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Alignment and pseudospin symmetry

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Pseudospin symmetry has been invoked as a possible explanation for the ''quantized alignment'' observed in some superdeformed ''identical'' bands. The clearest case involves the 1/2[301] orbital (a pseudospin singlet, $\tilde{\Lambda}=0$), where pseudospin alignment provides an explanation for some identical bands in the mass-150 region. Pseudospin doublets ($\Omega=\tilde{\Lambda}\pm 1/2$) can also generate quantized alignments and such an explanation, with $\tilde{\Lambda}=1$, has recently been proposed for the band in ^{191}Au identical to ^{192}Hg. The present work examines whether the data on the normally deformed nuclei support such an interpretation and concludes that pseudospin alignment is plausible for states with $\tilde{\Lambda}=0$ or 1. [S0556-2813(98)50504-3]

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The concept of pseudospin symmetry [1,2] has proved useful in describing the ''normal parity'' states of nuclei. In this scheme the total angular momentum j of a particle is unchanged, but is decomposed into pseudo-orbital and pseudo-(intrinsic) spin components, which are related to the normal ones by $\tilde{l}=l\pm 1$ and $\tilde{s}=-s$. Perhaps the most striking success of the pseudospin scheme is the explanation of closely spaced doublets that occur in nuclear energy levels and are particularly apparent on any (Nilsson-type) plot of calculated level energies. In the spherical limit, these doublets have quantum numbers: $j_1=l_1+1/2$, $j_2=l_2-1/2=j_1+1$, and $l_2=l_1+2$ (e.g., $\{s_{1/2},d_{3/2}\}$ or $\{f_{7/2},h_{9/2}\}$). In the pseudospin scheme these become just the pseudo spin-orbit partners: $j=\tilde{l}\pm 1/2$, where $\tilde{l}=l_1+1$ ($\tilde{p}_{1/2,3/2}$ or $\tilde{g}_{7/2,9/2}$ in the above examples). With deformation, doublets persist with quantum numbers, $\Omega=\tilde{\Lambda}\pm 1/2$. It is the very weak pseudo spin-orbit coupling that provides the natural explanation for these doublets; however, it is not yet clear why this coupling is so weak. A deeper understanding of this phenomenon is a topic of considerable current interest; see, for example, the recent discussion by Ginocchio [3]. The implications of pseudospin for (quasi)particle motion in rotating nuclear potentials were discussed by Bohr *et al.* [4] and the alignment of pseudospin (due to the weak spin-orbit interaction) has subsequently been invoked as a possible explanation for the ''quantized alignment'' observed in superdeformed ''identical'' bands [5,6]. The present note examines some aspects of such an explanation.

The term ''identical band'' has been used in several different ways. Some have applied it to pairs of bands having the same dynamical moments of inertia (i.e., the same *separation* between gamma-ray energies) while others have restricted its use to pairs of bands having the same actual gamma-ray energies. The latter definition is considerably more restrictive and implies that the aligned spin, or alignment, is integer (including zero) between pairs of bands in nuclei of the same type (both even mass or both odd mass) and half integer between bands in nuclei of different types. The aligned spin in this context is just the difference in spin between the two bands at the same rotational frequency, and, since the rotational frequency is half the collective $E2$ gamma-ray energy, that means at the same gamma-ray energy. This second identical band definition requires ''quantized alignments,'' i.e., integer or half integer values.

While initially a sizable fraction of the known rotational bands in superdeformed nuclei were identical in this definition, the much larger number currently known and the much higher precision in the gamma-ray energies has made the existence of such identical bands as a special ''category'' controversial. It is not the purpose here to discuss whether such bands comprise a special category or not, but rather to consider whether pseudospin is likely to play any role in generating quantized alignments in superdeformed nuclei. That will be done by comparing with what we know about such alignments in normally deformed nuclei.

