition and are not part of the radiative correction.

with

$$g_v^{\rm low}(E_e) = 3\ln\left(\frac{\Lambda}{M_p}\right)+\frac{3}{4}+\mathcal{A},$$
$$\mathcal{A} = \frac{1}{2}\beta\ln\left(\frac{1+\beta}{1-\beta}\right)-1+2\ln\left(\frac{\lambda}{m}\right)\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right]$$
$$+\frac{3}{2}\ln\left(\frac{M_p}{m}\right)-\frac{1}{\beta}\left[\frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)\right]^2+\frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right), \tag{19}$$

where $\beta = p_e/E_e$ and m is the electron mass. Note the presence of the infrared divergence in ln(λ) with a coefficient of exactly the right magnitude and sign to cancel the corresponding term in the bremsstrahlung result, Eq. (15). The imposed cutoff is denoted Λ and is taken to be of the order of the nucleon mass.

In the high-energy regime, $k \gg M_p$, the exchanged photon is not aware of the hadronic structure, but rather the electromagnetic coupling occurs at the quark level. The four relevant graphs are given in Fig. 2, where particles 1 and 2 are quarks and particles 3 and 4 are leptons. Because this is the charged-current weak interaction, we only display the quark that changes flavor, the other quarks in the deuteron remaining spectators. In this limit, the approximation is made that the momenta and masses of the external particles can be put to zero. Then the radiative correction is independent of electron energy:

$$g_v^{\rm high,\gamma} = 8\pi^2 m_W^2\{-3I(Q_1Q_3+Q_2Q_4)$$
$$+\tfrac{3}{4}J(Q_1-Q_2)(Q_4-Q_3)\}, \tag{20}$$

where Q_1, Q_2, Q_3, and Q_4 are the charges of particles 1, 2, 3, and 4, and m_W the W-boson mass. The loop integral, I, is

$$I = \int\frac{d^4k}{(2\pi)^4 i}\frac{1}{k^2(k^2+M_p^2)(k^2+m_W^2)}$$
$$= \frac{1}{8\pi^2}\frac{1}{(m_W^2-M_p^2)}\ln\left(\frac{m_W}{M_p}\right)\to\frac{1}{8\pi^2}\frac{1}{m_W^2}\ln\left(\frac{m_W}{M_p}\right), \tag{21}$$

where the nucleon mass term is included in the denominator to regularize the integral. The other loop integral, J, is divergent. The presence of a divergent integral is not a concern. The same graphs occur for muon decay, but in this case the value of the graphs is incorporated into the definition of the weak coupling constant, G. Thus, we are only interested in radiative corrections that differ in the semileptonic and purely leptonic sectors. The term J is universal, identical for both beta decay and muon decay, because its coefficient depends only on (Q_1-Q_2) and (Q_4-Q_3), the charge differences at the vertices. The term in I, however, appears only for beta decay; it is zero for muon decay ($Q_1=0$, $Q_2=-1$, $Q_3=-1$, $Q_4=0$). We evaluate this term by writing $Q_1+Q_2=2\overline{Q}$, $Q_1-Q_2=1$, $Q_3=-1$, and $Q_4=0$, where $\overline{Q}$ is the average charge of the quarks partaking in the charge-changing weak interaction. We obtain

$$g_v^{\mathrm{high},\gamma}=3\left(\overline{Q}+\frac{1}{2}\right)\ln\left(\frac{m_W}{M_p}\right). \tag{22}$$

To join the low-energy and high-energy regimes, it is assumed the ultraviolet cutoff in the low-energy regime can be set to the nucleon mass, the same mass that was used in the high-energy regime to regularize the integral, I.

