PHYSICAL REVIEW C VOLUME 57, NUMBER 5 MAY 1998

Alternative view of collective bands in actinide nuclei

B. Buck and A. C. Merchant
Department of Physics, University of Oxford, Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, United Kingdom

S. M. Perez
Department of Physics, University of Cape Town, Private Bag, Rondebosch 7700, South Africa

(Received 24 July 1997)

A binary cluster model reproduces the quasirotational excitation energies and enhanced $E2$ transition rates of the low-lying 0^+ and 0^- bands in actinide nuclei. The characteristic $J^\pi = 0^+, 2^+, 4^+, \ldots$ and $1^-, 3^-, 5^-, \ldots$ patterns of the bands emerge naturally without invoking *ad hoc* symmetry arguments. The calculations yield very similar radial wave functions for levels of a given band up to large J^π, implying a common intrinsic state for each band. The model can then be extended to reproduce further phenomena more usually associated with the deformed rotor picture. [S0556-2813(98)50105-7]

PACS number(s): 21.60.Ev, 21.60.Gx, 23.20.Js, 27.90.+b

The idea of a permanently deformed rotating nucleus has long been the standard model for explaining the occurrence of the striking spectral band patterns observed in several parts of the periodic table [1]. Each band is supposed to possess a common underlying intrinsic state and the levels correspond to rotations of the system. But the model needs to be supplemented by *ad hoc* postulates about symmetries of the nucleus before a satisfactory account of the spectra emerges. In particular, the ubiquitous 0^+ bands with $J^\pi = 0^+, 2^+, 4^+, \ldots$ are assumed to arise from a nuclear shape with axial and equatorial symmetry in order to eliminate the unwanted odd-J states, and similarly for the 0^- bands with missing even-J's.

Once such assumptions are granted the model enables coherent treatment of numerous phenomena, e.g., a generation of higher bands in terms of beta and gamma vibrations coupled to the rotations, the Nilsson model for particles in deformed wells, band mixing, decoupling effects and details of gamma decays, and Coulomb excitation. A drawback is that it has always been difficult to account for the observed energy spacings in the bands even when they are clearly of a rotational nature given by $E(J) = E(0) + J(J+1)/2I$. That is, there is no generally accepted dynamical theory for the moment of inertia I, which is observed to depart markedly from both rigid body and fluid body values. Also, it appears experimentally that effective values of I change with J in many nuclei, sometimes leading to dramatic backbending effects in the band spectra. So there is a need for a theory which gives a dynamical reason for the observed spectra while preserving the basic ideas of the rotor model.

Brink *et al.* [2] have shown that the underlying assumptions of the standard model can be reduced just to the postulation of core nuclei with some assigned intrinsic state upon which are built degenerate states with spins $0^+, 2^+, 4^+, \ldots$ (and/or $1^-, 3^-, 5^-, \ldots$). The state functions for a particle interacting with the core bands emerge naturally with appropriate K values and are in the usual strong-coupling form independently of the assumed potentials. Introduction of a kinetic energy operator $L^2/2I$ then leads to a quantum derivation of the cranking formulas. The argument easily generalizes to describe the coupling of excited core states to the assumed bands so as to produce many extra band structures.

Although the above treatment required no symmetry arguments it left open the question of how the fundamental core bands arise in the first place. Here we discuss a simple model which furnishes the missing dynamics by producing the correct spin sequences, energy values, and transition strengths in the basic bands while also yielding essentially a common intrinsic state for each band. The results are cleanest in the trans-Pb region of deformed nuclei and we confine our remarks to that neighborhood.

We have recently shown that the lowest-lying 0^+ and 0^- bands of nuclei in the actinide region can be well reproduced by a binary cluster-core model [3,4]. Our original motivation for this was to study exotic decay modes from ground states of such nuclei, but we were surprised to discover that good results for half-lives could be obtained by taking the emitted neutron rich ions to have large preformation factors in the parent nuclei [5]. This suggested that the model be taken seriously for other properties. Specifically, we have applied the model to ^{222}Ra, ^{228}Th, ^{232}U, and ^{236}Pu, assuming a common ^{208}Pb core for each nucleus [3], to ^{222,224,226}Ra, assuming a common ^{14}C cluster for each isotope [4], and to ^{232,234,236,238}U using a common ^{24}Ne cluster for each isotope [6]. Most recently we have given a similarly successful account of 23 isotopes of Rn, Ra, Th, U, Pu and Cm [7]. A particularly striking result of these studies is that the observed jumps in $B(E2)$ strengths with increasing atomic number follow simply from the increasing cluster charges [3,7,8].

