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Minimal relativity and $^3S_1$-$^3D_1$ pairing in symmetric nuclear matter

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We present solutions of the coupled, nonrelativistic $^3S_1$-$^3D_1$ gap equations for neutron-proton pairing in symmetric nuclear matter, and estimate relativistic effects by solving the same gap equations modified according to minimal relativity and using single-particle energies from a Dirac-Brueckner-Hartree-Fock calculation. As a main result we find that relativistic effects decrease the value of the gap at the saturation density $k_F = 1.36\ \mathrm{fm}^{-1}$ considerably, in conformity with the lack of evidence for strong neutron-proton pairing in finite nuclei. [S0556-2813(98)51003-5]

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The size of the neutron-proton (np) $^3S_1$-$^3D_1$ energy gap in symmetric nuclear matter has been a much debated issue since the first calculations of this quantity appeared. While solutions of the BCS equations with bare nucleon-nucleon (NN) forces give a large energy gap of several MeV at the saturation density $k_F = 1.36\ \mathrm{fm}^{-1}$ [1–4], there is little empirical evidence from finite nuclei for such strong np pairing correlations. One possible resolution of this problem lies in the fact that all these calculations have neglected contributions from the so-called induced interaction. Fluctuations in the isospin and the spin-isospin channel will probably make the pairing interaction more repulsive, leading to a substantially lower-energy gap [5]. Another often neglected aspect is that all nonrelativistic calculations of the nuclear matter equation of state (EOS) with two-body NN forces fitted to scattering data fail to reproduce the empirical saturation point, seemingly regardless of the sophistication of the many-body scheme employed. For example, a Brueckner-Hartree-Fock (BHF) calculation of the EOS with one of the Bonn potentials would typically give saturation at $k_F = 1.6$–$1.8\ \mathrm{fm}^{-1}$. In a nonrelativistic approach it seems necessary to invoke three-body forces to obtain saturation at the empirical equilibrium density. This leads one to be cautious when talking about pairing at the empirical nuclear matter saturation density when the energy gap is calculated within a pure two-body force model, as this density will be below the calculated saturation density for this two-body force, and thus one is calculating the gap at a density where the system is theoretically unstable. One even runs the risk, as pointed out in Ref. [6], that the compressibility is negative at the empirical saturation density, which means that the system is unstable against collapse into a nonhomogeneous phase. A three-body force need not have dramatic consequences for pairing, which after all is a two-body phenomenon, but still it would be of interest to know what the $^3S_1$-$^3D_1$ gap is in a model in which the saturation properties of nuclear matter are reproduced. If one abandons a nonrelativistic description, the empirical saturation point can be obtained within the Dirac-Brueckner-Hartree-Fock (DBHF) approach, as first pointed out by Brockmann and Machleidt [7]. This might be fortuitous, since, among other things, important many-body effects are neglected in the DBHF approach. Nevertheless, we found it interesting to investigate $^3S_1$-$^3D_1$ pairing in this model and compare our results with a corresponding nonrelativistic calculation.

The first ingredient in our calculation is the self-consistent evaluation of single-particle energies in symmetric nuclear matter starting from the meson-exchange potential models of Machleidt and co-workers [8]. For the nonrelativistic (NR) calculations we use the BHF method, while the DBHF scheme is used in the relativistic (R) calculation. Details of both approaches are found in Refs. [7,9,10]. Since BHF and DBHF are computationally very similar, we will here content ourselves with giving a brief description of the DBHF method. In this scheme, the single-particle energies and binding energy of nuclear matter is obtained using a medium renormalized NN potential G defined through the solution of the G-matrix equation

$$G(\omega) = V + VQ\frac{1}{\omega - QH_0Q}QG(\omega), \tag{1}$$

where ω is the unperturbed energy of the interacting nucleons, V is the free NN potential, H_0 is the unperturbed energy of the intermediate scattering states, and Q is the Pauli operator preventing scattering into occupied states. Only ladder diagrams with two-particle intermediate states are included in Eq. (1). In this work we solve Eq. (1) using the Bonn A potential defined in Table A.2 of Ref. [8]. This potential model employs the Thompson [7,11] reduction of the Bethe-Salpeter equation, and is tailored for relativistic nuclear structure calculations. For the nonrelativistic calculation we employ the Bonn A potential with parameters from Table A.1 in Ref. [8]. This model employs the Blankenbecler-Sugar (BbS) reduction of the Bethe-Salpeter equation, and is therefore suited for nonrelativistic calculations. For further details, see Refs. [7,8,10].