We next consider in the high-energy regime the exchange of Z-vector bosons according to the same graphs in Fig. 2, with Z bosons replacing photons. Again the approximation is used that the Z boson is not aware of the hadronic structure and couples directly with the quarks. Then the external momenta and masses can be put to zero, and the radiative correction becomes independent of electron energy. The result is

$$\begin{aligned} g_v^{\mathrm{high},Z}=-\frac{8\pi^2 m_Z^2}{s^2}\Bigg\{\Bigg[&-\frac{5}{2}+\frac{5}{2}(Q_1-Q_2+Q_4-Q_3)s^2 \\ &+(4Q_1Q_3+4Q_2Q_4-Q_1Q_4-Q_2Q_3)s^4\Bigg]I \\ &-[1-(Q_1-Q_2+Q_4-Q_3)s^2+(Q_1-Q_2) \\ &\times(Q_4-Q_3)s^4]J\Bigg\}, \end{aligned} \tag{23}$$

where $s=\sin\theta_W$ and m_Z the Z-boson mass. The loop integral, I, is

$$\begin{aligned} I&=\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{k^2(k^2+m_W^2)(k^2+m_Z^2)} \\ &=\frac{1}{8\pi^2}\frac{1}{m_Z^2-m_W^2}\ln\left(\frac{m_Z}{m_W}\right), \end{aligned} \tag{24}$$

while again the loop integral, J, is divergent. The coefficient of J depends on constants or the charge differences at the vertices, (Q_1-Q_2) and (Q_4-Q_3), and so is universal. Considering only terms that differ between semileptonic and leptonic decays as contributing to the radiative correction, we obtain

$$g_v^{\mathrm{high},Z}=s^2\frac{m_Z^2}{m_Z^2-m_W^2}3\left(\overline{Q}+\frac{1}{2}\right)\ln\left(\frac{m_Z}{m_W}\right)=3\left(\overline{Q}+\frac{1}{2}\right)\ln\left(\frac{m_Z}{m_W}\right) \tag{25}$$

on putting $m_W=m_Z\cos\theta_W$. The complete virtual correction is a sum of three terms:

$$\begin{aligned} g_v(E_e)&=g_v^{\mathrm{low}}(E_e)+g_v^{\mathrm{high},\gamma}+g_v^{\mathrm{high},Z} \\ &=3\left(\overline{Q}+\frac{1}{2}\right)\ln\left(\frac{m_Z}{M_p}\right)+\frac{3}{4}+\mathcal{A}. \end{aligned} \tag{26}$$

Before continuing, it is essential to demonstrate that this technique of handling the virtual corrections conforms to the more complete treatment given for the electron spectrum in nuclear beta decay discussed by Sirlin [18]. Thus we compute the bremsstrahlung graph for beta decay and add to it the virtual corrections $g_v^{\mathrm{low}}(E_e)$, $g_v^{\mathrm{high},\gamma}$, and $g_v^{\mathrm{high},Z}$ given in Eqs. (19), (22), and (25). We put the result in terms of Sirlin's function $g(E_e,E_0)$ defined in Eq. (20b) of Ref. [14], and to avoid problems in notation we call it $G(E_e,E_0)$ here. The maximum electron energy in beta decay is written E_0. After a little algebra we obtain for the radiative correction for beta decay

$$\begin{aligned} g^{\mathrm{beta}}(E_e,E_0)&=g_b^{\mathrm{beta}}(E_e,E_0)+g_v(E_e) \\ &=\frac{1}{2}G(E_e,E_0)+3\left(\overline{Q}+\frac{1}{2}\right)\ln\left(\frac{m_Z}{M_p}\right) \\ &\quad+3\ln\left(\frac{\Lambda}{M_p}\right)+\frac{9}{8}. \end{aligned} \tag{27}$$

We compare this with the result obtained by Sirlin [18] of

$$\begin{aligned} g^{\mathrm{beta}}(E_e,E_0)=\frac{1}{2}G(E_e,E_0)+\frac{3}{2}\ln\left(\frac{m_W}{M_p}\right)+\frac{1}{2}\ln\left(\frac{m_W}{m_A}\right) \\ +C-2\ln\left(\frac{m_W}{m_Z}\right)+\frac{1}{2}\mathcal{A}_g. \end{aligned} \tag{28}$$

