

## Chemical potential dependence of $\pi$ and $\rho$ properties

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(Received 28 January 1998)

Using a confining, Dyson-Schwinger equation model of QCD at finite temperature ( $T$ ) and chemical potential ( $\mu$ ), we study the behavior of  $\langle \bar{q}q \rangle$ ,  $m_\pi$ ,  $f_\pi$  and the masses of the  $\rho$ - and  $\phi$ -mesons. For each of these quantities there is a necessary anticorrelation between its response to  $T$  and its response to  $\mu$ . ( $-\langle \bar{q}q \rangle$ ) and  $f_\pi$  decrease with  $T$  and increase with  $\mu$ ;  $m_\pi$  is almost insensitive to  $T$  and  $\mu$  until very near the border of the confinement domain; the mass of the longitudinal component of the vector mesons increases with  $T$  and decreases with  $\mu$ . At  $T=0$ , the  $\rho$ -meson mass is reduced by approximately 15% at nuclear matter density. These results are a consequence of the necessary momentum-dependence of the dressed-quark self-energy. [S0556-2813(98)51006-0]

PACS number(s): 11.10.Wx, 12.38.Mh, 24.85.+p, 11.10.St

High-energy heavy-ion ( $A$ - $A$ ) experiments, which produce systems with large baryon density, are an important preliminary step in the search for a quark-gluon plasma. An outcome of these experiments is the observation that the dilepton yield in the region below the  $\rho$ -resonance is approximately five-times greater than that seen in proton induced ( $p$ - $A$ ) reactions [1]. One model calculation [2] shows that this enhancement can be explained by a medium-induced reduction of the  $\rho$ -meson's mass, another [3] that it follows from an increase in the  $\rho$ -meson's width. A decrease in the  $\rho$ -meson's mass is consistent with the QCD sum rules analysis of Ref. [4], but inconsistent with that of Ref. [5], which employs a more complex phenomenological model for the in-medium spectral density used in matching the two sides of the sum rule. In Ref. [5] there is no shift in the  $\rho$ -meson mass, but a significant increase in its width. The consistency between Refs. [3] and [5] is not surprising, since, in contrast to Ref. [4], they both rely heavily on effective Lagrangians with elementary hadron degrees-of-freedom. Herein, using a simple, Dyson-Schwinger equation (DSE) model of QCD, we reconsider the response of the  $\rho$ -meson's mass to increasing baryon density. In focusing on dressed-quark and -gluon degrees of freedom, our study has similarities to that of Ref. [4].

DSE's provide a nonperturbative, continuum framework for analyzing quantum field theories. While they have been used extensively at  $T=0=\mu$  in the study of dynamical chiral symmetry breaking (DCSB) and confinement [6], and the calculation of hadronic observables [7], their application at finite  $T$  and  $\mu$ , though straightforward, is in its infancy.

We begin with the specification of a model dressed-gluon propagator

$$g^2 D_{\mu\nu}(\vec{p}, \Omega_k) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{|\vec{p}|^2 + \Omega_k^2} \right) 2\pi^3 \frac{\eta^2}{T} \delta_{k0} \delta^3(\vec{p}), \quad (1)$$

where  $(p_\mu) := (\vec{p}, \Omega_k)$ ,  $\Omega_k = 2k\pi T$  is the boson Matsubara frequency and  $\eta$  is a mass-scale parameter. Equation (1) is an extension to finite- $T$  of the model for  $D_{\mu\nu}$  introduced in Ref. [8]. It has the feature that the infrared enhancement of

the dressed-gluon propagator suggested by the DSE studies of Refs. [9] is manifest. However, it represents poorly the behavior of  $D_{\mu\nu}(\vec{p}, \Omega_k)$  away from  $|\vec{p}|^2 + \Omega_k^2 \approx 0$ . Nevertheless, this limitation leads only to easily identifiable artefacts and hence does not preclude the judicious use of the model. Another feature of Eq. (1) is that it specifies a model with the merit of simplicity: it provides for the reduction of integral DSEs [e.g., the gap equation and Bethe-Salpeter equations (BSE's)] to algebraic equations, which facilitates the elucidation of many of the qualitative features of more sophisticated models.