The present interest in pseudospin comes from the fact that it can generate quantized alignments. In the simplest case of $\Omega=1/2$ bands, those with $\Lambda=1$ have zero decoupling parameter in the asymptotic limit, while those with $\Lambda=0$ have decoupling parameter ± 1. This latter case has been shown [6] to be equivalent to the (quantized) alignment of the intrinsic spin along the rotation axis—i.e., an alignment $\pm 1/2$. In the pseudospin scheme, these values are reversed; therefore, for example, the state $[301]1/2$ becomes $[\tilde{2}\tilde{0}\tilde{0}]1/2$ and the alignment, which would be zero in the asymptotic limit, becomes $\pm 1/2$ in the pseudospin limit, the sign depending on the signature. In their 1982 paper, Bohr *et al.* [4] showed that, in general, the deformed rare-earth nuclei follow the pseudospin value for this decoupling parameter (alignment) much more closely than the asymptotic value and later, in 1990, Nazarewicz *et al.* [5] used the above proton orbital as the explanation for identical superdeformed bands in ^{152}Dy and ^{151}Tb. These bands have the same transition energies, requiring exactly half-integer alignment, which the $[\tilde{2}\tilde{0}\tilde{0}]1/2$ orbital can provide. Since that time several other cases of identical bands involving this particular orbital have been observed in this region of nuclei. More recently a band in ^{151}Dy with transition energies midway

between those of ^{152}Dy has been proposed [7] to be based on the neutron orbital, [411]1/2, which becomes $[\tilde{3}\tilde{1}\tilde{0}]1/2$ in the pseudospin representation. Such transition energies imply a decoupling parameter of -1 which is precisely what is expected since the sign of the decoupling parameter is given by $(-1)^{\tilde{N}}$. Until very recently these have been the only cases where identical bands having nonzero quantized alignments had explanations that could plausibly be directly related to pseudospin.

Also in 1990 it was pointed out [6] that the role of pseudospin in generating identical bands might extend beyond $\Omega=1/2$ bands and this possibility was explored further [8] a few months later. If the pseudospin doublets have the structure, $\Omega=\tilde{\Lambda}\pm1/2$, they will be mixed by the Coriolis force (i.e., generate pseudospin, $\tilde{\Sigma}$, alignment) through the matrix element $\langle\tilde{N}n_3\tilde{\Lambda}\tilde{\Sigma}\pm1|j_\pm|\tilde{N}n_3\tilde{\Lambda}\tilde{\Sigma}\rangle$ from which the operative piece is the pseudospin-flip matrix element, $\langle\tilde{s}_\pm\rangle$, whose value is equal to one. (The other piece of the $j_\pm$ operator, $\tilde{l}_\pm$, vanishes for these pseudospin doublets which are based on the same $\tilde{\Lambda}$.) However, if other $\tilde{\Lambda}$ values are included, $\langle\tilde{N}n_3'\tilde{\Lambda}\pm1\tilde{\Sigma}|\tilde{l}_\pm|\tilde{N}n_3\tilde{\Lambda}\tilde{\Sigma}\rangle$ could generate $\tilde{\Lambda}$ alignment, and the relative size of these two sources of alignment is important. The alignment, at a fixed spin, depends initially on $\langle j_\pm\rangle^2/\Delta E$, where $\langle j_\pm\rangle$ is the relevant matrix element and ΔE is the separation between admixed levels. The energy spacing between the $\tilde{\Lambda}$ and $\tilde{\Lambda}\pm1$ states is expected to be roughly 2.5 MeV for the cases we will consider, whereas the pseudospin doublet energy separation is only about 0.15 MeV for $\tilde{\Lambda}=1$. The relevant matrix elements for $\tilde{\Lambda}$ alignment are several (~3) times larger than those for $\tilde{\Sigma}$ alignment so that these two sources of alignment could be nearly comparable. However, since $\Delta E_{\tilde{\Lambda}}$ is large compared with the pairing gap parameter Δ, sizable reductions in these matrix elements are expected (e.g., given by the BCS factor, $U_iU_j+V_iV_j$, where U and V are the usual pair-occupation probabilities). On the other hand, $\Delta E_{\tilde{l}\tilde{s}}$ is small compared with Δ; thus, no reduction of the spin-flip matrix elements for the doublets is expected, even if the two states are on opposite sides of the Fermi level, causing their real energy separation to become very small, in which case the alignment quickly saturates at $\pm1/2$. Thus, rather complete $\tilde{\Sigma}$ alignment could often precede any significant $\tilde{\Lambda}$ alignment, so that over an appropriate spin range (which determines the strength of the Coriolis interaction) the $\tilde{\Sigma}$ alignment could be nearly pure. This is less likely to occur when the Fermi level does not fall between the doublet states.