The first two terms are the universal photonic contributions arising from the weak vector current, the third and fourth terms are the asymptotic and nonasymptotic photonic corrections induced by the weak axial-vector current, the fifth term arises from the Z-boson exchange graphs, while the sixth term is a small perturbative QCD correction estimated by Marciano and Sirlin [24] to be $\mathcal{A}_g=-0.37$. It is convenient to gather the leading logarithms together and recast Eq. (28) as

$$\begin{aligned} g^{\mathrm{beta}}(E_e,E_0)&=\frac{1}{2}G(E_e,E_0)+2\ln\left(\frac{m_Z}{M_p}\right) \\ &\quad+\frac{1}{2}\ln\left(\frac{M_p}{m_A}\right)+C+\frac{1}{2}\mathcal{A}_g \\ &=\frac{1}{2}G(E_e,E_0)+2\ln\left(\frac{m_Z}{M_p}\right)+0.55. \end{aligned} \tag{29}$$

In giving a numerical value to the last three terms we have taken $m_A=1260$ MeV, and C from the calculations of Towner [25], $C=0.881$. This constant term is much smaller than the first two terms. We can now compare this expression from Sirlin [18], Eq. (29), with our expression Eq. (27). We see there is accord in the leading two terms if the cutoff

TABLE I. Values of the radiative correction, $(\alpha/\pi)g(E_e,E_\nu)$, Eq. (30) (in percent units) as a function of the neutrino energy, E_ν, and electron recoil kinetic energy, T_e.

	Neutrino energy (MeV)							
T_e	7	8	9	10	11	12	13	14
5.00	1.38	2.50	3.13	3.60	4.01	4.38	4.73	5.08
5.50	−1.17	1.88	2.68	3.21	3.65	4.03	4.39	4.72
6.00		0.98	2.17	2.81	3.29	3.69	4.06	4.39
6.50		−1.70	1.53	2.37	2.92	3.35	3.73	4.08
7.00			0.61	1.86	2.52	3.01	3.41	3.77
7.50			−2.19	1.22	2.08	2.64	3.09	3.46
8.00				0.28	1.57	2.25	2.75	3.15
8.50				−2.63	0.93	1.82	2.39	2.84
9.00					−0.03	1.31	2.01	2.51
9.50					−3.04	0.66	1.57	2.16
10.00						−0.32	1.06	1.77
10.50						−3.41	0.40	1.34
11.00							−0.59	0.83
11.50							−3.76	0.16
12.00								−0.85
12.50								−4.09

Λ is taken as the nucleon mass, M_p (as already chosen), and $\overline{Q}$ is taken to be $\overline{Q}=\frac{1}{6}$, the average charge of an up and down quark (again as posited). The expressions differ in the small constant terms. We can therefore improve our estimate of the virtual radiative correction by adding the constant $\frac{1}{2}\ln(M_p/m_A)+C+\frac{1}{2}\mathcal{A}_g-9/8=-0.57$.

The complete radiative correction for neutrino scattering on deuterium for the CC reaction is the sum of the bremsstrahlung, Eq. (15), and virtual, Eq. (26), corrections with the additional small constant term just discussed

$$g(E_e,E_\nu)=g_b(E_e,E_\nu)+g_v(E_e)-0.57. \tag{30}$$

In Table I we give values of $(\alpha/\pi)g(E_e,E_\nu)$ for the case when the bremsstrahlung photon has not been detected. The value of $Q_{\rm max}$ is set equal to $y\equiv\Delta+E_\nu-E_e$ in Eq. (15). Results are given for neutrino energies that span the ^{8}B solar neutrino spectrum and for recoil electron kinetic energy, $T_e=E_e-m$, that exceed the likely detection threshold of $T_{\rm min}=5$ MeV. In these ranges the radiative correction is seen to vary from $+5\%$ to -4%, with the correction decreasing for increasing T_e.