Using Eq. (1) and the rainbow-truncation for the dressed-quark-gluon vertex:  $\Gamma_\mu^a(k, p) = \frac{1}{2} \lambda^a \gamma_\mu$ , the DSE for the dressed-quark propagator in our model for QCD at finite- $T$  and  $\mu$  (QCD $_T^\mu$ ) is [10]

$$S^{-1}(\vec{p}, \omega_k) = S_0^{-1}(\vec{p}, \omega_k) + \frac{1}{4} \eta^2 \gamma_\nu S(\vec{p}, \omega_k) \gamma_\nu, \quad (2)$$

where  $S_0^{-1}(\vec{p}, \omega_k) := i\vec{\gamma} \cdot \vec{p} + i\gamma_4 \omega_{k+} + m$ , with  $\omega_{k+} = \omega_k + i\mu$ ,  $\omega_k = (2k+1)\pi T$  is the fermion Matsubara frequency and  $\mu$  is the chemical potential, and  $m$  is the current-quark mass. (In our Euclidean formulation,  $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$  with  $\gamma_\mu^\dagger = \gamma_\mu$ .) The solution of Eq. (2) has the general form

$$S(\vec{p}_k) = \frac{1}{i\vec{\gamma} \cdot \vec{p} A(\vec{p}_k) + i\gamma_4 \omega_{k+} C(\vec{p}_k) + B(\vec{p}_k)}, \quad (3)$$

$$= -i\vec{\gamma} \cdot \vec{p} \sigma_A(\vec{p}_k) - i\gamma_4 \omega_{k+} \sigma_C(\vec{p}_k) + \sigma_B(\vec{p}_k), \quad (4)$$

where  $\vec{p}_k := (\vec{p}, \omega_{k+})$ , and Eq. (2) entails that the scalar functions introduced in Eq. (3) satisfy

$$\eta^2 m^2 = B^4 + mB^3 + (4\vec{p}_k^2 - \eta^2 - m^2)B^2 - m(2\eta^2 + m^2 + 4\vec{p}_k^2)B, \quad (5)$$

$$A(\vec{p}_k) = C(\vec{p}_k) = \frac{2B(\vec{p}_k)}{m + B(\vec{p}_k)}. \quad (6)$$

In the chiral limit ( $m=0$ ) the character of the solution of Eq. (2) is transparent. There are two qualitatively distinct solutions. The ‘‘Nambu-Goldstone’’ solution, for which

$$B_0(\vec{p}_k) = \begin{cases} \sqrt{\eta^2 - 4\vec{p}_k^2}, & \text{Re}(\vec{p}_k^2) < \frac{\eta^2}{4}, \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

$$C_0(\vec{p}_k) = \begin{cases} 2, & \text{Re}(\vec{p}_k^2) < \frac{\eta^2}{4}, \\ \frac{1}{2}(1 + \sqrt{1 + 2\eta^2/\vec{p}_k^2}), & \text{otherwise,} \end{cases} \quad (8)$$

describes a phase of this model in which: (1) chiral symmetry is dynamically broken because one has a nonzero quark mass-function;  $B(\vec{p}_k)$ , in the absence of a current-quark mass, and (2) the dressed-quarks are confined because the propagator prescribed by these functions does not have a Lehmann representation. The alternative ‘‘Wigner’’ solution, for which

$$\hat{B}_0(\vec{p}_k) \equiv 0, \quad \hat{C}_0(\vec{p}_k) = \frac{1}{2}(1 + \sqrt{1 + 2\eta^2/\vec{p}_k^2}), \quad (9)$$

describes a phase of the model in which chiral symmetry is not broken and the dressed-quarks are not confined.

With these two phases, characterized by qualitatively different, momentum-dependent modifications of the quark propagator, this model of  $\text{QCD}_T^\mu$  can be used to explore simultaneously both chiral symmetry restoration and deconfinement. That and the calculation of its equilibrium thermodynamic properties are the subject of Ref. [11]. The model exhibits coincident, first-order deconfinement and chiral symmetry restoration for all  $\mu \neq 0$ , but the coincident transitions are second order for  $\mu=0$ . The extreme points on the phase boundary are ( $T=0$ ,  $\mu \approx 0.28\eta$ ) and ( $T \approx 0.16\eta$ ,  $\mu=0$ ).

