Novel scaling behavior for the multiplicity distribution under second-order quark-hadron phase transition

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Deviation of the multiplicity distribution P_q in a small bin from its Poisson counterpart p_q is studied within the Ginzburg-Landau description for a second-order quark-hadron phase transition. The dynamical factor $d_q \equiv P_q/p_q$ for the distribution and the ratio $D_q \equiv d_q/d_1$ are defined, and novel scaling behaviors between D_q are found which can be used to detect the formation of quark-gluon plasma. The study of d_q and D_q is also very interesting for other multiparticle production processes without a phase transition. [S0556-2813(98)05808-7]

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The multiplicity distribution is one of the most important and most easily accessible experimental quantities in high-energy leptonic and hadronic collisions. From the well-known KNO scaling and its violation [1,2] to the novel scaling form [3] investigated very recently the distribution shows a lot about the dynamical features for the processes. Local multiplicity distributions have been studied for many years in terms of a variety of phase space variables [4], and substantial progress has been made recently in deriving analytical QCD predictions for those observables [5]. Based on assuming the validity of the local parton-hadron duality hypothesis, those analytical predictions for the parton level can be compared to experimental data. A global and local study of multiplicity fluctuations [6] shows, however, that the theoretical predictions have a significant deviation from experimental data. The significant deviation of theoretical predictions from experimental data indicates that we know only a little about multiparticle production processes since the hadronization process in soft QCD is far from being understood.

In this paper we try to investigate the multiplicity distribution in some small two-dimensional kinetic region (which can be the rapidity and transverse momentum or azimuthal angle, for example) in high-energy heavy-ion collision processes. In such collisions a new matter form, quark-gluon plasma (QGP), may be formed which subsequently cools and decays into the experimentally observed hadrons; thus the system undergoes a quark-hadron phase transition. The hadrons produced in such processes may, in principle, carry some relic information about their parent state. Thus the investigation of the multiplicity distribution may be interesting and useful for probing the formation of QGP. In this paper we are limited to discussing the multiplicity distribution under the assumption of a second-order phase transition, within the Ginzburg-Landau description for the phase transition. Within the same description for quark-hadron phase transitions, the scaled factorial moments have been studied by many authors for second-order [7,8] and first-order [9–12] phase transitions, and a universal scaling exponent $\nu \simeq 1.30$ is given in [7,8,11,12].

It is useful to point out that the study of multiplicity fluctuations in photon production at the threshold of lasing, which shows a similar type of phase transition [13], is already in its mature age, although the theory and experiment for a quark-hadron phase transition are still in their infancy. As explained in [7–12] the Ginzburg-Landau model, which has been used in describing superconducting transitions and other macroscopic second-order phase transitions, can also be used to describe the multiplicity fluctuations in both second- and first-order phase transitions. In [12] multiplicity distributions are studied for both first- and second-order phase transitions. The authors showed that for a second-order phase transition the probability P_q of finding q hadrons in the small bin under investigation decreases monotonically with the increase of q regardless of the value of the bin width and that for a first-order phase transition P_q is a decreasing function of q for a small bin width whereas the shape of the distribution changes with the increase of the bin width. Thus the shape of the distribution was claimed as a tool for finding the order of the phase transition.

In this paper we first show that the criterion in [12] based on the shape of the multiplicity distribution for the order of the phase transition is equivocal. This is easily seen once one considers the trivial case without dynamical fluctuations. For such a case, the multiplicity distribution p_q is a Poisson one:

$$p_q(\overline{s}) = \frac{\overline{s}^q}{q!} \exp(-\overline{s}) \quad , \tag{1}$$

with $\overline{s}$ the mean multiplicity. From this distribution one has

$$\frac{p_{q+1}}{p_q} = \frac{\overline{s}}{q+1} \quad . \tag{2}$$

If $\overline{s} < 2.0$, p_q is a monotonically decreasing function of q, whereas p_q changes its shape for $\overline{s} > 2.0$. Using the same parameters as in [12] $\overline{s}$ is calculated and listed in Table I. Thus one can see that the shapes of multiplicity distributions given in [12] are similar to those of Poisson ones. In real experiments, one can always choose a bin width to ensure a mean multiplicity larger than 2.0; then one cannot tell

TABLE I. Mean multiplicities $\overline{s}$ for second-order ($B=+1$) and first-order ($B=-1$) phase transitions for different bin widths. x is a parameter (different from the quantity used in this paper) associated with the bin width δ, parameter s in [12]. The mean multiplicities are calculated using Eq. (13) in [12].

x	1	2	4	6
$B=+1$	0.226	0.429	0.804	1.155
$B=-1$	0.342	0.797	2.172	4.582

whether the distribution is shaped due to statistical fluctuations or due to the dynamics in the phase transitions. So one cannot give the order of the quark-hadron phase transition just from the general shape of the distributions, and detailed information is needed. This result is not surprising, because nondominant dynamical fluctuations can only modify the shape of the statistical distribution to some extent but cannot change its general behavior drastically.