C. Radiative corrections in the NC reaction

For the neutral current, Eq. (1), there are no bremsstrahlung corrections from the leptons, while the bremsstrahlung corrections from the hadrons will be neglected, consistent with the approximation made for the CC reaction. For the virtual correction, there are no graphs with photon exchanges, since both leptons in the graph are the uncharged neutrinos. However, the four graphs in Fig. 2 can contribute for Z-boson exchanges. We again evaluate these graphs in the limit that the Z boson is not aware of the hadronic structure and couples directly to the quarks. Then the momenta and masses of the external particles are put to zero. But, unlike the case of virtual corrections in the CC reaction discussed in Sec. III B, these graphs are not completely independent of the hadronic structure. This is because the Z-boson-hadron coupling is not simply proportional to $(1+\gamma_5)$. Rather, for the bare graph the hadron covariant is

$$H_\mu^{\rm bare}=i\overline{u}_1\gamma_\mu(g_V^{(h)}+g_A^{(h)}\gamma_5)u_2, \tag{31}$$

with $g_V^{(h)}=\pm1-4Q_1\sin^2\theta_W$, $g_A^{(h)}=\pm1$, the upper sign for an up quark the lower sign for a down quark, while for the radiative-correction graphs the hadron covariant has two Z-boson-hadron couplings plus an extra γ_5[2]

$$H_\mu^{\rm rad\text{-}corr}=i\overline{u}_1\gamma_\mu\gamma_5(g_V^{(h)}+g_A^{(h)}\gamma_5)^2u_2. \tag{32}$$

The $H_\mu^{\rm rad\text{-}corr}$ therefore is not simply proportional to $H_\mu^{\rm bare}$ as was the case in the CC reaction. However, for the particular case of neutrino absorption on deuterium, we assume the transition is from a ^{3}S state to a ^{1}S state, a pure Gamow-Teller transition. Thus in the hadron covariants, we retain only the axial-vector pieces. If we define $H_\mu^{(A)}=i\overline{u}_1\gamma_\mu\gamma_5u_2$ then the two hadron covariants,

$$H_\mu^{\rm bare}\to g_A^{(h)}H_\mu^{(A)},$$

$$H_\mu^{\rm rad\text{-}corr}\to(g_V^{(h)2}+g_A^{(h)2})H_\mu^{(A)}, \tag{33}$$

are simply proportional to each other and the dependence on hadronic structure for the radiative correction is just limited to ratios of coupling constants. Then we can proceed as with the CC reaction. The T-matrix element for the bare reaction is

[2]The extra γ_5 comes from the replacement of a product of three gamma matrices by the product of one gamma matrix and γ_5.

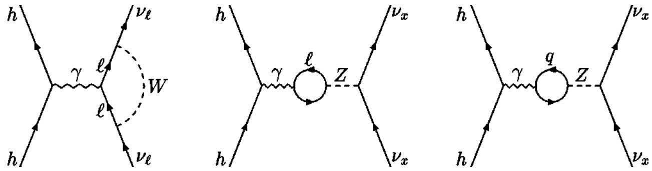

FIG. 3. Other possible radiative corrections for the NC neutrino-hadron scattering, but which give zero contribution for pure Gamow-Teller Transitions.

$$T_{fi}^{\mathrm{bare}} = -\frac{G}{2\sqrt{2}} g_A^{(h)} H^{(A)} \cdot L, \tag{34}$$

where the lepton covariant for neutrinos is as before: $L_\mu = i\overline{u}_3 \gamma_\mu (1+\gamma_5) u_4$. The radiative-correction T-matrix element, $T_{fi}^{\mathrm{rad\text{-}corr}}$, is proportional to T_{fi}^{bare}, and their ratio defines the radiative correction, g_v^{NC}:

$$T_{fi}^{\mathrm{bare}} + T_{fi}^{\mathrm{rad\text{-}corr}} = T_{fi}^{\mathrm{bare}} \left(1 + \frac{\alpha}{2\pi} g_v^{\mathrm{NC}} \right). \tag{35}$$