The vacuum quark condensate is proportional to the matrix trace of the chiral-limit dressed-quark propagator. Using Eqs. (3), (7), and (8), we obtain the following expression, valid in the domain of confinement and DCSB:

$$-\langle \bar{q}q \rangle = \eta^3 \frac{8N_c}{\pi^2} \bar{T} \sum_{l=0}^{l_{\max}} \int_0^{\bar{\Lambda}_l} dy y^2 \text{Re}(\sqrt{\frac{1}{4} - y^2 - \bar{\omega}_{l+}^2}), \quad (10)$$

with:  $\bar{T} = T/\eta$ ,  $\bar{\mu} = \mu/\eta$ ,  $\bar{\omega}_{l+} = \omega_{l+}/\eta$ , and  $\bar{\omega}_{l_{\max}}^2 \leq \frac{1}{4} + \bar{\mu}^2$ ,  $\bar{\Lambda}_l^2 = \frac{1}{4} + \bar{\mu}^2 - \bar{\omega}_l^2$ . For  $T=0 = \mu$ ,  $(-\langle \bar{q}q \rangle) = \eta^3/(80\pi^2) = (0.11\eta)^3$ . In Fig. 1 we see that  $(-\langle \bar{q}q \rangle)$  decreases with  $T$ , but *increases* with increasing  $\mu$ , up to a critical value of  $\mu_c(T)$ , when it drops discontinuously to zero.<sup>1</sup> The increase of  $(-\langle \bar{q}q \rangle)$  with  $\mu$  must be expected in the confinement domain because confinement entails that each additional

quark must be locally paired with an antiquark, thereby increasing the density of condensate pairs. This vacuum rearrangement is manifest in the behavior of the necessarily momentum-dependent scalar part of the quark self-energy,  $B(\vec{p}_k)$ .

Our primary interests are the bound state properties of  $\pi$ - and  $\rho$ -mesons. The  $\pi$  has been much studied and it follows [15] from the axial-vector Ward-Takahashi identity that, writing the  $\pi$  Bethe-Salpeter amplitude as

$$\Gamma_\pi(p; P) = i\gamma_5 B_0(p^2), \quad (11)$$

one can obtain a reliable approximation in the calculation of those  $\pi$  observables for which the dominant, intrinsic momentum scale is less than  $10 \text{ GeV}^2$ . Using Eq. (11) then [12,13] in the domain of confinement and DCSB

$$f_\pi^2 m_\pi^2 = \langle m\bar{q}q \rangle_\pi, \quad (12)$$

$$\begin{aligned} \langle m\bar{q}q \rangle_\pi &= \eta^4 \frac{8N_c}{\pi^2} \bar{T} \sum_{l=0}^{l_{\max}} \int_0^{\bar{\Lambda}_l} dy y^2 \\ &\times \text{Re}\{\bar{B}_0(\bar{\sigma}_{B_0} - \bar{B}_0[\bar{\omega}_{l+}^2 \bar{\sigma}_C^2 + y^2 \bar{\sigma}_A^2 + \bar{\sigma}_B^2])\}, \end{aligned} \quad (13)$$

where  $|\vec{p}| = \eta y$  and  $B_0(\vec{p}_l) := \eta \bar{B}_0(\eta y, \eta \bar{\omega}_l)$ , etc. The right-hand side of Eq. (13) is zero for  $m=0$  and increases linearly with  $m$ , for small- $m$ . In Eq. (12) the canonical normalization constant for the  $\pi$  Bethe-Salpeter amplitude is

