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Semiclassical description of the shears mechanism and the role of effective interactions

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By deriving the angle between the proton and neutron spin vectors $\vec{j}_\pi$ and $\vec{j}_\nu$ in the shears bands in ^{198,199}Pb, we present a semiclassical analysis of the $B(M1)$ and $B(E2)$ transition probabilities as a function of the shears angle. This provides a semiempirical confirmation of the shears mechanism proposed by Frauendorf using the tilted-axis-cranking model. In addition, we propose that the rotational-like behavior observed for these bands may arise from a residual proton-neutron interaction. [S0556-2813(98)51303-9]

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The observation of cascades of magnetic dipole ($M1$) transitions in neutron-deficient Pb nuclei [1] has generated great interest in the nuclear structure community. These regular sequences of $M1$ transitions show an energy spectrum that follows $\Delta E(I)=E(I)-E(I_b)\sim A(I-I_b)^2$ where I_b is the spin of the bandhead. This apparent rotational behavior has been difficult to understand in terms of the rotational model because of the rather small deformation ($\beta_2 \lesssim 0.1$) that can be expected for these bands on the basis of the small $E2/M1$ branching ratios observed [$B(E2)/B(M1)$ are typically $\lesssim 0.025-0.05\ (e\mathrm{b}/\mu_N)^2$] [2]. The large values of the ratio of the moment of inertia to the reduced $E2$ transition probability, $\mathcal{J}^{(2)}/B(E2)$, have also been interpreted in the literature as a fingerprint for a different origin of the inertia in these bands.

Using the tilted-axis-cranking (TAC) model, Frauendorf [3] interpreted these sequences as arising from the coupling of $h_{9/2}$ and $i_{13/2}$ protons and $i_{13/2}$ neutron-holes. The total angular momentum is generated by aligning the proton and neutron spin vectors, $\vec{j}_\pi$ and $\vec{j}_\nu$, in a way that resembles the closing of a pair of shears, hence the name usually given to these structures: shears bands. In a recent experiment with GAMMASPHERE, lifetimes of states in the shears bands in ^{198,199}Pb isotopes have been determined by Doppler shift attenuation method (DSAM) measurements [4]. The deduced $B(M1)$ values provide a sensitive test of the mechanism responsible for the generation of angular momentum and are in excellent agreement with the TAC predictions [3,4]. In addition, the orbitals involved and their perpendicular coupling at the bandhead have been confirmed recently by a g-factor measurement in ^{197}Pb [5].

In the first part of this Rapid Communication we will present a global analysis of the $B(M1)$ and $B(E2)$ values. This analysis is based on a schematic model of the coupling of two long j vectors ($\vec{j}_\pi,\vec{j}_\nu$); under this simple assumption we will show that the picture of the shears mechanism is indeed consistent with the overall behavior of the experimental data. In the second part, we will discuss the form of the residual interaction that, within the framework of this simple system, may give rise to a rotational-like behavior.

Following the nomenclature introduced in Fig. 1, we start by defining θ_π and θ_ν as the angles of the proton and neutron spin vectors with respect to the total angular momentum, $\vec{I}=\vec{j}_\nu+\vec{j}_\pi$. The shears angle θ that corresponds to a given state in the band can be derived using the semiclassical expression

$$\cos\theta=\frac{\vec{j}_\nu\cdot\vec{j}_\pi}{|\vec{j}_\nu||\vec{j}_\pi|}=[I(I+1)-j_\nu(j_\nu+1)-j_\pi(j_\pi+1)]/2[j_\nu(j_\nu+1)j_\pi(j_\pi+1)]^{1/2}. \quad (1)$$

Since the $B(M1)$ values are proportional to the square of the component of the magnetic moment perpendicular to the spin vector [6,7] they should show a characteristic drop as the shears close (i.e., $\theta\approx 90°\rightarrow\theta\approx 0°$). From the simple geometry specified in Fig. 1, this dependence is given by

$$B(M1,I\rightarrow I-1)=\frac{3}{4\pi}\frac{1}{2}\vec{\mu}_\perp^2=\frac{3}{4\pi}g_{\mathrm{eff}}^2 j_\pi^2\frac{1}{2}\sin^2\theta_\pi\quad[\mu_N^2] \quad (2)$$

as a function of the proton angle $\theta_\pi=\theta_\pi(\theta)$ and where we have introduced an effective gyromagnetic factor, $g_{\mathrm{eff}}=g_\pi-g_\nu$. The relation between θ and θ_π is given by the formula $\tan\theta_\pi=j_\nu\sin\theta/(j_\pi+j_\nu\cos\theta)$.