Note that in the approximation of the high-energy regime, g_v^{NC} is independent of lepton energies. The result is

$$g_v^{\mathrm{NC}} = -\frac{3}{4c^2 s^2} \frac{(g_V^{(h)2} + g_A^{(h)2})}{g_A^{(h)}} 8\pi^2 m_Z^2 I, \tag{36}$$

where $c^2 = \cos^2\theta_W$ and $s^2 = \sin^2\theta_W$. The loop integral, I, is defined as

$$\begin{aligned} I &= \int \frac{d^4k}{(2\pi)^4 i} \frac{1}{(k^2+M_p^2)(k^2+m_Z^2)^2} \\ &= \frac{1}{8\pi^2} \frac{1}{(m_Z^2 - M_p^2)} \left[\frac{1}{2}(m_Z^2 - M_p^2) - M_p^2 \ln\left(\frac{m_Z}{M_p}\right) \right] \\ &\to \frac{1}{16\pi^2} \frac{1}{m_Z^2}. \end{aligned} \tag{37}$$

The nucleon mass has again been used to regularize the integral. It remains to decide how to choose the coupling constants $g_V^{(h)}$ and $g_A^{(h)}$ for the case of neutrino absorption on deuterium. We will make the very naive assumption that the radiative correction in deuterium is simply three times the sum of the radiative corrections for an up quark and a down quark evaluated from Eq. (36). The result is

$$g_v^{\mathrm{NC}} = 3\left(1 - \frac{s^2}{c^2}\right), \tag{38}$$

which provides a correction of about $+0.4\%$ for the NC reaction rate.

There are a further set of graphs that could contribute to the radiative correction of a neutral current reaction, shown in Fig. 3. The first two show lepton intermediate states and are considered electroweak corrections, while the third has quark intermediate states and are QCD corrections. These graphs give the principal NC radiative correction in neutrino-electron scattering [23]. However, we notice that in each case the coupling at the hadron vertex is with a photon and so is of purely vector character. As we have already discussed, for the deuteron situation we require axial-vector coupling at the hadron vertex. Therefore none of these graphs will lead to a radiative correction for a pure Gamow-Teller transition.

D. Bremsstrahlung photon detected

For the SNO detector the real bremsstrahlung photon emitted in the CC reaction, $\nu_e + d \to p + p + e^- + \gamma$, will, in principle, be detected. So the procedure outlined in Sec. III A for obtaining the radiative correction by integrating over the photon energy cannot be followed. Suppose that SNO can detect photons of energy greater than some threshold, say ω, and that ω is less than or equal to the threshold for the detection of the recoil electrons, $\omega \leqslant T_{\mathrm{min}}$. Further, for simplicity of the following analysis, we will assume that photons and electrons are detected with equal efficiency and that it is the sum of the energy deposited by the photons and electrons that is observed in the SNO detector.

Then, starting from Eq. (10), we change variables from $dE_e dQ$ to $dX dE$, where X is the sum $E_e + Q$ and $E = E_e$, and integrate over E from m to X to obtain the differential cross section as a function of X

$$\frac{d\sigma_{\mathrm{CC}}^\gamma}{dX} = \frac{1}{16\pi^5} \int_m^X dE \frac{\beta(E) E^2 (X-E)}{64 M_d E_\nu E_1 E_2 E} \times M_p [M_p(\Delta + E_\nu - X)]^{1/2} \int_{-1}^{+1} dx T^\gamma, \tag{39}$$

where T^γ is given in Eq. (11), and $\beta(E) = [1 - m^2/E^2]^{1/2}$. For the bare reaction without photon emission, Eq. (7), the differential cross section is

$$\frac{d\sigma_{\mathrm{CC}}}{dX} = \frac{G^2}{4\pi^3} V_{ud}^2 g_A^2 M_p [M_p(\Delta + E_\nu - X)]^{1/2} \beta(X) X^2 |I|^2, \tag{40}$$

where the electron energy has been set to X, the total energy deposited in the absence of a photon. The radiative correction is then defined to be

$$\frac{d\sigma_{\mathrm{CC}}}{dX} + \frac{d\sigma_{\mathrm{CC}}^\gamma}{dX} = \frac{d\sigma_{\mathrm{CC}}}{dX} \left(1 + \frac{\alpha}{\pi} g_b(X) \right) \tag{41}$$