$$\begin{aligned} f_\pi^2 &= \eta^2 \frac{2N_c}{\pi^2} \bar{T} \sum_{l=0}^{l_{\max}} \int_0^{\bar{\Lambda}_l} dy y^2 \\ &\times \text{Re}\{\bar{B}_0^2(\bar{\sigma}_A^2 - 2[\bar{\omega}_{l+}^2 \bar{\sigma}_C \bar{\sigma}'_C + y^2 \bar{\sigma}_A \bar{\sigma}'_A + \bar{\sigma}_B \bar{\sigma}'_B] \\ &- \frac{4}{3} y^2 \{[\bar{\omega}_{l+}^2 (\bar{\sigma}_C \bar{\sigma}''_C - (\bar{\sigma}'_C)^2) + y^2 (\bar{\sigma}_A \bar{\sigma}''_A - (\bar{\sigma}'_A)^2) \\ &+ \bar{\sigma}_B \bar{\sigma}''_B - (\bar{\sigma}'_B)^2]\}), \end{aligned} \quad (14)$$

with  $\bar{\sigma}' := \partial \sigma(y, \bar{\omega}_{l+})/\partial y$ , which provides a quantitatively accurate approximation to the leptonic decay constant.<sup>2</sup>

In the chiral limit, we have from Eqs. (6)–(8) that

$$\bar{\sigma}_{B_0} = B_0, \quad \bar{\sigma}'_{B_0} = -\frac{2}{B_0}, \quad \bar{\sigma}''_{B_0} = -\frac{4}{B_0^3}, \quad (15)$$

$\bar{\sigma}_C = C = \bar{\sigma}_A$  and  $\bar{\sigma}'_C = 0 = \bar{\sigma}'_A$ . The simplicity of the model is again manifest in these identities, which yield

$$f_\pi^2 = \eta^2 \frac{16N_c}{\pi^2} \bar{T} \sum_{l=0}^{l_{\max}} \frac{\bar{\Lambda}_l^3}{3} (1 + 4\bar{\mu}^2 - 4\bar{\omega}_l^2 - \frac{8}{5}\bar{\Lambda}_l^2). \quad (16)$$

<sup>1</sup>These results are in qualitative and semiquantitative agreement with ( $T=0$ ,  $\mu \neq 0$ ) [12] and ( $T \neq 0$ ,  $\mu=0$ ) [13] studies of a more sophisticated model that better represents the behavior of  $D_{\mu\nu}$  in the ultraviolet. The  $\mu$ -dependence is also qualitatively identical to that observed in a random matrix theory with the global symmetries of the QCD partition function [14].

<sup>2</sup>The relation between the normalization of the  $\pi$  Bethe-Salpeter amplitude and the leptonic decay constant is discussed in Ref. [15]. The demonstration [16] that Eq. (14) provides an accurate estimate of the  $\pi$  decay constant when using Eq. (11) is an antecedent to Ref. [15].

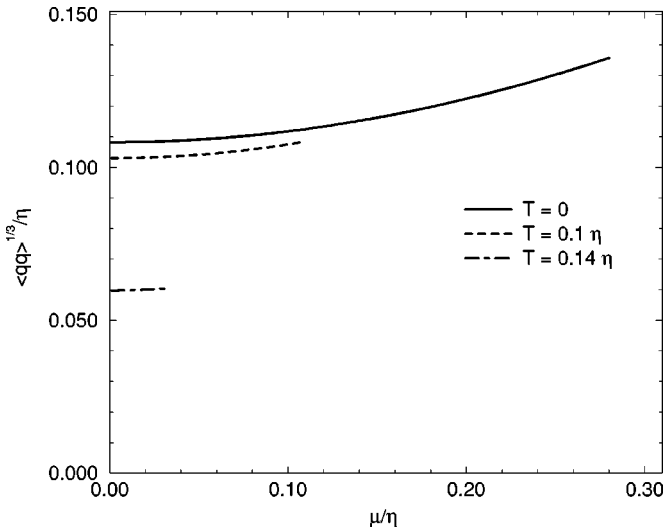


FIG. 1. The quark condensate, Eq. (10), as a function of  $\mu$  for a range of values of  $T$ . In all existing studies, in which the quark mass function has a realistic momentum dependence, it increases with  $\mu$  and decreases with  $T$ . At the critical chemical potential,  $\mu_c(T)$ ,  $(-\langle \bar{q}q \rangle)$  drops discontinuously to zero, as expected of a first-order transition. For  $\mu=0$  it falls continuously to zero, exhibiting a second-order transition at  $T_c(\mu=0)=0.16\eta$ .