Nevertheless, it should be pointed out that the study of the multiplicity distribution is still very interesting and useful for processes with the onset of quark-hadron phase transitions. In Ginzburg-Landau theory, the multiplicity distribution turns out to be a Poisson one if the field is purely coherent. Conversely, the distribution turns into a negative binomial one if the field is completely chaotic. In reality, one can assume multiplicity production arising from a mixture of chaotic and coherent fields, and so the multiplicity distribution in real processes is not a Poisson one or a negative binomial one, and the deviation of the distribution from a Poisson one is due to dynamical fluctuations. The real quantity concerned is the deviation of the experimental P_q from its theoretical Poisson counterpart p_q. Thus studying the deviation may reveal features of the dynamical mechanism involved. Let the probability of having q hadrons in a certain bin be P_q; the deviation of P_q from its Poisson counterpart p_q can be measured by the ratio $d_q=P_q/p_q$. For the definition of d_q to make sense, it is necessary to let the mean multiplicity $\overline{s}$ for P_q and p_q be the same. Dynamical fluctuations are shown to exist if the ratio is far from 1.0, either much larger or much smaller, for some q. The ratio d_q can be called the dynamical factor, since it is 1.0 unless there are dynamical fluctuations in the process. In the Ginzburg-Landau description for second-order phase transitions P_q is given by [8]

$$P_q(\delta)=Z^{-1}\int \mathcal{D}\phi p_q(\delta^2|\phi|^2)e^{-F[\phi]}, \tag{3}$$

where $Z=\int\mathcal{D}\phi e^{-F[\phi]}$ is the partition function, $p_q(\overline{s})$ the Poisson distribution with average $\overline{s}$, and $F[\phi]$ the free energy functional:

$$F[\phi]=\delta^2[a|\phi|^2+b|\phi|^4].$$

It is instructive to note that a free energy functional with $O(N)$ QCD order parameter was studied in [14]. The free energy functional used here is different from that in [14] because of the consideration that we are now only interested in final state charged hadrons (most of them are $\pi^{\pm}$) so that a two-component order parameter is enough (which is written as a complex number) for our purpose. One can see that the functional used here can be derived from that in [14] by integrating out all other components and neglecting higher-order powers of remaining components in the exponential. One more simplification used in present functional is that the derivative term is neglected since former studies (see the last two papers in [7] for details) find that the term has little contribution to the universal exponent which is a measure of the fluctuations. Because of this simplification, the non-Gaussian functional integral can be treated as a normal integral and can be evaluated directly.

With the free energy functional above the system is in the plasma state for $a>0$ (the order parameter $|\phi_0|^2$ corresponding to the minimum of $F[\phi]$ is zero) and in the hadron phase for $a<0$ (the order parameter $|\phi_0|^2>0$). In real experiments the temperature at which hadrons are emitted from the source is unknown and may be different from event to event. So we treat a as a free parameter and discuss only the case with $a<0$ in the following since in the quark phase with $a>0$ only a few hadrons can be produced through fluctuations. From the distribution of Eq. (3) one gets the mean multiplicity for $a<0$,

$$\overline{s}=x\frac{J_1(|a|x)}{J_0(|a|x)}, \tag{4}$$

with $J_n(\alpha)\equiv\int_0^\infty dy\ y^n\exp(-y^2+\alpha y)$, $x=\delta/\sqrt{b}$ representing the bin width δ, and $a\propto T-T_c$ representing the temperature when the phase transition takes place. For a small phase space bin the mean multiplicity in the bin is proportional to x and thus can be very small. Under such circumstances the distribution must be concentrated around P_0, and both P_q and p_q for $q>1$ must be very small; so a direct comparison between them could induce a large uncertainty. This demands that the bin width in a real experimental analysis should be large enough to ensure that the mean multiplicity in the bin will not be too small (larger than 0.5, say). Of course, smaller bins can be used if a precise determination of both P_q and p_q can be obtained from high-statistical experimental data. For a zero bin width the relevant results are rather sensitive to the cascading production of particles through resonances, and so an extremely small bin width should be avoided.