and

$$g_b(X) = \frac{2}{\beta(X) X} F(X) \ln\left(\frac{X-m}{\lambda}\right) - \frac{2}{\beta(X) X} \int_m^X dE \frac{F(X) - F(E)}{X - E} + \frac{1}{2} \frac{(X J_1 - J_2)}{\beta(X) X^2}, \tag{42}$$

where

$$J_1 = \int_m^X dE \ \ln\left(\frac{1+\beta}{1-\beta}\right),$$

TABLE II. Values of the radiative correction, $(\alpha/\pi)g(X)$, Eq. (44) (in percent units) as a function of X, where X is the sum of the electron and photon energies.

$X=$	7	8	9	10	11	12	13	14
	4.14	4.23	4.31	4.39	4.47	4.55	4.62	4.69

$$J_2=\int_m^X dEE\,\ln\left(\frac{1+\beta}{1-\beta}\right), \tag{43}$$

$$F(E)=\beta E\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right].$$

Note the integrals over dE and dx are handled delicately to isolate the logarithmic pole in λ, the photon mass. To this expression we add the radiative correction from the virtual graphs, Eq. (26), which remain unchanged except that the variable E_e is now replaced by X. The logarithmic singularity in λ is exactly cancelled in the sum $g_b(X)+g_v(X)$.

Note, also, the radiative correction no longer depends explicitly on the neutrino energy. The only dependence on E_ν in the bremsstrahlung differential cross section, Eq. (10), occurred in the factor $(\Delta+E_\nu-E_e-E_k)^{1/2}$. For the bare reaction without photon emission, Eq. (7), the factor is $(\Delta+E_\nu-E_e)^{1/2}$. It is the ratio of these two factors that provides the neutrino-energy dependence to the radiative correction. However, when the photon is detected with equal efficiency with the electron such that only the total energy deposited in the detector is recorded, then the factor from the bremsstrahlung differential cross section, Eq. (39), is $(\Delta+E_\nu-X)^{1/2}$, while the factor in the bare cross section, Eq. (40), is also $(\Delta+E_\nu-X)^{1/2}$, because in the latter case the total energy deposited in the detector, X, is equal to E_e. So the explicit dependence on E_ν drops out in the ratio.

In Table II we give values of $(\alpha/\pi)g(X)$, where

$$g(X)=g_b(X)+g_v(X), \tag{44}$$

for the case where the bremsstrahlung photon has been detected with equal efficiency with the recoil electrons for a range of energies between $T_{\min}+m\leqslant X\leqslant\Delta+E_\nu^{\max}$. The radiative correction is very nearly constant at around 4.4%. It increases only slightly with increasing energy, X, by +0.08% per MeV. A constant radiative correction will have no effect on the SNO observables $\langle T_e\rangle$ and $\langle T_e^2\rangle$, but gives a constant shift to the ratio, $N_{\rm CC}/N_{\rm NC}$, the number of charged-current to neutral-current counts.

IV. RESULTS AND DISCUSSION

The SNO experiment [1] will observe the charged-current neutrino-deuterium reaction, Eq. (2), by measuring the Cerenkov light emitted by the recoiling electron. The electron kinetic energy, T_e, is distributed between 0 and $\Delta+E_\nu-m$, where E_ν is the neutrino energy, Δ is the mass difference $M_d-2M_p=-0.891$ MeV, and m is the electron mass. The threshold for electron detection has been set at $T_{\min}=5$ MeV, below which the signal-to-noise ratio is likely to be poor. In observing the Cerenkov light, the distribution of the measured recoil kinetic energy, T_e, around the true kinetic energy, T_e', can be described by a resolution function of the form [2,3]

$$R(T,T')=\frac{1}{\Delta_{T'}(2\pi)^{1/2}}\exp\left[-\frac{(T'-T+\delta)^2}{2\Delta_{T'}^2}\right]. \tag{45}$$

The bias term δ accounts for a possible uncertainty in the absolute energy calibration and the energy dependent width, $\Delta_{T'}$, scales as $\sqrt{T'}$ due to photon statistics

$$\Delta_{T'}=\Delta_{10}\sqrt{T'/(10\ \text{MeV})}, \tag{46}$$

where Δ_{10} is the energy resolution width at 10 MeV. For SNO, the parameters have been set at $\Delta_{10}=1.1\pm0.11$ MeV and $\delta=\pm100$ keV.