Without rotation, $\tilde{\Sigma}$ is coupled parallel or antiparallel to $\tilde{\Lambda}$ by the weak pseudospin-orbit force. The Coriolis force realigns $\tilde{\Sigma}$ parallel or antiparallel with the rotation axis, resulting, when complete, in two signature-degenerate bands the lower of which has alignment $+1/2$ for both signatures, and the upper has alignment $-1/2$. These bands can give rise to identical bands between odd-mass and even-even nuclei in the same way as the $\Omega=1/2$ bands do, and one such case has recently been suggested for bands in ^{191}Au and ^{192}Hg by Schuck *et al.* [9] In this case the orbitals proposed were [530]1/2 and [532]3/2 or $[\tilde{4}\tilde{3}\tilde{1}]1/2,3/2$ and the band in ^{191}Au does, indeed, have both signatures rather than just one as in the ^{151}Tb case (above). Thus it appears that an alignment of $\pm1/2$ might be generated by this $\tilde{\Lambda}=1$ case as well as for the $\tilde{\Lambda}=0$ case described above. If this is true, then there might be examples in the normally deformed nuclei where similar things happen, and the aim of the present work is to examine whether this does indeed occur.

In searching through regions of well-deformed nuclei, the occurrence of 1/2-3/2 pseudospin doublets near the ground state is rare because such pairs systematically tend to occur at the edges of such regions, where deformation changes or other factors make analysis difficult. This is true, in particular, for the above pair of orbitals. The only good and reasonably extended set of data we found in the *Table of Isotopes* [10] was on the pair of neutron orbitals [510]1/2 and [512]3/2 or $[\tilde{4}\tilde{1}\tilde{1}]1/2,3/2$, which fortunately seems likely to be similar to the $[\tilde{4}\tilde{3}\tilde{1}]1/2,3/2$ pair. These orbitals are lowest-lying in the W and Os nuclei having neutron numbers N of 109, 111, and 113. The Coriolis mixing has been previously studied in these nuclei, but not, as far as we could find, with any analysis of the alignment behavior. The expectations, as described above, are rather simple. The mixing, if complete, should produce alignments of $\pm1/2$ in the two bands, or alignment exactly one relative to each other. This would result in equal transition energies in the two bands, but connecting states that are $1\hbar$ higher in the lower band. This is a very simple property and it should be reasonably independent of pairing (blocking) effects and $\tilde{\Lambda}$ alignment since these are expected to be similar in the two bands.

In a quantitative evaluation of the mixing of the 1/2 and 3/2 bands in the $[\tilde{4}\tilde{1}\tilde{1}]1/2,3/2$ case, one must take into account the energy separation of the two bands. The mixing will not be complete if the bands are too far apart in energy even if $\langle\tilde{s}_\pm\rangle$ is one as expected. To illustrate the data we have plotted in Fig. 1 the relative alignment between the two bands i versus a quantity $2H_{\rm Cor}/\Delta E$ where $H_{\rm Cor}$ is the Coriolis matrix element (see below) and ΔE is the energy separation of the *final* (admixed) states of a given spin. To get the relative alignment we compared transition-energy differences (from states differing in spin by $1\hbar$) in the two different bands with those in the same bands. We used only $E2$ gamma rays since this avoids any variations due to the decoupling parameter (although these are small), and the alignment i is given by