In order to avoid excessive detail we now discuss our ideas by making particular reference to the 0^+ ground state band of ^{234}U for which there are extensive data on level energies and $E2$ transitions [9,10]. We have previously modelled the actinide ^{232}U in terms of the binary decomposition ^{208}Pb+^{24}Ne, as suggested by the exotic decay mode of this nucleus together with the systematics of $B(E2\downarrow: 2^+ \rightarrow 0^+)$ values in this region. The exotic decay of ^{234}U suggests the decomposition ^{210}Pb+^{24}Ne, or possibly the combination ^{206}Hg+^{28}Mg. The former is preferred because of the simi-

 © 1998 The American Physical Society

larity of its $B(E2)$ value of 238 ± 10 W.u. to that of ^{232}U, which is 241 ± 21 W.u. [10,11]. As mentioned above, in our model similar $B(E2)$ values imply the same cluster charges. Hence we choose for ^{234}U the interacting nuclear pair ^{210}Pb and ^{24}Ne as the basic dynamical entity to be considered.

As a result of numerous cluster calculations throughout the periodic table (ranging from $^{8}\mathrm{Be}=\alpha+\alpha$ to $^{236}\mathrm{Pu}=^{208}\mathrm{Pb}+^{28}\mathrm{Mg}$) we have found a form for the cluster-core potential energy which gives a good first order approximation to many properties of these systems [12]. This interaction, as a function of the distance r between the constituent nuclei, is a weighted sum of (SW) and $(\mathrm{SW})^3$ shapes, where SW stands for Saxon-Woods:

$$V(r)=-V_0\left[\frac{x}{\{1+\exp[(r-R_0)/a]\}}+\frac{1-x}{\{1+\exp[(r-R_0)/3a]\}^3}\right]. \quad (1)$$

In Eq. (1) we select parameter values as in previous calculations [4,6], namely

$$V_0=53.9A_c \text{ MeV}, \quad x=0.36, \quad a=0.73 \text{ fm}, \quad (2)$$

where for $^{210}\mathrm{Pb}+^{24}\mathrm{Ne}$ the cluster mass $A_c=24$ and the radius parameter R_0 is chosen as explained below. The internuclear interaction is completed by the simplified Coulomb potential arising from a point cluster charge relative to a uniform spherical charge distribution of the core, with radius equal to R_0.

A remarkable feature of the above potential is that solution of the corresponding Schrödinger equation leads naturally to bands of bound and quasibound states which are labeled by a global quantum number $G=2n+L$, where n is the node number and L is the angular momentum of a band member. The energies of levels within a band generally increase with L, though this may cease to hold higher in the band. This is in sharp contrast to results from other potentials of the simple Saxon-Woods or folded forms, in which the spectra are inverted. The point of these observations is that for $G=$ even (odd) the band states have $L=$ even (odd) only. Thus this interaction produces just the right kind of spectra for possible interpretation as the basic 0^+ and 0^- bands occurring in many nuclei. Even more to the point, the radial wave functions for the low-lying states of each band, with many nodes, are all closely similar over the greater part of their extent, and hence each band has what amounts to the same intrinsic state structure.

To buttress our hypotheses further we now need to discuss the physical reasons for our choice of the G values, and corresponding R_0 values, which are found to give good simultaneous accounts of energy spacings and $B(E2)$ strengths in a band. Appropriate G values are selected to satisfy the demands of the Pauli principle. This is straightforward in light nucleus combinations such as the closed-shell core + alpha particle, but can only be done approximately for other binary systems. The principal consideration is that the nucleons in the cluster must all be outside the Fermi-surface of the core. A simple oscillator counting argument has proved generally effective. If the N cluster nucleons have excitation quanta relative to the core given by $(2n_i+l_i)$ and correlate with each other to give a cluster with M internal quanta then the total G value for the relative orbital motion of the binary system should satisfy $G=2n+L\geqslant\Sigma_i(2n_i+l_i)-M$. This is called the Wildermuth condition [13]. It leads, for the example of an α particle orbiting ^{16}O as a model of the first 0^+ and 0^- bands of ^{20}Ne, to $M=0$ and $G=8$ and 9, respectively. We have found for the actinides that it is adequate to take $G=5A_c$ and, using this value of G ($=120$ here), we find $R_0=6.720$ fm as the radius which gives the observed Q value (corrected for electron shielding) of 59.026 MeV for exotic decay of the ground state of ^{234}U by emission of ^{24}Ne. The model ground state has $L=0$ and $n=60$ nodes.