The DBHF is a variational procedure where the single-particle energies are obtained through an iterative self-consistency scheme. To obtain the relativistic single-particle energies, we solve the Dirac equation for a nucleon in the nuclear medium, with $c = \hbar = 1$,

$$[\not{p} - m + \Sigma(p)]\tilde{u}(p,s) = 0, \tag{2}$$

where m is the free nucleon mass and $\tilde{u}(p,s)$ is the Dirac spinor for positive energy solutions, $p=(p^0,\mathbf{p})$ being a four momentum and s the spin projection. The self-energy $\Sigma(p)$ for nucleons can be written as

$$\Sigma(p)=\Sigma_S(p)-\gamma_0\Sigma^0(p)+\boldsymbol{\gamma}\cdot\mathbf{p}\Sigma^V(p). \tag{3}$$

Since $\Sigma^V \ll 1$ [7,12], we approximate the self-energy by

$$\Sigma\approx\Sigma_S-\gamma_0\Sigma^0=U_S+U_V, \tag{4}$$

where U_S is an attractive scalar field and U_V is the timelike component of a repulsive vector field. The Dirac spinor then reads

$$\tilde{u}(p,s)=\sqrt{\frac{\tilde{E}_p+\tilde{m}}{2\tilde{m}}}\begin{pmatrix}\chi_s\\ \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\tilde{E}_p+\tilde{m}}\chi_s\end{pmatrix}, \tag{5}$$

where χ_s is the Pauli spinor and terms with tilde like $\tilde{E}_p=\sqrt{\mathbf{p}^2+\tilde{m}^2}$ represent medium modified quantities. Here we have defined [7,12] $\tilde{m}=m+U_S$. The single-particle energies $\tilde{\varepsilon}_p$ can then be written as

$$\tilde{\varepsilon}_p=\langle p|\boldsymbol{\gamma}\cdot\mathbf{p}+m|p\rangle+u_p=\tilde{E}_{\mathbf{p}}+U_V, \tag{6}$$

where the single-particle potential u_p is given by $u_p=U_S\tilde{m}/\tilde{E}_p+U_V$ and can in turn be defined in terms of the G-matrix

$$u_p=\sum_{p'\leq k_F}\frac{\tilde{m}^2}{\tilde{E}_{\mathbf{p}'}\tilde{E}_p}\langle pp'|G(\omega=\tilde{\varepsilon}_p+\tilde{\varepsilon}'_p)|pp'\rangle, \tag{7}$$

where p,p' represent quantum numbers like momentum, spin, isospin projection, etc., of the different single-particle states and k_F is the Fermi momentum. Equations (6) and (7) are solved self-consistently starting with adequate values for the scalar and vector components U_S and U_V. The energy per particle can then be calculated from

$$\frac{\mathcal{E}}{A}=\frac{1}{A}\sum_{p'\leq k_F}\frac{\tilde{m}m+\mathbf{p}'^2}{\tilde{E}_{\mathbf{p}'}}+\frac{1}{2A}\sum_{p'p''\leq k_F}\frac{\tilde{m}^2\langle p'p''|G(\tilde{E}=\tilde{\epsilon}'_p+\tilde{\epsilon}_{p''})|p'p''\rangle_{\mathrm{AS}}}{\tilde{E}_{\mathbf{p}'}\tilde{E}_{\mathbf{p}''}}-m. \tag{8}$$

In Fig. 1 we show the EOS obtained in our nonrelativistic and relativistic calculations. The nonrelativistic one fails to meet the empirical data, while the relativistic calculation very nearly succeeds.