Because of the normalization of both P_q and p_q, the dynamical factor d_q must be larger than 1.0 for some q and less than 1.0 for some other q if there exist dynamical fluctuations. One can easily derive

$$d_q(x)=\frac{J_q(x(|a|-1))}{J_0(x|a|)}\frac{J_0^q(x|a|)}{J_1^q(x|a|)}\exp(\overline{s}). \tag{5}$$

The dependence of d_q on q for different x and $|a|$ is shown in Fig. 1 by choosing $|a|=1.0$ and 2.0, $-\ln x=-1.0$, 0.0, 1.0, 2.0, 3.0, respectively. From this figure, one can see that the general shapes of d_q are similar for different choices of $|a|$ but depend strongly on the bin width x. In detail, for large x (small $-\ln x$ or high mean multiplicity) d_1 is quite large while $d_{q>1}$ are smaller than 1.0. For small x (large $-\ln x$ or low mean multiplicity), however, d_1 is smaller than but close to 1.0 while $d_{q>1}$ are larger than 1.0, indicating that two or more particles are more likely to be in the same

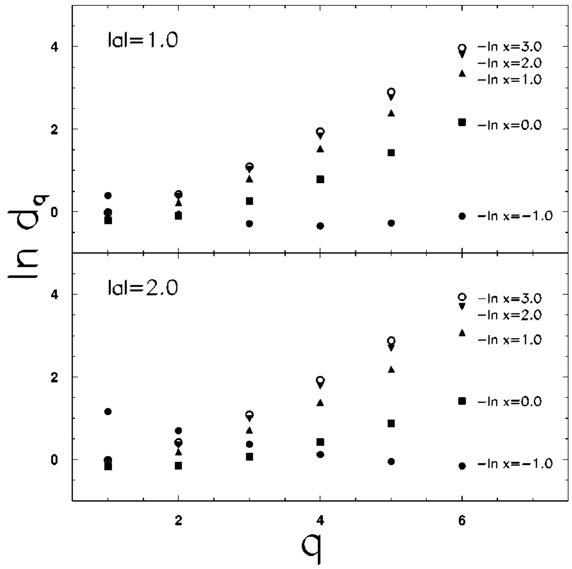

FIG. 1. Dependence of the dynamical factor d_q on q for $|a|=1.0$ and 2.0, for $-\ln x=-1.0,0.0,1.0,2.0,3.0$.

small bin than for the pure statistical case. This phenomenon may be associated with the cluster effect or minijets in quark-hadron phase transitions. For small x the values for d_q are independent of $|a|$. From Fig. 1 one can also see that the dependence of d_q on x is quite complicated. For some large q, d_q is monotonically increasing with the decrease of x, but for small q, d_q first decreases and then increases with the decrease of x. Complicated behaviors can be seen for $p_q(x)$, considering the fact that $\overline{s}$ is an increasing function of x while $p_q(\overline{s})$ changes its behavior at $\overline{s}=q$. But the ratio $p_q(x)/p_1(x)$ is an increasing function of x for $q>1$. Moreover, there exists a scaling law between p_q:

$$\frac{p_q(\overline{s})}{p_1(\overline{s})}=\frac{2^{q-1}}{q!}\left(\frac{p_2(\overline{s})}{p_1(\overline{s})}\right)^{q-1}. \tag{6}$$

Thus it may be more interesting to study the dependence on x of

$$D_q\equiv\frac{d_q}{d_1}=\frac{P_q/P_1}{p_q/p_1} \tag{7}$$

instead of d_q, and one may expect some scaling behavior of D_q when the resolution is changed. Now we turn to study D_q for second-order quark-hadron phase transitions. If there is no dynamical reason, $P_q=p_q$, one can see that D_q for all q can have only one value, 1.0, no matter how large or small the bin width is. So from the range of values D_q takes, one can evaluate the strength of dynamical fluctuations. D_q can be expressed in terms of $J_n(\alpha)$ as

$$\ln D_q=(q-1)\ln\frac{J_0(|a|x)}{J_1(|a|x)}+\ln\frac{J_q(x(|a|-1))}{J_1(x(|a|-1))}. \tag{8}$$