With the cluster-core potential of Eq. (1) completely defined, the energies and wave functions of all levels in the ground state band, having $J^\pi=0^+,2^+,4^+,\ldots G^+$, can be calculated. Also, using the semiclassical method [5], the exotic decay half-life for emission of ^{24}Ne from the 0^+ state comes out as 2.55×10^{25} s, to be compared with the experimental value of $(8.37\pm6.10)\times10^{25}$ s. Other important observables are the reduced matrix elements of the electric quadrupole $(E2)$ operator, given by

$$\langle J_f\|M(E2)\|J_i\rangle=\sqrt{\frac{(2J_i+1)(2J_f+1)}{4\pi}}\alpha_2\langle J_f0J_i0|20\rangle\langle r^2\rangle, \quad (3)$$

where J_i and J_f are initial and final state angular momenta, and the $\langle r^2\rangle$ integral is computed from the corresponding radial functions. The factor α_2 is given by

$$\alpha_2=\frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}, \quad (4)$$

with Z_i, A_i, the charges and masses of cluster and core. Although the cluster model ensures a high degree of collectivity an effective charge ϵ needs to be introduced by replacing Z_i with $Z_i+\epsilon A_i$, in order to compensate for the likely underestimation of the surface peaking of the radial wave functions in our local potential model. Here we use $\epsilon=0.2$ throughout.

We now compare our results with those from other approaches. We first convert our excitation energies E_J to notional frequencies of rotation using the customary relation

$$\hbar\omega(J)=1/2(E_{(J+1)}-E_{(J-1)}). \quad (5)$$

Figure 1 is a plot of $\hbar\omega(J)$ against J using values obtained from the experiment, from our calculation, and from a rotation-vibration model (RVM) [9]. Figure 2 shows the $E2$ reduced matrix elements, with values from the same sources, versus J_i. The quality of fit to the data obtained from the cluster model is clearly superior to that from the RVM. We further note that a rigid rotor model would yield a linear plot in Fig. 1, and hence a very poor fit; but it would, with a suitably chosen quadrupole deformation parameter β_2, give $E2$ values closely similar to ours in Fig. 2 and thus a good fit to the data [9]. This highlights the difficulties of previous

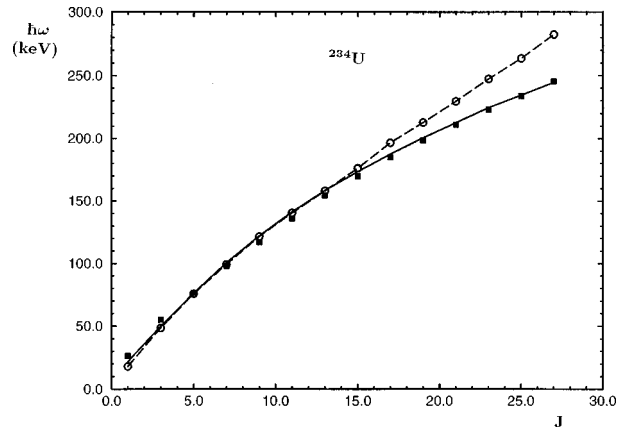

FIG. 1. Comparison of experimental values (solid line) of rotational frequency $\hbar\omega(J)$ as a function of J [see Eq. (5)] with our theoretical cluster model calculations (solid squares) and with RVM predictions (dashed line and open circles) for ^{234}U [9].

models, which do not provide consistent fitting of energies and transition rates. These remarks also hold true for other actinide nuclei [4,6,7,9].