The three SNO observables are $\langle T_e\rangle$, $\langle T_e^2\rangle$ or its variance σ_0^2, and the ratio $N_{\rm CC}/N_{\rm NC}$. These quantities are defined as

$$\langle T_e\rangle=\frac{1}{N_{\rm CC}}\int_{T_{\min}}dT_eT_e\int dE_\nu\lambda(E_\nu)P_{ee}(E_\nu)\times\int dT_e'R(T_e,T_e')\frac{d\sigma_{\rm CC}}{dT_e'}(E_\nu),$$

$$\langle T_e^2\rangle=\frac{1}{N_{\rm CC}}\int_{T_{\min}}dT_eT_e^2\int dE_\nu\lambda(E_\nu)P_{ee}(E_\nu)\times\int dT_e'R(T_e,T_e')\frac{d\sigma_{\rm CC}}{dT_e'}(E_\nu),$$

$$\sigma_0^2=\langle T_e^2\rangle-\langle T_e\rangle^2, \tag{47}$$

$$N_{\rm CC}=\int_{T_{\min}}dT_e\int dE_\nu\lambda(E_\nu)P_{ee}(E_\nu)\times\int dT_e'R(T_e,T_e')\frac{d\sigma_{\rm CC}}{dT_e'}(E_\nu),$$

$$N_{\rm NC}=\int dE_\nu\lambda(E_\nu)\sigma_{\rm NC}(E_\nu),$$

where $\lambda(E_\nu)$ is the spectrum of ^{8}B solar neutrinos, and $P_{ee}(E_\nu)$ is the survival probability that electron neutrinos prepared in the sun remain as electron neutrinos when detected on earth. The above integrands should also include detector efficiencies, $\epsilon_{\rm CC}(T_e')$ and $\epsilon_{\rm NC}(E_\nu)$. If the efficiencies are assumed to be energy independent, then the moments $\langle T_e\rangle$ and $\langle T_e^2\rangle$, are independent of $\epsilon_{\rm CC}$, while the ratio of CC to NC events, $N_{\rm CC}/N_{\rm NC}$, will scale as the ratio of efficiencies. The ^{8}B neutrino spectrum we take from Bahcall *et al.* [26] and the neutrino survival probabilities from Bahcall and Krastev [27], who have given the following neutrino-oscillation solutions: a purely vacuum oscillation (VAC) solution with neutrino mass-mixing parameters Δm^2 and $\sin^2 2\theta$ given by 6.0×10^{-11} eV^2 and 0.96, and two (best-fit) Mikheyev-Smirnov-Wolfenstein (MSW) solutions at small and large mixing angle (SMA and LMA) with parameters $(\Delta m^2,\sin^2 2\theta)$ given by $(5.4\times10^{-6}$ eV^2, $7.9\times10^{-3})$ and $(1.7\times10^{-5}$ eV^2, 0.69), respectively.

For the standard (STD) case with no neutrino oscillations, $P_{ee}=1$, Bahcall *et al.* [2,3] have calculated the following values for the SNO observables:

$$\langle T_e\rangle = 7.658 \pm 0.070\ \text{MeV},$$
$$\sigma_0^2 = 3.04 \pm 0.15\ \text{MeV}^2, \qquad (48)$$
$$N_{\rm CC}/N_{\rm NC} = 1.882 \pm 0.079,$$

where the 1σ errors are due to (a) statistics of 5000 CC events and 1354 NC events, (b) uncertainty in the neutrino-deuterium cross-sections, (c) uncertainty in the ^{8}B neutrino spectrum, (d) energy resolution, and (e) the absolute energy calibration. If SNO performs as expected, the measurement of $\langle T_e\rangle$ should distinguish a VAC solution from the no-oscillation (STD) solution and possibly resolve the SMA solution; while in the $N_{\rm CC}/N_{\rm NC}$ measurement all three neutrino-oscillation solutions are well resolved from the STD situation.