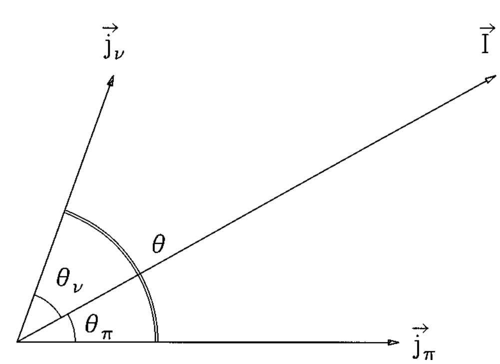

FIG. 1. Schematic drawing of the angular momentum coupling of neutrons and protons in an $M1$ band.

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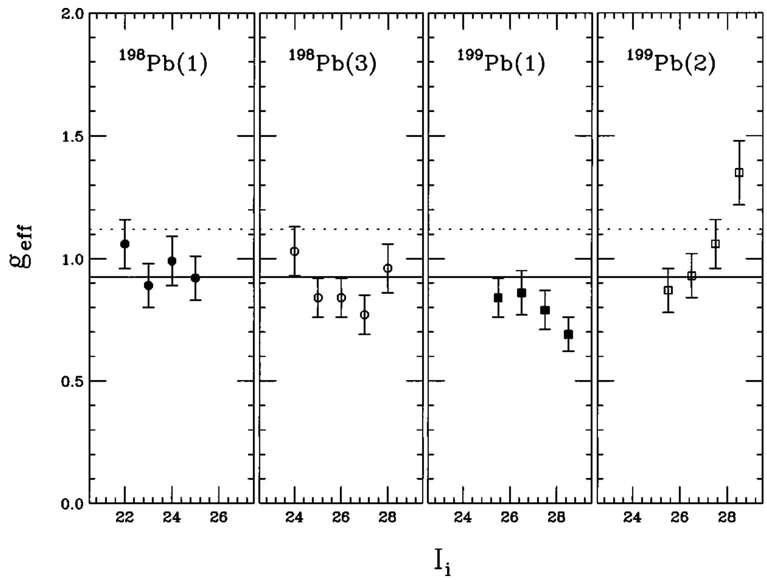

FIG. 2. Effective g-factors obtained for the bands in ^{198,199}Pb. The labels follow those used in Ref. [3]. The solid line is the average value and the dotted line is the value expected from the configurations $(\pi h_{9/2} \otimes \pi i_{13/2})_{11^-}$ and $(\nu i_{13/2})^{-2}_{12^+}$

In our analysis we fix the proton configuration to $j_\pi = 11$ (corresponding to the 11^- K isomer seen in several of the Pb nuclei) and the neutron spin j_ν is determined to reproduce the spin of the bandhead state in each band, assuming a perpendicular coupling [5]. The angle of the blade θ for higher spins is then obtained from Eq. (1). From the measured $B(M1)$'s we derive $g_{\rm eff}$ for the different bands in ^{198,199}Pb using Eq. (2) and these values are shown in Fig. 2. They seem consistent with those expected for nuclei in this region [8] where, for example, using the measured values for the configurations $(\pi h_{9/2} \otimes \pi i_{13/2})_{11^-}$ and $(\nu i_{13/2})^{-2}_{12+}$ we estimate $g_{\rm eff} \approx 1.12$. Although we may expect slight differences between individual bands, the approximate constancy of $g_{\rm eff}$ indicates that expression (2) provides a reasonable description of the experimental data. Indeed, this is shown in a different way in Fig. 3(a) by plotting $B(M1)$'s as a function of the shears angle together with the results of Eq. (2) using the average $g_{\rm eff}=0.92$

Following Refs. [6, 7] we can also derive a similar expression for $B(E2, I \rightarrow I-2)$ values, which are proportional to the square of the $\mathcal{M}(E2, \mu=2)$ component of the electric quadrupole tensor. Here we have

$$B(E2, I \rightarrow I-2) = \frac{5}{16\pi} (eQ)^2_{\rm eff} \frac{3}{8} \sin^4 \theta_\pi \quad [e^2 {\rm b}^2] \qquad (3)$$

in terms of $(eQ)_{\rm eff} = e_\pi Q_\pi + (j_\pi / j_\nu)^2 e_\nu Q_\nu$ that takes into account the contributions from protons and neutrons. The $B(E2)$'s also drop as the shears close and should go to zero because the charge distribution becomes symmetric around the rotation axis. As shown in Fig. 3(b), the overall angle dependence is reproduced by Eq. (3) with an average $(eQ)_{\rm eff} \approx 6.5\, e$b. Assuming that $(eQ)_{\rm eff}$ is determined from the contributions of ~ 2 protons and ~ 2 neutrons with radii $\langle r^2_\pi \rangle \sim \langle r^2_\nu \rangle \sim 7^2$ fm^2 we need an $E2$ polarization charge [6] $(e_{\rm pol})_{E2} \approx 3$ to reproduce the experimental value. This rather large polarization charge may indicate the contribution of more particles to the estimates of $(eQ)_{\rm eff}$ and/or collective contributions from the core not included in Eq. (3).

