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Isoscalar and isovector dipole mode in drip line nuclei in comparison with β-stable nuclei

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The isoscalar and the isovector dipole mode of drip line and β-stable nuclei are investigated, using the self-consistent Hartree-Fock plus the random-phase approximation with Skyrme interactions. Including simultaneously both the isoscalar and the isovector correlation, the RPA response function is estimated in the coordinate space so as to take into account properly the continuum effect. The spurious component is carefully taken away from the calculated strength. In β-stable nuclei such as ^{208}Pb the frequency of the isovector giant dipole resonance (IVGDR) is lower than that of the isoscalar giant dipole resonance (compression mode). In contrast, in lighter drip line nuclei a major part of the isoscalar compression dipole strength lies at an energy much lower than the energy of the IVGDR. [S0556-2813(98)50203-8]

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The dynamical response of drip line nuclei to various external fields is expected to show an interesting exotic structure, due to the presence of the low-lying threshold strength unique in those nuclei. Performing the Hartree-Fock (HF) calculation with Skyrme interactions and then using the random-phase approximation (RPA), we have studied the response functions of drip line nuclei [1]. Taking into account both the isoscalar (IS) and isovector (IV) correlation in the RPA [2], which is solved in coordinate space using Green's functions, in the present work we study the IS dipole mode (compression mode) in comparison with the IV dipole mode. Taking away carefully the IS spurious (center of mass) component from the calculated dipole spectra, we compare the obtained dipole response of drip line nuclei with that of β-stable nuclei.

The IV giant dipole resonance (IVGDR) is well established and the oldest one among various giant resonances in nuclei [3]. The IS dipole resonance was theoretically studied already more than 20 years ago [3,4]. And, numerical calculations were made for some doubly closed β-stable nuclei [5,6], for which hadron inelastic scattering experiments could be easily done. Recently, the observation of an IS giant dipole resonance (ISGDR), the IS dipole compression mode, in ^{208}Pb was reported. In Ref. [7] the peak of the ISGDR was identified at $E_x = 22.5$ MeV, using the (α,α') cross sections at forward angles. In β-stable nuclei the ISGDR may well be expected at such a high energy. In contrast, it is an interesting open question whether a considerable amount of the IS compression dipole strength will appear in the low-energy threshold region, since it was pointed out [1,2] that the response functions for various noncompression multipoles show in general a large transition strength in the low-energy region just above the threshold.

We study the RPA strength function

$$S(E) \equiv \sum_n |\langle n|D|0\rangle|^2 \delta(E-E_n) = \frac{1}{\pi} \text{Im Tr}(D^\dagger G_{\text{RPA}}(E) D). \tag{1}$$

In Eq. (1) D represents the one-body operators

$$D_\mu^{\lambda=1,\tau=1} = \sum_i \tau_z(i) r_i Y_{1\mu}(\hat{r}_i) \quad \text{for isovector dipole strength} \tag{2}$$

and

$$D_\mu^{\lambda=1,\tau=0} = \sum_i r_i^3 Y_{1\mu}(\hat{r}_i) \quad \text{for isoscalar dipole strength.} \tag{3}$$

The transition density for an excited state $|n\rangle$,

$$\rho_{n0}^{\text{tr}}(\vec{r}) \equiv \langle n|\sum_{i=1}^{A} \delta(\vec{r}-\vec{r}_i)|0\rangle, \tag{4}$$

can be calculated from the RPA response, and the radial transition density $\rho_n^{\text{tr}}(r)$ is defined by

$$\rho_{n0}^{\text{tr}}(\vec{r}) \equiv \rho_n^{\text{tr}}(r) Y_{\lambda\mu}(\hat{r}). \tag{5}$$

If the numerical calculation of the self-consistent RPA could be achieved with perfect accuracy, then the spurious center-of-mass (c.m.) state would be degenerate with the ground state and the excitation spectra would be free from the spurious component. In practice, it is difficult to take away completely the weak dipole strength contributed by the spurious component, especially from the low-lying rather isolated RPA peaks, which come from some particular particle-hole $(p\text{-}h)$ configurations. Thus, for the IV dipole strength we use the operator

$$\overline{D}_\mu^{\lambda=1,\tau=1} = -\frac{2N}{A} \sum_i^{\text{proton}} r_i Y_{1\mu}(\hat{r}_i) + \frac{2Z}{A} \sum_i^{\text{neutron}} r_i Y_{1\mu}(\hat{r}_i), \tag{6}$$

which is obtained from Eq. (2) with the condition that the dipole field should only depend on coordinates relative to the center of mass. In the higher energy region the calculated RPA response function is almost exactly the same as the one obtained by using $D_\mu^{\lambda=1,\tau=1}$ in Eq. (2).