Characteristic in Eq. (16) is the combination  $\mu^2 - \omega_l^2$ , which entails that, whatever change  $f_\pi$  undergoes as  $T$  is increased, the *opposite* occurs as  $\mu$  is increased. Without calculation, Eq. (16) indicates that  $f_\pi$  will *decrease* with  $T$  and *increase* with  $\mu$ , and this provides a simple elucidation of the calculated results in Refs. [12, 13]. Figure 2 illustrates this behavior for  $m \neq 0$ .

In Fig. 2 we also plot  $m_\pi$ , from Eq. (12). It is *insensitive* to changes in  $\mu$  and only increases slowly with  $T$ , until  $T$  is very near the critical temperature. This insensitivity is the result of mutually canceling increases in  $\langle m\bar{q}q \rangle_\pi$  and  $f_\pi$ , and is a feature of studies that preserve the momentum-dependence of the confined, dressed-quark degrees of freedom in meson bound states.

With  $\eta=1.37$  GeV and  $m=30$  MeV, one obtains  $f_\pi=92$  MeV and  $m_\pi=140$  MeV at  $T=0=\mu$ . That large values of  $\eta$  and  $m$  are required is a quantitative consequence of the inadequacy of Eq. (1) in the ultraviolet: the large- $p^2$  behavior of the scalar part of the dressed-quark self-energy is incorrect. This defect is remedied easily [15] without qualitative changes to the results presented here [17].

With the vector Ward identity unable to assist with a significant simplification of the bound state problem,  $\rho$ -meson properties are more difficult to study: one must solve directly the vector-meson Bethe-Salpeter equation. The ladder truncation of the kernel in the inhomogeneous axial-vector vertex equation and the rainbow truncation of the quark DSE form an axial-vector Ward-Takahashi identity preserving pair [10]. It follows that the ladder BSE is accurate for flavor-nonsinglet pseudoscalar and vector bound states of equal-mass quarks because of a cancellation in these channels between diagrams of higher order in the skeleton expansion of which this pair of truncations is the lowest order term.

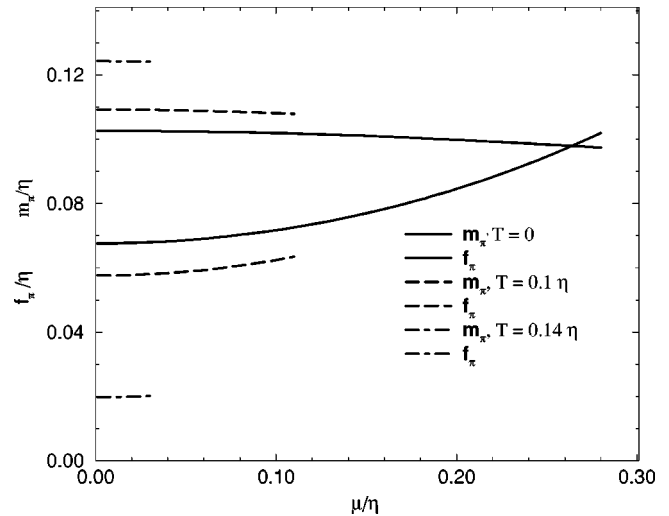


FIG. 2. The pion mass, Eq. (12), and weak decay constant, Eq. (14), as a function of  $\mu$  for a range of values of  $T$ .  $m_\pi$  falls slowly and uniformly with  $\mu$  [ $m_\pi(T=0, \mu_c)=0.95m_\pi(T=0, \mu=0)$ ], but increases with  $T$ . Such a decrease is imperceptible if the ordinate has the range in Fig. 3.  $f_\pi$  increases with  $\mu$  and decreases with  $T$  [ $f_\pi(T=0, \mu_c)=1.51f_\pi(T=0, \mu=0)$ ].