In what follows we would like to address the question of how do we get rotational behavior from this coupling scheme. In other words, what kind of effective interaction is needed so that the dynamics of the system can be represented with a rotational-like spectrum. Knowing the angle θ between $\vec{j}_\pi$ and $\vec{j}_\nu$ and the level energies it is also possible to obtain information about the nature of this effective interaction, $V_{\pi\nu}$, between the protons and the neutrons [9]. If we restrict ourselves to spatial forces, by symmetry arguments we can expand this interaction in even multipoles as [10]

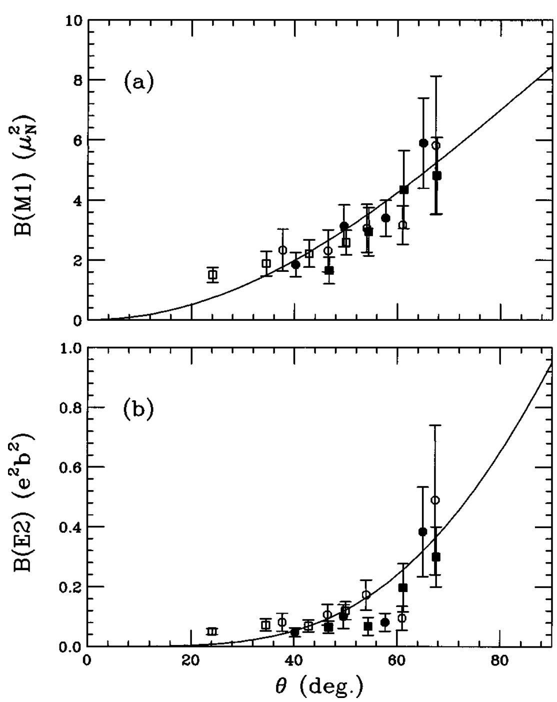

FIG. 3. (a) $B(M1)$ values as a function of the shears angle. The solid line is the result of Eq. (2) with $g_{\rm eff}=0.92$. (b) $B(E2)$ values as a function of the shears angle; the solid line is from Eq. (3) using $(eQ)_{\rm eff}=6.5\, e$b. Symbols are as follows: solid-circle, ^{198}Pb(1); circle, ^{198}Pb(3); solid-square, ^{199}Pb(1); square, ^{199}Pb(2).

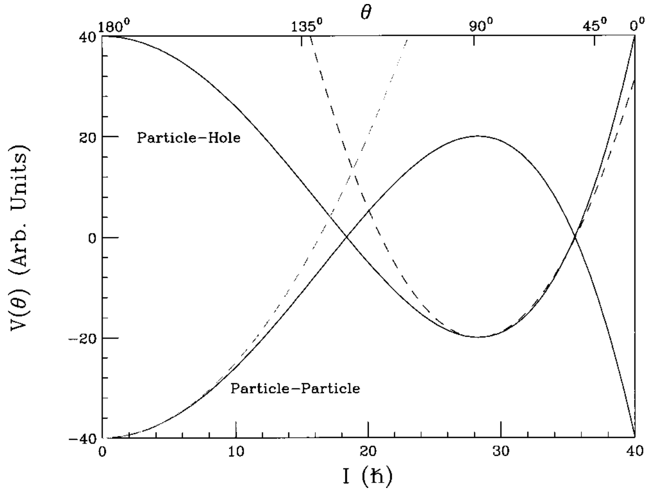

FIG. 4. Particle-particle (hole) potential as a function of angular momentum and θ for an interaction of the form $V_2P_2(\theta)$. The dashed lines correspond to a rotational approximation. The example is for $j_\pi = j_\nu = j = 20$.

$$V_{\pi\nu}(\theta) = V_0 + V_2 P_2(\theta) + \cdots , \qquad (4)$$

and we will now show that the P_2 term can give rise to a rotational spectrum. Let us assume for simplicity that we have a neutron and a proton of the same j coupled to spin I and interacting via a term of the form $V_2P_2(\theta)$. The energy along the band is given only by the change in potential energy due to the recoupling of the angular momenta and therefore

$$\Delta E(I) \propto \langle jjI|V(\theta)|jjI\rangle = V_2 \frac{3\cos^2 \theta(I) - 1}{2}. \qquad (5)$$