$$i=\{\Delta E_\gamma(\text{avg})-\{E_\gamma[(I+2)\rightarrow I]_l - E_\gamma[(I+1)\rightarrow(I-1)]_u\}\}/\Delta E_\gamma(\text{avg}), \quad (1)$$

where subscripts u and l mean upper and lower bands, respectively, and ΔE_γ(avg) is the average of the difference between the same $E2$ transitions within each band. The Coriolis matrix element is

$$H_{\rm Cor}=-\langle\Omega|j_\pm|\Omega\mp1\rangle\langle K\mp1|I_\mp|K\rangle\hbar^2/2\mathcal{I}, \quad (2)$$

and for the pseudospin doublet, $\langle j_\pm\rangle$ reduces to $\langle\tilde{s}_\pm\rangle$ with the expected value 1; $\langle K\mp1|I_\mp|K\rangle=[(I\mp K)(I\pm K+1)]^{1/2}$; and $\mathcal{I}$ is the moment of inertia which we take to be the average for the two bands at the spin where ΔE is evaluated. The abscissa, $2H_{\rm Cor}/\Delta E$, should vary between zero (if

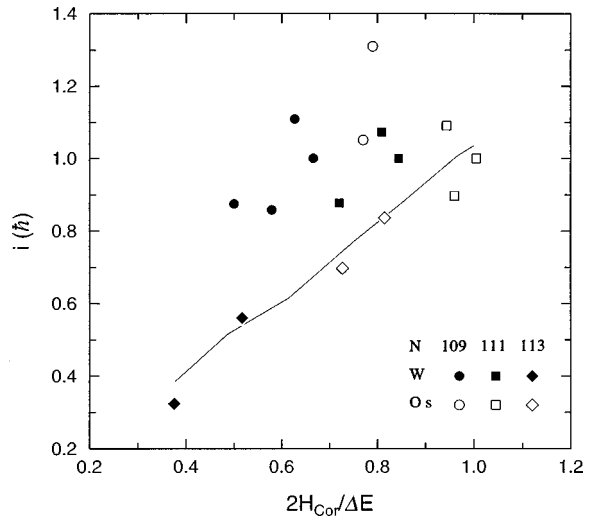

FIG. 1. Relative alignment of the pseudospin doublet bands in some tungsten and osmium isotopes plotted against $2H_{\mathrm{Cor}}/\Delta E$, where H_{Cor} is the Coriolis matrix element for mixing the bands (with $\langle j_{\pm}\rangle = 1$) and ΔE is the final (observed) energy separation of the mixed levels. Different points for the same nucleus correspond to different spin states. The line represents a calculation of these quantities for a $\tilde{\Lambda}=1$ pseudospin doublet having various spins and initial energy separations.