A ladder BSE using the  $T=0$  limit of Eq. (1) was introduced in Ref. [8]. It has one notable pathology: the bound state mass is determined only upon the additional specification that the constituents have zero relative momentum. This specification leads to a conflict with the axial-vector Ward-Takahashi identity, which relates the momentum dependence of the  $\pi$  Bethe-Salpeter amplitude to the functions in the quark propagator [15]. We find this to be an artifact of implementing the delta-function limit discontinuously; i.e., the identities [15] between the scalar functions in the  $\pi$  Bethe-Salpeter amplitude, and  $A(p^2)$  and  $B(p^2)$  are manifest for any finite-width representation of the delta-function, as this width is reduced continuously to zero. In other respects this ladder BSE provides a useful qualitative and semiquantitative tool for analyzing features of the pseudoscalar and vector meson masses. For example, Goldstone's theorem is manifest, in that the  $\pi$  is massless in the chiral limit, and also  $m_\pi^2$  rises linearly with the current-quark mass. Further, there is a naturally large splitting between  $m_\pi$  and  $m_\rho$ , which decreases slowly with the current-quark mass.

To illustrate this and determine the response of  $m_\rho$  to increasing  $T$  and  $\mu$ , we generalize the BSE of Ref. [8] to finite- $(T, \mu)$  as

$$\begin{aligned} \Gamma_M(\vec{p}_k; \check{P}_\rho) &= -\frac{\eta^2}{4} \text{Re}\{\gamma_\mu S(\vec{p}_k + \frac{1}{2}\check{P}_\rho) \Gamma_M(\vec{p}_k; \check{P}_\rho) S(\vec{p}_k - \frac{1}{2}\check{P}_\rho) \gamma_\mu\}, \\ & \end{aligned} \quad (17)$$

where  $\check{P}_\rho := (\vec{P}, \Omega_\rho)$ . The bound state mass is obtained by considering  $\check{P}_{\rho=0}$  and, in ladder truncation, the  $\rho$ - and  $\omega$ -mesons are degenerate.

As a test we first consider the  $\pi$  equation, which admits the solution

$$\Gamma_\pi(\vec{P}_0) = \gamma_5(i\theta_1 + \vec{\gamma} \cdot \vec{P}\theta_2) \quad (18)$$

and yields the mass plotted in Fig. 3. The mass behaves in qualitatively the same manner as  $m_\pi$  in Fig. 2, from Eq. (12), as required if Eq. (17) is to provide a reliable guide. In particular, it vanishes in the chiral limit.

In the case of the  $\rho$ -meson there are two components: longitudinal and transverse to  $\vec{P}$ . The BSE has a solution of the form

$$\Gamma_\rho = \begin{cases} \gamma_4 \theta_{\rho+} \\ \left( \vec{\gamma} - \frac{1}{|\vec{P}|^2} \vec{P} \vec{\gamma} \cdot \vec{P} \right) \theta_{\rho-} \end{cases}, \quad (19)$$

where  $\theta_{\rho+}$  labels the longitudinal and  $\theta_{\rho-}$  the transverse solution. The eigenvalue equation obtained from Eq. (17) for the bound state mass,  $M_{\rho\pm}$ , is

$$\frac{\eta^2}{2} \operatorname{Re}\{\sigma_S(\omega_{0+}^2 - \frac{1}{4}M_{\rho\pm}^2)^2 - [\pm\omega_{0+}^2 - \frac{1}{4}M_{\rho\pm}^2]\sigma_V(\omega_{0+}^2 - \frac{1}{4}M_{\rho\pm}^2)^2\} = 1. \quad (20)$$

The equation for the transverse component is obtained with  $[-\omega_{0+}^2 - \frac{1}{4}M_{\rho-}^2]$  in Eq. (20). Using the chiral-limit identities, Eq. (15), one obtains immediately that

$$M_{\rho-}^2 = \frac{1}{2}\eta^2, \text{ independent of } T \text{ and } \mu. \quad (21)$$

This is the  $T=0=\mu$  result of Ref. [8]. Even for nonzero current-quark mass,  $M_{\rho-}$  changes by less than 1% as  $T$  and  $\mu$  are increased from zero toward their critical values. Its insensitivity is consistent with the absence of a constant mass-shift in the transverse polarization tensor for a gauge-boson.