To further illustrate the relationship of our binary cluster model and the simplest rotation model, using a deformed *rigid* rotor, we show in Fig. 3 a comparison of the radial wave functions implied by our model for $J^\pi = 0^+, 4^+, 8^+, 12^+, 16^+$, and 20^+, which are low compared to the limiting $J = 120$. We see that outside the central region these functions are essentially indistinguishable. We have checked that this characteristic is robust under reasonable changes of potential, and note that it would almost certainly be accentuated if, as is likely, the model underestimates the surface peaking [14]. For many purposes therefore the levels of a band have a fixed intrinsic state function. The vital point here is that this does not imply any simple pattern for the corresponding excitation energies arising from our potential model, and in particular, we do not in general expect to find the $J(J+1)$ dependence which is predicted from the axially and equatorially symmetric rigid rotor picture. The energies given by

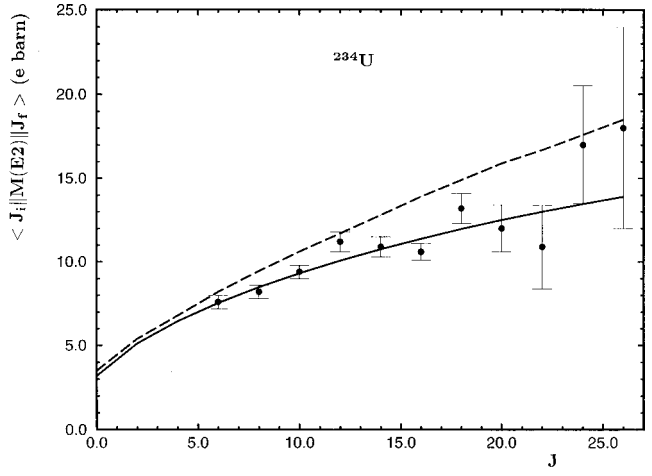

FIG. 2. Comparison of experimental values (solid circles with error bars) of the reduced quadrupole matrix elements $\langle J_f \| M(E2) \| J_i \rangle$ [see Eqs. (3) and (4)] as functions of angular momentum J_i with our theoretical cluster model calculations (solid line) and with RVM predictions (dashed line) [9].

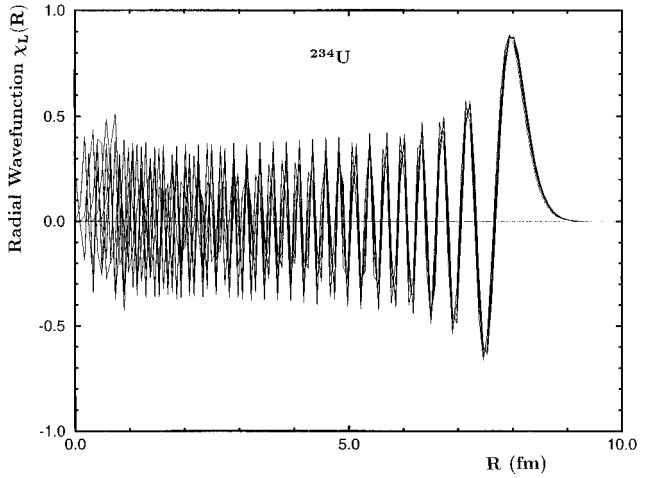

FIG. 3. Calculated radial wave functions of ^{210}Pb–^{24}Ne relative motion, representing states of ^{234}U with $J^\pi = 0^+, 4^+, 8^+, 12^+$, 16^+, and 20^+.

the cluster model depend on the detailed shape of the potential, and have been found to reproduce observations rather well, as Fig. 1 and previous work show [3,4,6].

However, when the central parts of the radial functions are unimportant, as for the $E2$ transitions with $J_i \ll 120$, we can to good approximation replace all these functions by a single one, say, that for $J^\pi = 0^+$. This radial function, together with the internal state vectors of the cluster and core, can then be taken as the single intrinsic state for a band, which thus mimics the assumed situation in rigid rotor models. Hence we recover the simple $E2$ strength pattern of a rigid rotor with fixed internal state. The values are proportional to squares of Clebsch-Gordan coefficients. Our model nicely reconciles the observed quasirotational energies (described by variable moments of inertia) with the correct $B(E2)$ systematics (which are implied by the rigid rotor picture).