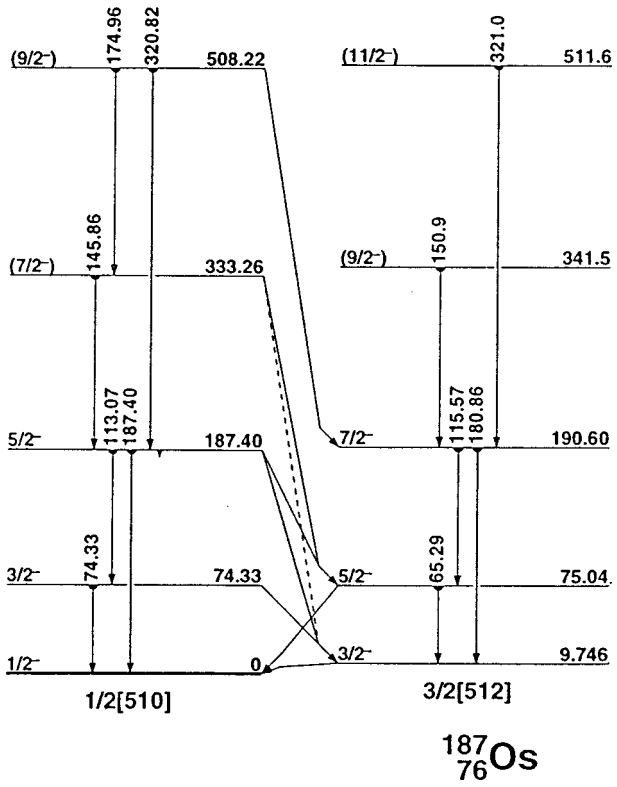

FIG. 2. The observed pseudospin doublet bands in ^{187}Os.

ΔE is large) and one, since degenerate states before mixing should be separated by $2H_{\mathrm{Cor}}$ after mixing.

The line in Fig. 1 is the calculated curve for $\langle \tilde{s}_{\pm}\rangle = 1$ and the points are the data for the W and Os isotopes. The data $N=111$ and 113 are in reasonably good agreement with the calculated curve, whereas the $N=109$ nuclei are somewhat too strongly mixed, having average alignments around 1.0 while the expected values average closer to 0.7. This larger mixing has been noted before and attributed [11] to hexadecapole deformation, but this would not explain the data for the higher-N nuclei which fall close to the line in Fig. 1 and seem likely to have similar hexadecapole moments. The data for $N=111$ show $1\hbar$ alignment rather convincingly, and since the Fermi level is nearly halfway between the two orbitals, the levels themselves differing by $1\hbar$ are nearly degenerate. This only happens when the levels in the two initial bands are nearly degenerate, whereas the $1\hbar$ alignment can also occur with larger initial energy separations. Figure 2 shows the lowest bands in ^{187}Os (references to the primary data for all the cases discussed are contained in the *Table of Isotopes* [10]), which is a striking example of this behavior. The nuclei with $N=113$ are too far apart in energy to align $\tilde{\Sigma}$ completely, but behave consistently with $\langle \tilde{s}_{\pm}\rangle = 1$ and should reach full $\tilde{\Sigma}$ alignment at higher spins. Overall, the agreement seems satisfactory, suggesting that the expected $\tilde{\Sigma}$ alignment does occur in these nuclei. One should remember, however, that this comparison is not sensitive to blocking effects on the moment of inertia nor to alignment of $\tilde{\Lambda}$ as these are likely to be similar in the two bands.

The next step is to look at the mean alignment of the two bands in a nucleus relative to an adjacent even-even nucleus, which would be zero if the alignments in each band were equal and of opposite sign ($\pm 1/2$ in the limit) and nothing else was happening. This would be necessary in order to explain the ^{191}Au identical band case. In fact, this mean alignment relative to the lower-mass even-even isotope is shown in Fig. 3 and averages about $+0.4$ for all the cases in Fig. 1, which is too large to produce identical bands. This extra alignment could be due either to pairing (blocking) effects or some alignment of $\tilde{\Lambda}$. Both of these should, in general, be smaller in the superdeformed nuclei due to the weaker pairing at higher spins and the wider spacing of the $\tilde{\Lambda}$ states at larger deformation. One might be able to distinguish between these two causes for the additional alignment by comparing with another band in the odd-mass nucleus that is expected to have similar pairing (blocking) effects to those of our pseudospin bands. There is a 7/2 [503] orbital close by in these nuclei and the average alignment of the pseudospin bands relative to this band is nearly zero, suggesting that the above deviation of $+0.4$ is largely a pairing effect. However, the alignment of the pseudospin bands relative to this 7/2 orbital varies considerably with mass number, so that its use as a comparison orbital is probably not justified. Based on these results we cannot rule out other sources of alignment in the ^{191}Au bands, although it seems likely that these are not large.