In Fig. 4 we show the dependence of this term as a function of I and θ for the particle-particle (hole-hole) and particle-hose cases. As can be seen, the minimum of the potential energy for the particle-particle case (V_2 negative) occurs at $\theta = 180°$, $I = 0$ where the overlap between the particles' wave functions is maximum. Since for $I < j$ we have $\sin\theta \approx (I/j)$, it follows Eq. (5) in that the low-spin members of the $(2j+1)$-multiplet are split approximately by I^2 as shown with the dashed curve. This result leads to the interesting prediction for the existence of excited shears bands in near spherical $N = Z$ nuclei where protons and neutrons occupy the same high-j particle-particle (hole-hole) orbits. It is however uncertain whether these bands will be low enough in excitation energy to be seen in (HI, xn) reactions. If we now turn our attention to the particle-hole channel, which is the situation in the Pb shears bands, the spatial interaction we are considering changes sign, i.e., $V \to -V$ [10] (V_2 positive), and the minimum now occurs at $\theta = 90°$, $I_{90} = \sqrt{2}j$. We then obtain $\Delta E \propto (I - I_{90})^2$, as observed in experiment with I_{90} representing the spin of the bandhead. It is clear that one can also couple $\vec{j}_\pi$ and $\vec{j}_\nu$ to angles greater than 90°. An inspection of Fig. 4 shows that these members of the multiplet will lie on the unfavored side of the parabola. The observation of these states would provide an important signature of the shears mechanism.

It has also been suggested by Frauendorf [11] that this new mechanism should give rise to $E2$ rotationallike bands in the so-called *antimagnetic rotor*. We note here that the particle-particle or hole-hole channels can provide the basic coupling schemes for these structures, where, having identical particles (holes), the allowed spins differ by $\Delta I = 2$.

A question that readily comes to mind concerns the mass dependence of the moment of inertia, $\mathcal{J}$, of these $M1$ bands and *a priori* there is not an obvious answer for the shears mechanism. However, from Eq. (5) we have $\Delta E \propto (V_2/j^2)I^2$ which gives $\mathcal{J} \propto j^2/V^2$, and because we expect the overall dependence $j \sim A^{1/3}$ and $V_2 \sim A^{-1}$ then $\mathcal{J} \sim A^{5/3}$ as in the case of normal rotational bands. Although the available information is still limited to a few examples that span a broad range of masses, they seem to confirm this prediction.

It is interesting to realize that a potential of the form $V = V_2P_2(\theta)$ can be expressed using the addition theorem for spherical harmonics [10] as

$$V \propto \sum_\mu Y_{2\mu}(\Omega_\pi) Y_{2\mu}(\Omega_\nu). \qquad (6)$$

Written in this form it resembles the quadrupole-quadrupole force [12], the basic ingredient of the long-range part of the nuclear force, which is responsible for the appearance of deformations in nuclei. We believe this provides a bridge between the TAC model that requires a deformed mean-field and our picture which is based only on a residual force.

Finally let us consider a spin-dependent force, represented, for example, by a $\vec{j}_\pi \cdot \vec{j}_\nu$ interaction. A P_1 term proportional to $\cos\theta$ can now appear in Eq. (4) and naturally give rise to a rotational spectrum [see Eq. (1)]. The importance of spin-dependent forces in nuclei is well established and of great interest. Whether they may manifest themselves in the shears mechanism remains an intriguing, yet open question.

In summary, we have presented a semiclassical analysis of the $B(M1)$ and $B(E2)$ values in the shears bands in ^{198,199}Pb. Based on the coupling of two spin vectors $\vec{j}_\pi$ and $\vec{j}_\nu$ we derive the angle between them in a given state of spin I, calculate the dependence of the reduced transition probabilities, and compare with the experimental results. This procedure seems to give a global consistent picture for both $B(M1)$ and $B(E2)$ values and provides additional support to the shears mechanism proposed by Frauendorf based on the TAC model.

It is proposed that a residual proton-neutron interaction may be responsible for the rotationallike motion observed for these bands. Indeed, we have shown in a simple schematic model that an effective interaction of the form $V_2P_2(\theta)$ can give rise to rotationallike bands with a spectrum that, for the particle-hole channel, follows approximately $\Delta E \propto (I - I_{90})^2$, in agreement with what is observed experimentally.

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