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Spin content of the nucleon in a valence and sea quark mixing model

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A dynamical valence and sea quark mixing model is shown to fit the baryon ground state properties as well as the spin content of the nucleon. The relativistic correction and the $q^3 \leftrightarrow q^3 q\bar{q}$ transition terms induced by the quark axial vector current $\bar{\psi}\vec{\gamma}\gamma^5\psi$ in this model space is responsible for the quark spin reduction. [S0556-2813(98)50801-1]

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The naive valence quark model after incorporating QCD effective one gluon exchange and phenomenological confinement interactions is quite successful in explaining hadron properties [1] and is encouraging in describing hadron interactions [2]. Therefore it seems to be a good model of hadron internal structure especially for the nucleon. The EMC measurement [3] shows only a small amount of the nucleon spin is carried by the quark spin. This surprising result challenges our understanding of nucleon structure and has stimulated a new round of nucleon structure studies. The vast literature can be found from the invited talks given at recent conferences [4]. We only mention a few which are relevant to the present discussion. Jaffe and Lipkin [5] proposed a toy model with q^3 and $q^3 q\bar{q}$ mixing to accommodate the EMC result. Hwang, Speth, and Brown [6] used the generalized Sullivan processes with phenomenological meson-baryon coupling vertices to explain the spin-flavor structure of the nucleon. Cheng and Li [7] used the chiral quark model to remedy the failures of the naive quark model. Ma and Brodsky [8] emphasized the relativistic reduction of the quark spin contribution due to the Melosh rotation and included a small amount of the intrinsic sea quark component caused by the energetically favored meson-baryon fluctuations to explain the violation of Ellis-Jaffe sum rule and Gottfried sum rule. Close [9] reiterated that the polarization asymmetry in the valence region confirms the naive valence quark model predictions and one should focus on the sea quark polarization especially the small x behavior.

There have been various suggestions to include the gluon spin and the quark and gluon orbital angular momentum contributions in the nucleon spin. However as clarified by Ji [10] and ourselves [11], in the usual decomposition of the nucleon spin,

$$\tfrac{1}{2} = \tfrac{1}{2}\Delta\Sigma + \Delta G + L_q + L_G,$$

$$\Delta\Sigma = \left\langle p+ \left| \int d^3x \bar{\psi}\gamma^3\gamma^5\psi \right| p+ \right\rangle,$$

$$\Delta G = \left\langle p+ \left| \int d^3x (E^1A^2 - E^2A^1) \right| p+ \right\rangle,$$

$$L_q = \left\langle p+ \left| \int d^3x \frac{1}{i}\psi^\dagger (x^1\partial^2 - x^2\partial^1)\psi \right| p+ \right\rangle,$$

$$L_G = \left\langle p+ \left| \int d^3x E^i (x^1\partial^2 - x^2\partial^1) A^i \right| p+ \right\rangle, \tag{1}$$

the terms, except the $\Delta\Sigma$ term, are neither separately gauge invariant nor Lorentz invariant. The gauge invariance is obvious, and the Lorentz invariance can be expressed as

$$\Delta\Sigma s^\mu = \left\langle ps \left| \int d^3x \bar{\psi}\gamma^\mu\gamma^5\psi \right| ps \right\rangle. \tag{2}$$

The quark and gluon contribution to the nucleon spin can be decomposed in the gauge invariant formalism as

$$\vec{J} = \int d^3x \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i}\vec{D}\psi + \int d^3x \vec{x} \times (\vec{E}\times\vec{B}). \tag{3}$$

Here $\vec{D}$ is the covariant derivative, but $\vec{r}\times(1/i)\vec{D}$ does not obey the angular momentum commutation relation.

The third term is the gluon contribution, including both the gluon spin and orbital angular momentum, and it is impossible to decompose this term into individually gauge invariant gluon spin and orbital angular momentum parts.

Due to these uncertainties we will concentrate our discussion on the contribution from the quark axial vector current operator $\int d^3x \bar{\psi}\vec{\gamma}\gamma^5\psi$. In the parton model manifested at infinite momentum frame

$$\Delta\Sigma = \int dx [q^\uparrow(x) - q^\downarrow(x)], \tag{4}$$

where $q^{\uparrow,\downarrow}(x)$ is the probability of finding a quark or antiquark with fraction x of the proton longitudinal momentum and polarization parallel or antiparallel to the proton spin.