For the longitudinal component, one obtains in the chiral limit:

$$M_{\rho+}^2 = \frac{1}{2}\eta^2 - 4(\mu^2 - \pi^2 T^2). \quad (22)$$

The characteristic combination  $[\mu^2 - \pi^2 T^2]$  again indicates the anticorrelation between the response of  $M_{\rho+}$  to  $T$  and its response to  $\mu$ , and, like a gauge-boson Debye mass, that  $M_{\rho+}^2$  rises linearly with  $T^2$  for  $\mu=0$ . The  $m \neq 0$  solution of Eq. (20) for the longitudinal component is plotted in Fig. 3. As signaled by Eq. (22),  $M_{\rho+}$  increases with increasing  $T$  and decreases as  $\mu$  increases.<sup>3</sup>

We have stated that contributions from skeleton diagrams not included in the ladder truncation of the vector meson BSE do not alter the calculated mass significantly because of cancellations between these higher order terms [10]. This is illustrated explicitly in two calculations: Ref. [18], which shows that the  $\rho \rightarrow \pi\pi \rightarrow \rho$  contribution to the real part of the  $\rho$  self-energy; i.e., the  $\pi$ - $\pi$  induced mass-shift, is only  $-3\%$ ; and Ref. [19], which shows, for example, that the contribution to the  $\omega$ -meson mass of the  $\omega \rightarrow 3\pi$ -loop is negligible. Therefore, ignoring such contributions does not introduce uncertainty into estimates of the vector meson mass based on Eq. (17).

<sup>3</sup>There is a 25% difference between the value of  $\eta$  required to obtain the  $T=0=\mu$  values of  $m_\pi$  and  $f_\pi$ , from Eqs. (13) and (14), and that required to give  $M_{\rho\pm} = 0.77$  GeV. This is a measure of the quantitative accuracy of our algebraic model.

Equation (20) can also be applied to the  $\phi$ -meson. The transverse component is insensitive to  $T$  and  $\mu$ , and the behavior of the longitudinal mass,  $M_{\phi+}$ , is qualitatively the same as that of the  $\rho$ -meson: it increases with  $T$  and decreases with  $\mu$ . Using  $\eta = 1.06$  GeV, the model yields  $M_{\phi\pm} = 1.02$  GeV for  $m_s = 180$  MeV at  $T=0=\mu$ .

In a 2-flavor, free-quark gas at  $T=0$ , the baryon number density is  $\rho_B = 2\mu^3/(3\pi^2)$ , by which gauge nuclear matter density,  $\rho_0 = 0.16$  fm<sup>-3</sup>, corresponds to  $\mu = \mu_0 := 260$  MeV  $= 0.245\eta$ . At this chemical potential our model yields

$$M_{\rho+}(\mu_0) \approx 0.75M_{\rho+}(\mu=0), \quad (23)$$

$$M_{\phi+}(\mu_0) \approx 0.85M_{\phi+}(\mu=0). \quad (24)$$

The study of Ref. [12] indicates that a better representation of the ultraviolet behavior of the dressed-gluon propagator expands the horizontal scale in Fig. 3, with the critical chemical potential increased by 25%. Based on this we judge that a more realistic estimate is obtained by evaluating the mass at  $\mu'_0 = 0.20\eta$ , which yields

$$M_{\rho+}(\mu'_0) \approx 0.85M_{\rho+}(\mu=0), \quad (25)$$

$$M_{\phi+}(\mu'_0) \approx 0.90M_{\phi+}(\mu=0), \quad (26)$$

a small, quantitative modification. The difference between Eqs. (23) and (25), and that between Eqs. (24) and (26), is a measure of the theoretical uncertainty in our estimates in each case. This reduction in the vector meson masses is quantitatively consistent with that calculated in Ref. [4] and conjectured in Ref. [20]. At the critical chemical potential for  $T=0$ ,  $M_{\rho+} \approx 0.65M_{\rho+}(\mu=0)$  and  $M_{\phi+} \approx 0.80M_{\phi+}(\mu=0)$ .