Having obtained in-medium single-particle energies, we proceed to solve the coupled gap equations for ${}^3S_1$-${}^3D_1$ pairing. Employing an angle-average approximation, these can be written [3]

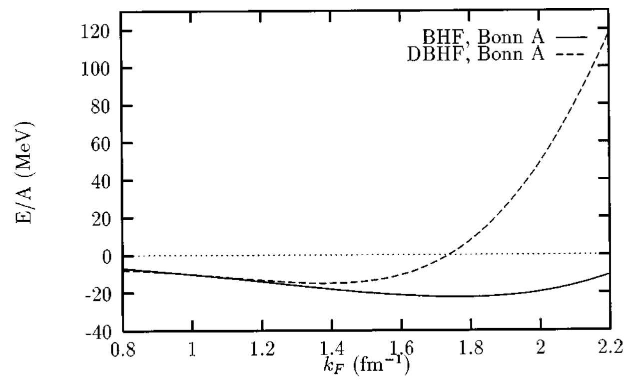

FIG. 1. EOS for symmetric nuclear matter with the NN potentials and many-body methods described in the text.

$$\Delta_0(k)=-\int_0^\infty dk'k'^2\frac{1}{E(k')}[V_{00}(k,k')\Delta_0(k')-V_{02}(k,k')\Delta_2(k')], \tag{9}$$

$$\Delta_2(k)=-\int_0^\infty dk'k'^2\frac{1}{E(k')}[-V_{20}(k,k')\Delta_0(k')+V_{22}(k,k')\Delta_2(k')], \tag{10}$$

where the subscripts 0 and 2 denote S and D states, respectively, $V_{ll'}$ is the free momentum-space NN interaction in the relevant channel, Δ_0 and Δ_2 are the S and D state gap functions, respectively, and $E(k)$ is the quasiparticle energy given by

$$E(k)=\sqrt{(\epsilon_k-\mu)^2+\Delta_0(k)^2+\Delta_2(k)^2}, \tag{11}$$

where μ is the chemical potential. The quantity

$$D_F=\sqrt{\Delta_0(k_F)^2+\Delta_2(k_F)^2} \tag{12}$$

will in the following be referred to as the energy gap, according to the conventional definition [1–4]. For ${}^3S_1$-${}^3D_1$ pairing it is also necessary to solve the equation for particle number conservation

$$\rho\equiv\frac{2k_F^3}{3\pi^2}=\frac{1}{\pi^2}\int_0^\infty dkk^2\left(1-\frac{\epsilon_k-\mu}{E(k)}\right) \tag{13}$$

for μ self-consistently together with Eqs. (9) and (10).

In the nonrelativistic calculation, we have used the Bonn A potential with parameters from Table A.1 in Ref. [8]. The results are shown in Fig. 2. We found a large energy gap at the empirical saturation density, around 6 MeV at $k_F=1.36$ fm^{-1}, in agreement with earlier nonrelativistic calculations [1–4].

Recently, several groups have developed relativistic formulations of pairing in nuclear matter [13–15], and have applied them to ${}^1S_0$ pairing. The models are of the Walecka-type [12] in the sense that meson masses and coupling constants are fitted so that the mean-field EOS of nuclear matter meets the empirical data. In this way, however, the relation of the models to free-space NN scattering becomes somewhat unclear. An interesting result found in Refs. [13–15] is

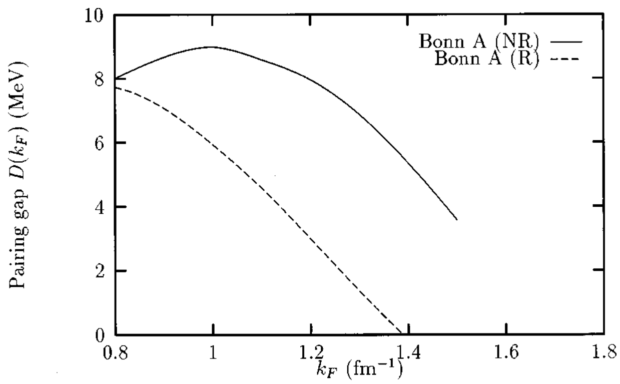

FIG. 2. $^3S_1$-$^3D_1$ energy gap in nuclear matter calculated in a nonrelativistic (full line) and a relativistic (dashed line) approach.

that the $^1S_0$ energy gap vanishes at densities slightly below the empirical saturation density. This is in contrast with nonrelativistic calculations which generally give a relatively small, but nonvanishing $^1S_0$ gap at this density, see for instance [16–19].