It is a quite intuitive impression from Eq. (4) that the counterpart of $\Delta\Sigma$ in the nonrelativistic constituent quark model is:

$$\Delta\Sigma^{NR} = \int d^3p [q^\uparrow(\vec{p}) - q^\downarrow(\vec{p})], \tag{5}$$

where $q^{\uparrow,\downarrow}(\vec{p})$ is the probability of finding a quark or antiquark of momentum $\vec{p}$ and polarization parallel or antiparallel to the proton spin in the nonrelativistic constituent quark model manifested at the proton rest frame.

This misidentifying of Eq. (5) is the root of the confusion related to the nucleon spin structure. We will show that Eq. (5) is true only for a static valence (q^3) quark model. For any realistic QCD inspired quark model, Eq. (5) is not true.

In the following discussion we will still use the term "quark spin contribution to the nucleon spin." In fact we are always talking about the matrix element of the quark axial vector current operator, which is the quantity measured in the deep inelastic scattering. To evaluate the axial vector current operator (2) in a nonrelativistic constituent quark model, we assume the quark field operator ψ can be directly related to the constituent quark degree of freedom. This is a usual assumption in such a model calculation, but needs to be studied further [12]. The next step is simple, but seems to be missed in a few model calculations [13]. The nonrelativistic reduction of the current operator includes not only the Pauli spin operator, but also a relativistic correction term

$$\int d^3x\,\overline{\psi}\vec{\gamma}\gamma^5\psi = \sum_{s's}\int d^3p\,\chi^\dagger_{s'}\left(\vec{\sigma}+\frac{\vec{\sigma}\cdot\vec{p}}{2E(E+m)}[\vec{\sigma},\vec{\sigma}\cdot\vec{p}]\right)\chi_s a^+_{ps'}a_{ps}. \tag{6}$$

This kind of relativistic reduction was discussed earlier in a pure phenomenological manner [14]. Applying this to the Isgur model [1], we have

$$\Delta\Sigma=\left(1-\frac{1}{3m^2b^2}\right)\sim 0.68. \tag{7}$$

Therefore the matrix element of the axial vector current operator, which is called the quark spin contribution in the literature, in a nonrelativistic model is not $\Delta\Sigma=1$, but around 0.70. This is due to quark Fermi motion in a confined region b. Only in the static $SU_6^{\sigma f}$ model, i.e., the case in which all the internal quark momenta $\vec{p}=0$, has one $\Delta\Sigma=1$. This result is similar to that of Ma and Brodsky [8] based on Melosh rotation and a light cone formalism.

The world average value of $\Delta\Sigma$ is [15]

$$\begin{aligned}\Delta\Sigma(Q^2\sim 3\ \mathrm{GeV}^2)&=\Delta u+\Delta d+\Delta s\\&=0.81(\pm 0.01)-0.44(\pm 0.01)\\&\quad-0.10(\pm 0.01)\\&=0.27(\pm 0.04).\end{aligned} \tag{8}$$

A possible contribution to the remaining difference ($\Delta\Sigma=0.68-0.27$) is the intrinsic sea quark component of the nucleon [5–9]. In the following we use a dynamical valence and sea quark mixing model [16] to study this problem.

In order to keep the successful part of the naive valence quark model, we assume a model Hamiltonian quite similar to that of the Isgur model [1]. However a new ingredient, the sea quark excitation interaction, is introduced in order to mix the $q^3q\bar{q}$ configuration with the q^3 valence part. Such a Hamiltonian should be written in a second quantized formalism, but we still use first quantization with an understanding that the one and two body operators include different particle numbers in different sub-Hilbert spaces:

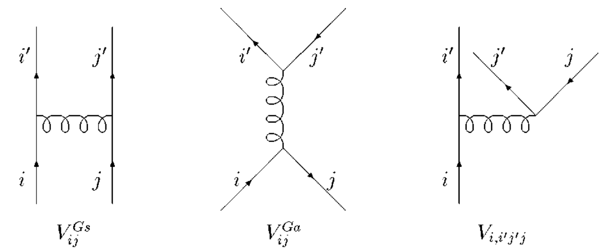

FIG. 1. Quark interaction diagrams.