We have analyzed a simple, DSE model of QCD<sub>T</sub><sup>μ</sup> that preserves the momentum-dependence of gluon and quark dressing, which is an important qualitative feature of more sophisticated studies. The simplicity of the model means that many of its consequences can be demonstrated algebraically. For example, it elucidates the origin of an anticorrelation, found for a range of quantities, between their response to increasing  $T$  and that to increasing  $\mu$ , discovered in contrasting the studies in Refs. [12] and [13].

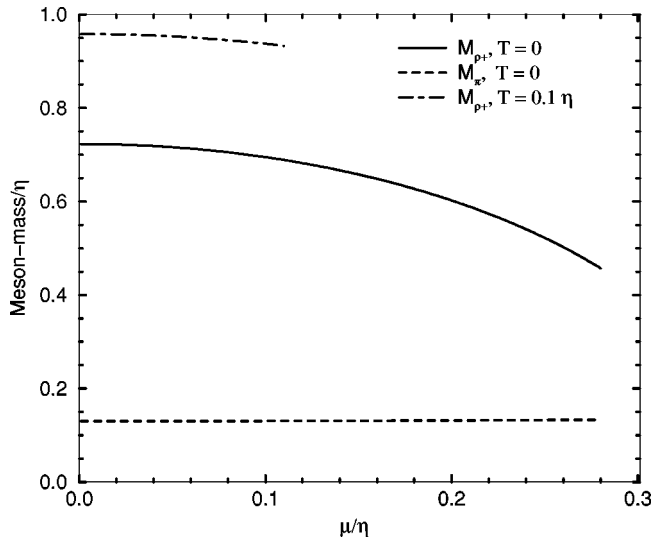


FIG. 3.  $M_{\rho^+}$  and  $m_\pi$  as a function of  $\bar{\mu}$  for  $\bar{T}=0,0.1$ . On the scale of this figure,  $m_\pi$  is insensitive to this variation of  $T$ . The current-quark mass is  $m=0.011\eta$ , which for  $\eta=1.06$  GeV yields  $M_{\rho^+}=770$  MeV and  $m_\pi=140$  MeV at  $T=0=\mu$ .

We find that both  $(-\langle\bar{q}q\rangle)$  and  $f_\pi$  decrease with  $T$  and increase with  $\mu$ , and this ensures that  $m_\pi$  is insensitive to increasing  $\mu$  and/or  $T$  until very near the edge of the domain of confinement and DCSB. The mass of the transverse component of the vector meson is insensitive to  $T$  and  $\mu$ , while the mass of the longitudinal component increases with in-

creasing  $T$ , but decreases with increasing  $\mu$ . This behavior is opposite to that observed for  $(-\langle\bar{q}q\rangle)$  and  $f_\pi$ , and hence the scaling law conjectured in Ref. [20] is inconsistent with our calculation, as it is with others of this type.

Our study has two primary limitations. First, we cannot calculate the width of the vector mesons in this model because the solution of Eq. (17) does not provide a realistic Bethe-Salpeter amplitude. We are currently working to overcome this limitation. Second, the reliable calculation of meson-photon observables at  $T=0=\mu$  only became possible with the determination [21] of the form of the dressed-quark-photon vertex. The generalization of this vertex to nonzero  $T$  and  $\mu$  is a necessary precursor to the study of these processes at  $T\neq 0\neq\mu$ .

We acknowledge useful conversations with D. Blaschke, Yu. Kalinovsky, and G. Poulis. For their hospitality and support during visits in which some of this work was conducted: P.M. and C.D.R. gratefully acknowledge the Department of Physics at the University of Rostock; S.S., the Physics Division at Argonne National Laboratory; and C.D.R. and S.S., the Bogoliubov Laboratory for Theoretical Physics and the Laboratory of Computing Techniques and Automation at the Joint Institute for Nuclear Research. This work was supported in part by the US Department of Energy, Nuclear Physics Division, under Contract No. W-31-109-ENG-38, the National Science Foundation under Grant No. INT-9603385, Deutscher Akademischer Austauschdienst, and benefited from the resources of the National Energy Research Scientific Computing Center.

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