With these results in mind, we found it interesting to consider relativistic effects on $^3S_1$-$^3D_1$ pairing in nuclear matter, which, to our knowledge, has not been done before. A simple way of doing this is to incorporate minimal relativity in the gap equation, thus using DBHF single-particle energies in the energy denominators and modifying the free NN interaction by a factor $\tilde{m}^2/\tilde{E}_k\tilde{E}_{k'}$ [20]. With this prescription, we obtained the results shown in Fig. 2 (dashed line). As can be seen, the gap at the empirical saturation density is reduced from 6 MeV to nearly zero.

Let us try to understand the difference between the relativistic and the nonrelativistic calculation. First of all, we point out that it is well known that the introduction of relativity in the many-body problem leads to increased repulsion at densities at and above the saturation density [21]. Even a slight increase in the repulsion might have large consequences for the energy gap, as the gap depends exponentially on the interaction at the Fermi surface. One can obtain a numerical estimate of the effect as follows. If one takes the weak-coupling limit of the gap equations, Eqs. (9) and (10), it is easy to show that one obtains the same form for the energy gap as for $^1S_0$ pairing,

$$D_F=2\,\delta\epsilon\exp\left(-\frac{1}{N(k_F)V_{\text{pair}}}\right), \tag{14}$$

where $\delta\epsilon$ is an appropriate energy interval, $N(k_F)=m^*k_F/2\pi^2\hbar^2$ is the density of states at the Fermi surface, m^* is the nucleon effective mass. The pairing interaction at the Fermi surface, V_{pair}, is given by

$$V_{\text{pair}}=\frac{\sqrt{(V_{SS}-V_{DD})^2+4V_{SD}^2}-V_{SS}-V_{SD}}{2}, \tag{15}$$

where $V_{SS}=V_{00}(k_F,k_F)$, $V_{SD}=V_{02}(k_F,k_F)=V_{20}(k_F,k_F)$, and $V_{DD}=V_{22}(k_F,k_F)$. With our relativistic approach to the gap equation, the corresponding weak-coupling expression for the gap is obtained by replacing V_{pair} with $\tilde{m}^2/\tilde{E}_{k_F}^2V_{\text{pair}}$ and using the relativistic expression for the density of states instead of the nonrelativistic one. We will now consider the saturation density, and take $k_F=1.4$ fm^{-1}. The nonrelativistic single-particle spectrum was parameterized as $\epsilon_k=k^2/2m^*+U_0$. At $k_F=1.4$ fm^{-1} the values of the parameters were $m^*/m=0.6751$, $U_0=-97.2755$ MeV. For the relativistic single-particle spectrum the relevant quantities were $U_S=-384.89$ MeV, $U_V=300.18$ MeV. We found $N_{\text{R}}(k_F=1.4\ \text{fm}^{-1})/N_{\text{NR}}(k_F=1.4\ \text{fm}^{-1})\approx 1$, and then

$$\frac{D_F^{\text{R}}}{D_F^{\text{NR}}}=\exp\left[-\frac{1}{N_{\text{NR}}(1.4)V_{\text{pair}}}\left(\frac{\tilde{E}_F^2}{\tilde{m}^2}-1\right)\right]$$

$$\approx\left(\frac{D_F^{\text{NR}}}{2\,\delta\epsilon}\right)^{1/4},$$

where the superscript R refers to relativistic quantities, NR to nonrelativistic ones. If we make the common choice $\delta\epsilon=\epsilon_F^{\text{NR}}$ and use $D_F\approx 6$ MeV, we obtain

$$\frac{D_F^{\text{R}}}{D_F^{\text{NR}}}\approx 0.5,$$

thus, the introduction of relativity in the gap equation suppresses the gap at the saturation density by a factor of two. This argument makes it reasonable that relativistic effects reduce the gap. That the reduction is larger in the full calculation than in this simple estimate is understandable, since we in the weak coupling approximation neglect the momentum-dependence of the interaction. More specifically, the repulsive high-momentum components are left out, and these will reduce the gap further.