$$\begin{aligned}H&=\sum_i\left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j}(V^c_{ij}+V^G_{ij})\\&\quad+\sum_{i<j}(V_{i,i'j'j}+V^\dagger_{i,i'j'j}),\\V^c_{ij}&=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r^2_{ij},\\V^{Gs}_{ij}&=\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{4\vec{\sigma}_i\cdot\vec{\sigma}_j}{3m_im_j}\right)\delta(\vec{r}_{ij})+\cdots\right],\\V^{Ga}_{ij}&=\alpha_s\left(\frac{\vec{\lambda}_i\cdot\vec{\lambda}^*_j}{2}\right)^2\left(\frac{1}{3}+\frac{\vec{f}_i\cdot\vec{f}^*_j}{2}\right)\\&\quad\times\left(\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{2}\right)^2\frac{2}{3}\frac{1}{(m_i+m_j)^2}\delta(\vec{r}_{ij}),\\V_{i,i'j'j}&=i\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\frac{1}{2r_{ij}}\left\{\left[\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\vec{\sigma}_j+\frac{i\vec{\sigma}_j\times\vec{\sigma}_i}{m_i}\right]\cdot\frac{\vec{r}_{ij}}{r^2_{ij}}\right.\\&\quad\left.-\frac{2\vec{\sigma}_j\cdot\vec{\nabla}_i}{m_i}\right\},\end{aligned} \tag{9}$$

where $\vec{\lambda}_i(\vec{f}_i)$ are the $SU_3^c(SU_3^f)$ Gellmann operators, the V^{Gs}_{ij}, V^{Ga}_{ij}, and $V_{i,i'j'j}$ correspond to the following diagrams of Fig. 1, respectively, the other symbols have their usual meaning.

Following the chiral quark model [17], the model Hilbert space is truncated to a subspace which includes all possible combinations of color singlet s-wave q^3 baryon states and ${}^1S_0\ q\bar{q}$ pseudoscalar meson states compatible with the quantum number of a baryon. The color, spin, flavor wave functions of the q^3 baryon core and the $q\bar{q}$ meson are the usual $SU_3^c\times SU_6^{\sigma f}$ ones. The internal orbital wave functions of q^3 and $q\bar{q}$ are assumed to be a Gaussian with a common size parameter b. The relative motion between q^3 baryon core and $q\bar{q}$ meson is assumed to be a p-wave to meet the posi-

TABLE I. Proton model wave function.

q^3	$N\eta$	$N\pi$	$\Delta\pi$	$N\eta'$	ΛK	ΣK	Σ^*K
−0.923	0.044	0.232	−0.252	0.065	0.109	−0.036	−0.106

TABLE II. Masses and magnetic moments of the baryon octect and decuplet. $m=330$ (MeV), $m_s=564$ (MeV), $b=0.61$ (fm), $\alpha_s=1.46$, $a_c=48.2$ (MeV fm^{-2}).

		p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Δ	Σ^*	Ξ^*	Ω
	M (MeV)	939		1116	1193		1346		1232	1370	1523	1659
Theor.	$E1$ (MeV)	2203		2323	2306		2409		2288	2306	2450	2638
	μ (μ_N)	2.780	−1.818	−0.522	2.652	−1.072	−1.300	−0.412				
	$\sqrt{\langle r^2\rangle}$ (fm)	0.802	0.124									
	M (MeV)	939		1116	1189		1315		1232	1385	1530	1672
Exp.	μ (μ_N)	2.793	−1.913	−0.613	2.458	−1.160	−1.250	−0.651				
	$\sqrt{\langle r^2\rangle}$ (fm)	0.836	0.34									

tive parity requirement of ground state baryons. For simplicity, it is assumed to be a p-wave Gaussian with the same b as that of the internal part. Essentially we use a shell model approximation, but the wave function of the center of mass is eliminated.

The model parameters, u,d quark mass m, s quark mass m_s, quark gluon coupling constant α_s, q^3 quark core baryon size b, and confinement strength a_c, are fixed by an overall fit to the ground state octet and decuplet baryon masses and the magnetic moments of the octet. The root mean square charge radius of proton is also fitted. A relativistic correction term (to the order of p^2/m^2) is included in the calculation of the nucleon charge radius.

Table I shows the wave function of the proton. The entry is the amplitude of the individual component. It is an example of our model wave functions of ground state baryons.