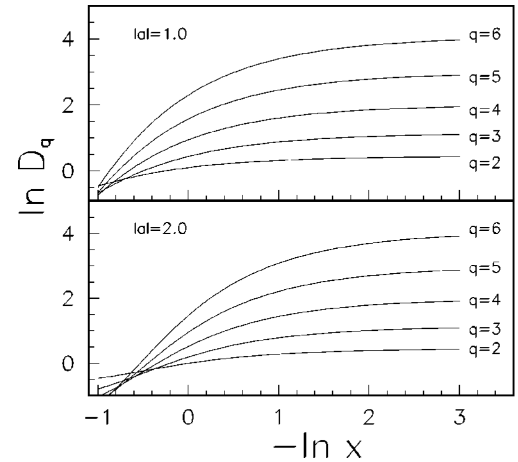

FIG. 2. Dependences of D_q on the bin width $-\ln x$ for $|a|=1.0$ and 2.0 for $q=2, 3, 4, 5, 6$.

Besides x, there is in the last expression another parameter $|a|$ which is a measure of how far from the critical temperature the hadronization process occurs and is unknown in current experiments. First let us fix $|a|$ to be 1.0. One can immediately see that, for any x, $D_q\propto D_2^{q-1}$, a power law satisfied by Poisson, binomial, and many other distributions in statistics. For any value of $|a|$, D_q increases monotonically with the decrease of x. This shows that the dynamical influence can be observed more easily in a high-resolution analysis. This can be understood physically, since different dynamical fluctuations may offset each other and become less obviously observable in a large bin analysis. The behaviors of $\ln D_q$ as functions of resolution, $-\ln x$, are shown in Fig. 2 for $|a|=1.0$ and 2.0 for $q=2,3,4,5,6$. For small $-\ln x$ values of D_q depend strongly on the parameter $|a|$, but they approach parameter $|a|$ independent values for large $-\ln x$. The similarity in the shapes of $\ln D_q$ as functions of $-\ln x$ suggests a power law for other $|a|$ between D_q and D_2 similar to the case we showed for $|a|=1.0$. $\ln D_q$ are reshown in Fig. 3 as functions of $\ln D_2$ with the same data as in Fig. 2. For both $|a|=1.0$ and 2.0 perfect linear

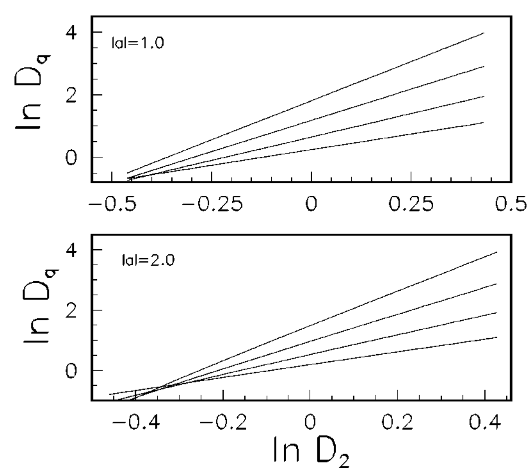

FIG. 3. Scaling behaviors between D_q and D_2 for $|a|=1.0$ and 2.0. The data are the same as in Fig. 2. The curves, from the lower to the upper, are for $q=3, 4, 5, 6$, respectively.

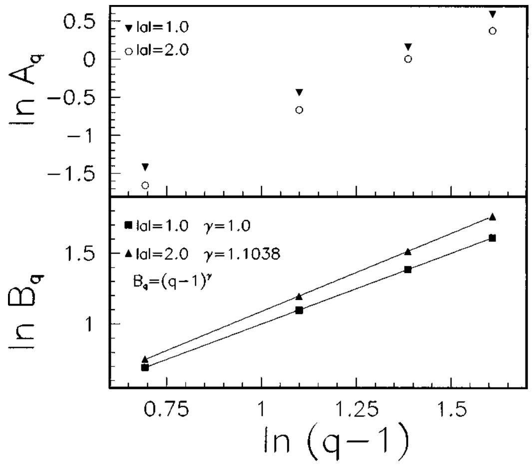

FIG. 4. Coefficients for the scaling between D_q and D_2, $\ln D_q = A_q + B_q \ln D_2$, as functions of $\ln(q-1)$ for $|a|=1.0$ and 2.0. Linear fitting curves are shown for $B_q=(q-1)^\gamma$.