It is also interesting to obtain the ratio $D_F^{\text{R}}/D_F^{\text{NR}}$ at the respective saturation densities for the relativistic and nonrelativistic calculations. The nonrelativistic EOS saturates at $k_F\approx 1.8$ fm^{-1}. At this density, we had numerical problems with solving the gap equations, something which may occur when the gap is small. However, using the weak-coupling expression for the gap, we could estimate $D_F^{\text{R}}(k_F=1.4\ \text{fm}^{-1})/D_F^{\text{NR}}(k_F=1.8\ \text{fm}^{-1})$. First we used the nonrelativistic gaps in the density range $k_F=1.2$–1.4 fm^{-1} to calculate $N_{\text{NR}}(k_F)V_{\text{pair}}(k_F)$, and fitted the results with a quadratic polynomial in k_F. From this fit we estimated $N_{\text{NR}}(k_F=1.8)V_{\text{pair}}(k_F=1.8)\approx 0.257$. Then

$$\frac{N_{\text{NR}}(1.8)V_{\text{pair}}(1.8)}{N_{\text{NR}}(1.4)V_{\text{pair}}(1.4)}=\frac{0.257}{0.333}$$

$$\Rightarrow V_{\text{pair}}(1.8)=\frac{N_{\text{NR}}(1.4)}{N_{\text{NR}}(1.8)}\,\frac{0.257}{0.333}V_{\text{pair}}(1.4)$$

$$\approx 0.695V_{\text{pair}}(1.4)$$

where we have used $m^*(1.4)/m=0.675$, $m^*(1.8)/m=0.5834$ in the densities of states. We then formed the ratio $D_F^{\text{R}}(1.4)/D_F^{\text{NR}}(1.8)$ and obtained

$$\frac{D_F^{\mathrm{R}}}{D_F^{\mathrm{NR}}} = \exp\left[-\frac{1}{N_{\mathrm{NR}}(1.8)V_{\mathrm{pair}}(1.4)} \times \left(\frac{N_{\mathrm{NR}}(1.8)}{N_{\mathrm{R}}(1.4)} \frac{\tilde{E}_F^2(1.4)}{\tilde{m}^2(1.4)} - \frac{1}{0.695} \right) \right] \approx 1,$$

since we found $N_{\mathrm{NR}}(1.8)/N_{\mathrm{R}}(1.4) \approx 1.15$, $\tilde{E}_F^2(1.4)/\tilde{m}^2(1.4) \approx 1.25$, thus making the expression in the inner parentheses ≈ 0. Although this argument is only indicative, it makes it reasonable to assume that the nonrelativistic gap will be very small at the calculated nonrelativistic saturation density.

In this work, we have presented nonrelativistic and relativistic calculations of the $^3S_1$-$^3D_1$ np gap in symmetric nuclear matter. The nonrelativistic calculations gives a large gap of approximately 6 MeV at the empirical saturation density. In the relativistic calculation we find that the gap is vanishingly small at this density. This is our main result. Nonrelativistic calculations with two-body interactions will in general give a saturation density which is too high, an example of which is shown in Fig. 1. Thus, in the nonrelativistic approach we are actually calculating the gap at densities below the theoretical saturation density, and one may question the physical relevance of a large gap at a density where the system is theoretically unstable. If one looks at the gap at the *calculated* saturation density, it is in fact close to zero. In the DBHF calculation we come very close to reproducing the empirical saturation density and binding energy, and when this is used as a starting point for a BCS calculation, we find that the gap vanishes, both at the empirical and the calculated saturation density. That the DBHF calculation meets the empirical points is perhaps fortuitous, as important many-body diagrams are neglected and only medium modifications of the nucleon mass are accounted for. However, a recent investigation where medium modifications of meson masses were included, showed that the results of the DBHF do not change very much, and in particular the saturation properties are still very good [22]. Nevertheless, the essential property which is needed in all nonrelativistic models to get to the empirical point is an increased repulsion at and around the empirical saturation density. Regardless of the mechanism, this may reduce the pairing gap dramatically. The main point we wish to make is thus that the inclusion of these additional repulsive effects may suppress pairing at the empirical saturation density. We should add that one should include higher-order many-body effects in the pairing interaction. The first correction to the bare force, coming from the so-called induced interaction, is probably repulsive, and will thus reduce the energy gap further. A study of this contribution is under way [23].

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