Table II summarizes our model predictions and the model parameters. These results show that it is possible to have a valence and sea quark mixing model which can describe, with the commonly accepted quark model parameters, the ground state octet and decuplet baryon properties as good as the successful naive valence quark model. Furthermore, the proton charge radius is reproduced as well. The first excited states are higher than 2 GeV. This is consistent with the fact that there is no pentaquark states observed below 2 GeV.

The spin structure of the proton is listed in Table III, where the matrix element of the axial vector current operator (2) in a spin up proton state is decomposed into particle number conserved components $q^3\leftrightarrow q^3, q^4\overline{q}\leftrightarrow q^4\overline{q}$ and particle number nonconserved components $q^3\leftrightarrow q^4\overline{q}$. The relativistic correction (6) has been taken into account in the calculation of the $q^3\leftrightarrow q^3$ matrix element. After antisymmetrization, it is impossible to separate the u,d valence and sea quark contribution of $q^3q\overline{q}$ components. Moreover in addition to the particle number conserved term (6), due to mixing of q^3 and $q^4\overline{q}$ components, the axial vector operator has a particle number nonconserved term between q^3 and $q^4\overline{q}$ components,

$$\int d^3x\,\overline{\psi}\vec{\gamma}\gamma^5\psi=\sum_{s,s'}\int d^3p\,\chi^\dagger_{s'}i\frac{\vec{\sigma}\times\vec{p}}{E}\chi_s a^+_{ps'}b^+_{-ps}, \tag{10}$$

where b^+_{-ps} is antiquark creation operator. This particle number nonconserved term (and its Hermitian conjugate) gives rise an additional contribution to the nucleon spin. It is this transition term which contributes negative Δq, which in turn reduces the $\Delta\Sigma$ of proton further. Physically, this transition term is similar to the generalized Sullivan processes which has been discussed in [6]. Adding these three contributions together, we obtain a spin distribution Δu, Δd, and Δs quite close to the world average result.

Our conclusion is that a nonrelativistic quark model with small amount of $q^3q\overline{q}$ component mixing is able to explain the $\Delta\Sigma(Q^2\sim 3\text{ GeV}^2)\sim 0.27$ measured in the deep inelastic scattering and at the same time keep a good fit to the baryon properties. The key point is to distinguish the quark spin sum which is 1 for a pure valence quark model from the matrix element of the quark axial vector current operator which is measured in the deep inelastic scattering. As for the nucleon spin, i.e., the total angular momentum of the nucleon, we should point out that it is still $\frac{1}{2}$ in our scheme. Because the content of quark orbital angular momentum in QCD is also different from that in nonrelativistic quark model, and if we make the nonrelativistic reduction of it, we will get relativistic correction terms as well. Simply speaking, these correction terms come from the small component of Dirac spinors. Furthermore, they are exactly the same, but with opposite sign as the correction terms from the quark axial vector current, therefore guarantee the nucleon spin to be $\frac{1}{2}$.

It should be mentioned that we have not adjusted the parameters very carefully for getting a perfect fit, since our aim is to show that the nucleon spin content measured in the deep inelastic scattering is understandable in a nonrelativistic quark model. Our model itself is a very rough one. Firstly, the q^3 and $q^3q\overline{q}$ mixing interaction is derived by an effective one gluon exchange, while the real interaction is quite likely to be nonperturbative. Secondly, in our model the pseudoscalar meson is approximated as a pure $q\overline{q}$ state and only the pseudoscalar meson is included in our truncated space, which is rather artificial. If the space is enlarged to include vector meson, we found that $N\omega, N\rho, \Delta\rho, \Lambda K^*$ components are mixed as strongly as

TABLE III. The spin contents of the proton.

	q^3	$q^3-q^4\overline{q}$	$q^4\overline{q}-q^4\overline{q}$	Sum	Exp.
Δu	0.773	−0.125	0.143	0.791	0.81
Δd	−0.193	−0.249	−0.043	−0.485	−0.44
Δs	0	−0.064	−0.002	−0.066	−0.10

the pseudoscalar ones and the fit is not better, but even worse. Another point worth mentioning is that the shell model approximation of the orbital wave function is questionable. In fact it should be a meson baryon continuum. The relativistic correction is also questionable quantitatively, since in our model the p/m is not small. Certainly much work should be done in the future.

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