The question to be answered here is: How much are these expectations modified when radiative corrections in the neutrino-deuterium cross sections are included? We consider two extremes: the case when the bremsstrahlung photons are not detected at all, and the case when the bremsstrahlung photons are detected with equal efficiency to the recoil electrons. In the first case, the integrands in Eq. (47) for the CC reaction are multiplied by the function $1+(\alpha/\pi)g(E_e,E_\nu)$, where $g(E_e,E_\nu)$ is the sum of the bremsstrahlung and virtual corrections, Eq. (30). In the second case, we consider the measured Cerenkov light in SNO to be a measure of the total energy deposited by recoil electrons and bremsstrahlung photons, and multiply the integrands for the CC reaction by $1+(\alpha/\pi)g(X)$, where $g(X)$ is defined in Eq. (44) and X is the total energy recorded by the Cerenkov detectors. In both cases, the NC cross section is multiplied by a constant, $1+(\alpha/\pi)g_v^{\rm NC}$, with $g_v^{\rm NC}$ given by Eq. (38).

In Table III we show the results of the changes to the SNO observables by the inclusion of radiative corrections. When the bremsstrahlung photons are not detected the SNO observables are shifted: $\langle T_e\rangle$ by -0.14%, σ_0^2 by -0.32%, and $N_{\rm CC}/N_{\rm NC}$ by $+0.5\%$, while when bremsstrahlung photons are detected, the shifts become $\langle T_e\rangle$ by $+0.03\%$, σ_0^2 by $+0.04\%$, and $N_{\rm CC}/N_{\rm NC}$ by $+3.7\%$. Only for the $N_{\rm CC}/N_{\rm NC}$ observable in the case when the photons are detected, are these shifts comparable to one standard deviation, where one standard deviation is considered to be the 1σ error given in Eq. (48).

TABLE III. Values of the SNO observables, $\langle T_e\rangle$, σ_0^2, $N_{\rm CC}/N_{\rm NC}$, for four different neutrino-oscillation scenarios, STD, SMA, LMA, VAC, with and without radiative corrections.

		No radiative corrrections	With radiative corrections: Photon undetected	Photon detected
STD	$\langle T_e\rangle$	7.658	7.648	7.660
	σ_0^2	3.04	3.03	3.05
	$N_{\rm CC}/N_{\rm NC}$	1.882	1.891	1.952
SMA	$\langle T_e\rangle$	7.875	7.864	7.877
	σ_0^2	3.17	3.16	3.17
	$N_{\rm CC}/N_{\rm NC}$	0.639	0.642	0.663
LMA	$\langle T_e\rangle$	7.654	7.644	7.656
	σ_0^2	3.04	3.03	3.05
	$N_{\rm CC}/N_{\rm NC}$	0.422	0.424	0.437
VAC	$\langle T_e\rangle$	8.361	8.349	8.363
	σ_0^2	3.24	3.23	3.24
	$N_{\rm CC}/N_{\rm NC}$	0.411	0.413	0.427

In summary, radiative corrections in neutrino-deuterium scattering have only a small impact on the observables likely to be measured in SNO. Thus, the anticipated discriminatory ability of SNO to uncover new physics is not compromised by our considerations here. Nevertheless it might be interesting for SNO in reaching its primary objective of measuring $N_{\rm CC}/N_{\rm NC}$ to investigate its ability to detect the internal bremsstrahlung photons and with what efficiency as this could have some impact on the neutrino-oscillation parameters that might be deduced from this SNO observable.

ACKNOWLEDGMENTS

I would like to thank the SNO group for a critical reading of the manuscript, and in particular mention Hay Boon Mak and Guy Jonkmans who found a serious error in the first draft. I would also like to thank J. N. Bahcall and E. Lisi for permission to use their computer code for the calculation of the neutrino-deuterium cross sections.

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