We have seen therefore, that a binary cluster model of ^{234}U gives rise dynamically to a $J^\pi = 0^+, 2^+, 4^+, \ldots$ ground state band, based on a common intrinsic state, with energy spacings closely matching the observed spectra. Similar re-

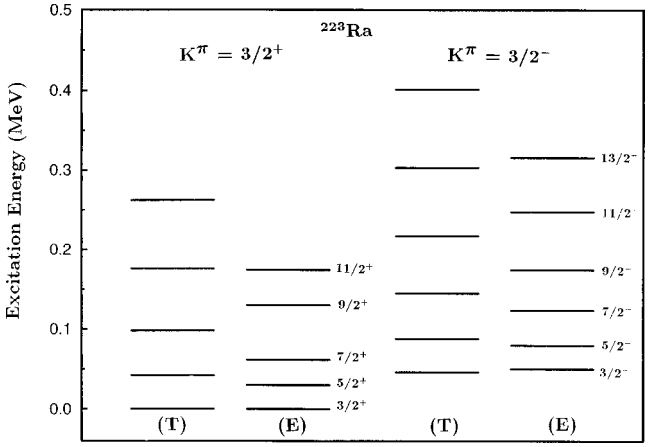

FIG. 4. Comparison of the measured energies (E) of the $K^\pi = 3/2^\pm$ parity doublet bands of ^{223}Ra with those calculated in our model (T) [15].

marks apply to the lowest 0^- band of this nucleus, described in our approach as the $J^\pi = 1^-, 3^-, 5^-, \ldots$ set of states belonging to a global quantum number one unit greater than that for the 0^+ band [4,6]. We stress again that although we have chosen ^{234}U to make our points, other actinides provide equally good illustrations [3,4,6,7,9].

Also of interest is that all the machinery is in place [2] to generate extensions of the model so as to describe additional band structures for comparison with other data on heavy deformed nuclei by coupling a valence particle to the above set of states. We have demonstrated this in detail in a calculation of the spectrum of ^{223}Ra [15], from which we reproduce in Fig. 4 our results for the $K^\pi = 3/2^+$ ground state band, and for its $K^\pi = 3/2^-$ parity doublet [16]. In our nontraditional treatment these parity doublets are the natural outcome of having a 0^- band in ^{222}Ra almost degenerate with the 0^+ band.

In conclusion, we have shown that a binary cluster model, using a deep potential well and large values of the global quantum number G, is not only able to reconcile many properties of the ground state bands of even-even nuclei, but also provides a solid foundation for generalizations describing other bands characteristic of heavy nuclei.

A.C.M. and S.M.P. would like to thank the U.K. Engineering and Physical Sciences Research Council for financial support. S.M.P. would also like to thank the S.A. Foundation for Research, and the University of Cape Town, for financial support.

[1] D.J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970), p. 1.
[2] D.M. Brink *et al.*, J. Phys. G **13**, 629 (1987).
[3] B. Buck, A.C. Merchant, and S.M. Perez, Phys. Rev. Lett. **76**, 380 (1996)
[4] B. Buck, A.C. Merchant, and S.M. Perez, Nucl. Phys. **A617**, 195 (1997).
[5] B. Buck *et al.*, J. Phys. G **20**, 351 (1994).
[6] B. Buck, A.C. Merchant, and S.M. Perez, Nucl. Phys. **A625**, 554 (1997).
[7] B. Buck, A.C. Merchant, and S.M. Perez (in preparation).
[8] B. Buck *et al.*, in *Proceedings of the 6th International Conference on Clusters in Nuclear Structure and Dynamics*, Strasbourg, France, edited by F. Haas (CRN, Strasbourg, 1994), p. 195.
[9] H. Ower *et al.*, Nucl. Phys. **A388**, 421 (1982).
[10] Y.A. Akovali, Nucl. Data Sheets **71**, 181 (1994).
[11] M.R. Schmorak, Nucl. Data Sheets **63**, 139 (1991).
[12] B. Buck, A.C. Merchant, and S.M. Perez, Nucl. Phys. **A614**, 129 (1997).
[13] B. Buck, H. Friedrich, and C. Wheatley, Nucl. Phys. **A275**, 246 (1976).
[14] K. Varga, R.G. Lovas, and L.J. Liotta, Phys. Rev. Lett. **69**, 37 (1992); Nucl. Phys. **A550**, 421 (1992).
[15] B. Buck, A.C. Merchant, and S.M. Perez, in *Proceedings of the Workshop on Clustering in Nuclear and Atomic Physics*, Rauischholzhausen, Germany, 1997 [Nuovo Cimento A (to be published)].
[16] R.K. Sheline, Y.S. Chen, and G.A. Leander, Nucl. Phys. **A486**, 306 (1988).