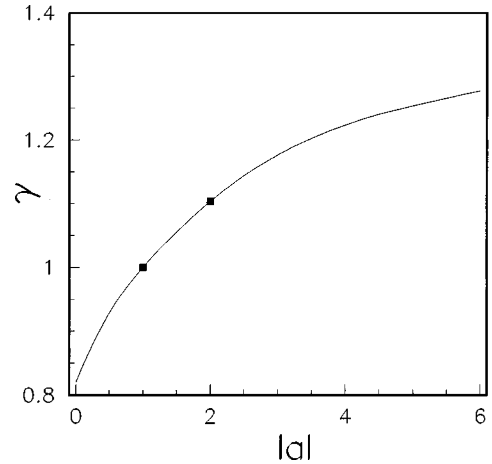

FIG. 5. Dependence of exponent γ on $|a|$. For large $|a|$, γ is about 1.34.

dependences of $\ln D_q$ on $\ln D_2$ can be seen. For other values of $|a|$ the similar linear dependence is checked to be true. Thus one has

$$\ln D_q = A_q + B_q \ln D_2, \tag{9}$$

with A_q and B_q depending on $|a|$. The fitted results of A_q and B_q from curves in Fig. 3 are shown in Fig. 4 as functions of $\ln(q-1)$ for $|a|=1.0$ and 2.0. It is obvious that both $\ln A_q$ and $\ln B_q$ have a linear dependence on $\ln(q-1)$ for fixed $|a|$. Especially, for the purpose of studying the power law, we investigate B_q and find that

$$B_q = (q-1)^\gamma \ , \tag{10}$$

with γ depending on $|a|$. For visualization, the linear fitting curves for $\ln B_q$ vs $\ln(q-1)$ are shown also in Fig. 4 for $|a|=1.0$ and 2.0. The slopes for $\ln A_q$ are about twice those for $\ln B_q$, and they increase with increasing $|a|$. When $|a|$ is zero, corresponding to the case in which hadrons are produced exactly at the critical point, numerical results show that γ is 0.819. With the increase of $|a|$, γ increases quickly. For sufficiently large $|a|$, when the difference between $|a|-1$ and $|a|$ can be neglected, corresponding to the case in which hadrons are produced at a temperature much below the critical point, one finds that $D_q(x)=F_q(|a|x)$, with F_q the scaled factorial moment which is given in [8] for second-order phase transitions as

$$F_q(x) = J_q(x)\ J_0^{q-1}(x)\ J_1^{-q}(x).$$

A similar relation between D_q and F_q is also true in the small x limit. In these limiting cases, the scaling of D_q is equivalent to that of the scaled factorial moments F_q, and one can get an exponent $\gamma=1.3424$ for large $-\ln x$ [15] or large $|a|$. The dependence of γ on $|a|$ is shown in Fig. 5. In real experiments, $|a|$ is not known for an event and may be increasing in the hadronization process. Thus some average over $|a|$ should be made. The smaller $|a|$, the less the number of produced particles. Thus the main contribution to P_q comes from events with large $|a|$ or with high multiplicities. For those events, one should get γ near 1.30, close to the universal exponent ν given in former studies of F_q for second-order phase transitions. For events with low multiplicity, one can get $\gamma > 0.819$. So the theoretical range for the exponent γ is (0.819, 1.3424), corresponding to a temperature range from $T=T_C$ to $T \ll T_C$.

In conclusion, two new quantities d_q and D_q are introduced to describe dynamical fluctuations in quark-hadron phase transitions. In the Ginzburg-Landau description for second-order quark-hadron phase transitions, d_q and D_q are investigated analytically, and it is found that D_q obeys a power law $D_q \propto D_2^{B_q}$, with $B_q=(q-1)^\gamma$. In experimental analysis, both d_q and D_q can be obtained quite easily. To get P_q one only needs to count the number of events with exact q hadrons in the bin. p_q is of Poisson type and can be calculated from the experimental $\overline{s}$. Simple algebras give d_q and D_q. The existence of dynamical fluctuations can be confirmed if d_q and D_q can take values very different from 1.0. The scaling between D_q and D_2 is a possible signal for the formation of QGP, because up to now no other dynamical reason is known to induce such a scaling. The value of the exponent γ can be used to measure the deviation of the temperature, at which the hadronization occurs, from the critical point. The study of D_q should be carried out in a real experimental analysis in the future to see whether QGP has been formed in current high-energy heavy-ion collisions. As a tool to study dynamical fluctuations d_q and D_q introduced in this paper may also be interesting in an experimental analysis of leptonic and hadronic interactions without quark-hadron phase transitions. The study of d_q and D_q in first-order quark-hadron phase transitions is in preparation.

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