A broader look at normally deformed pseudospin doublets turned up a few 3/2-5/2 pairs ($\tilde{\Lambda}=2$), but none of these aligned $1\hbar$ in the spin range observed (i.e., they fell into the lower left part of Fig. 1). There are data on some higher-Ω pairs, notably the 5/2[402]-7/2[404] pair ($\tilde{\Lambda}=3$) in some Ta and Re isotopes, but here the Coriolis matrix elements are observed to be reduced by as much as a factor of 2, thereby also reducing the limiting pseudospin alignment below $\frac{1}{2}\hbar$. A possible reason for this is that the separation between the pseudospin doublet states becomes larger as $\tilde{\Lambda}$ gets larger (it

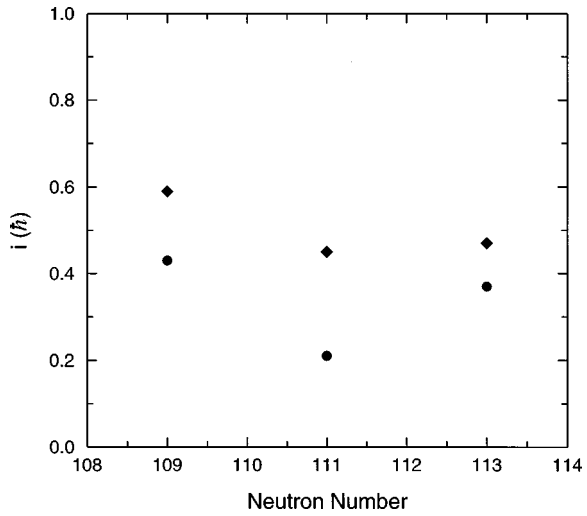

FIG. 3. The mean alignment of the pseudospin doublet bands relative to the adjacent lower-mass even-even nucleus plotted against neutron number. The diamonds are for tungsten data and the circles for osmium.

is proportional to $\tilde{\Lambda}$) and when this separation becomes comparable with the pairing gap parameter Δ, the matrix elements are reduced by the pairing (as discussed above). The larger energy separation also makes it less probable that the states will lie close to each other in the spectrum.

Finally we should point out that another potentially interesting set of data exists on the pair of orbitals $[400]1/2$ and $[402]3/2$ or $[\tilde{3}\tilde{0}\tilde{1}]1/2,3/2$ in the odd-mass Ir isotopes. A broad range of these nuclei have nearly degenerate levels differing in spin by $1\hbar$ which indicates a relative alignment of one. These nuclei have nonaxial shapes and a recent study [12] has shown that while the pseudospin symmetry remains good in triaxial nuclei, the near level degeneracy of pseudospin doublets is lost. However, even in the W and Os nuclei, for the levels themselves to be degenerate required the Fermi level to be in just the right place (only occurring for $N=111$) and this would presumably have to be the case for these Ir nuclei also. The real question is whether the more general observed property—the $1\hbar$ relative alignment—is due to pseudospin or not. Unfortunately, an analysis of the Coriolis effects in an asymmetric rotor is beyond the scope of the present work and we cannot answer this question.

In conclusion, based on this analysis, alignment of pseudospin ($\tilde{\Sigma}$) to give values very close to $\frac{1}{2}\hbar$ seems reasonably probable for states with $\tilde{\Lambda}=0$ or 1 (including the ^{191}Au superdeformed band), but become less likely for higher $\tilde{\Lambda}$ values. The Ir data raise the interesting question of whether such $\tilde{\Sigma}$ alignment can also occur in nuclei with nonaxial shapes. Other sources of alignment in these pseudospin doublets are expected to be smaller but cannot be excluded. This process can provide a plausible explanation for the identical bands in ^{191}Au and ^{192}Hg; however, with just this one case an accidental half-integer alignment of some other type cannot be ruled out.

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