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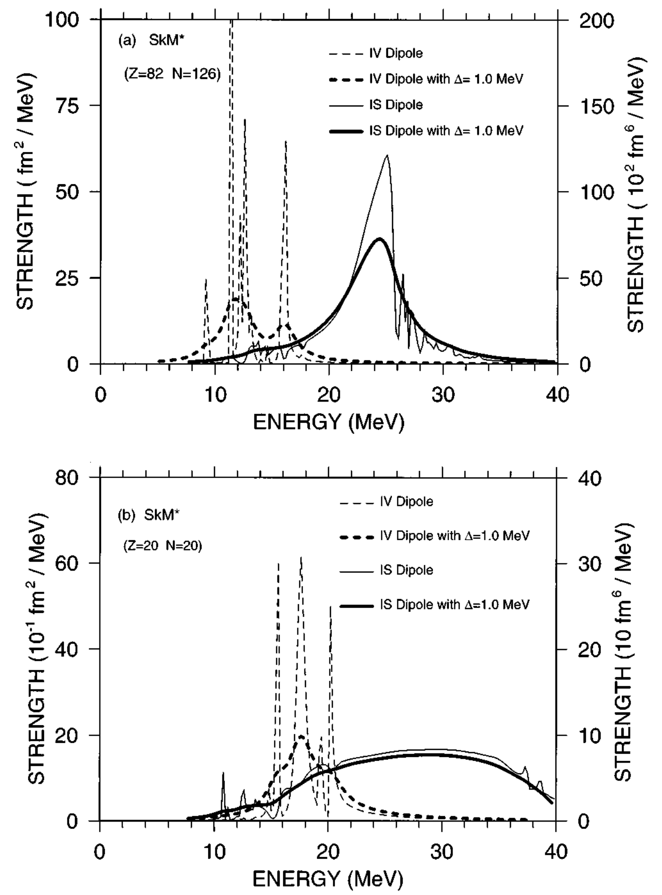

FIG. 1. The RPA IS and IV dipole strength in the β-stable nuclei, (a) ${}^{208}_{82}\mathrm{Pb}_{126}$ and (b) ${}^{40}_{20}\mathrm{Ca}_{20}$, as a function of excitation energy. The scale of the IS dipole strength is shown on the right-hand side, while that of the IV dipole strength is denoted on the left-hand side. The solid line expresses the IS dipole strength, while the dashed line denotes the IV dipole strength. The thick lines are obtained by averaging the calculated RPA strength (denoted by the respective thin lines) using Eq. (13) with $\Delta = 1$ MeV. The strength appearing below the threshold due to the averaging procedure has no meaning. The SkM* interaction is used.

For the IS dipole strength with the operator (3) a fairly small admixed spurious component may make an appreciable contribution, since the r dependence (r^3) of the operator (3) is different from that (r) of the c.m. operator. Thus, we have found that a more elaborate way of eliminating the spurious component is needed. For the energy region of a few MeV above the threshold, where the calculated strength for the c.m. operator

$$(\mathrm{CM})_{\mu}^{\lambda=1,\tau=0} = \sum_i r_i Y_{1\mu}(\hat{r}_i) \tag{7}$$

is small but sometimes non-negligible, we subtract the estimated radial transition density $\rho_{spr}^{\mathrm{tr}}(r)$ for the operator (7) from the one $\rho_n^{\mathrm{tr}}(r)$ for the operator (3) at the same energy, so as to guarantee the absence of the strength for the c.m. operator at each energy. It is observed that the radial dependence of $\rho_{spr}^{\mathrm{tr}}(r)$ depends somewhat on the energy and is often slightly different from $d\rho_0/dr$, where $\rho_0(r)$ expresses the HF ground-state density. On the other hand, for the higher energy region we have used the expression in which the radial dependence of $\rho_{spr}^{\mathrm{tr}}(r)$ is equal to that of $d\rho_0/dr$. Namely, we first calculate the radial transition density $\rho_n^{\mathrm{tr}}(r)$ for the operator (3) and determine the coefficient a so as to satisfy the condition that the calculated strength for the c.m. operator should vanish,

$$\int \left(\rho_n^{\mathrm{tr}}(r) - a\,\frac{d\rho_0}{dr}\right) r^3 dr = 0. \tag{8}$$

Then, we evaluate the strength function with the operator (3), using the radial transition density,

$$\rho_n^{\mathrm{tr}}(r) - a\,\frac{d\rho_0}{dr}, \tag{9}$$

which is now free from the spurious (c.m.) component. In fact, the above procedure using Eqs. (8) and (9) is equivalent to evaluating the strength function of the operator

$$\overline{D_{\mu}^{\lambda=1,\tau=0}} = \sum_i^{A} (r_i^3 - \eta r_i) Y_{1\mu}(\hat{r}_i) \tag{10}$$

where $\eta = \frac{5}{3}\langle r^2\rangle$, using the transition density $\rho_{n0}^{\mathrm{tr}}(\vec{r})$ that is calculated for the operator $D_{\mu}^{\lambda=1,\tau=0}$ in Eq. (3). In numerical calculations at all excitation energies we have very carefully taken away the dipole strength coming from the spurious component, which could be admixed. After carefully subtracting the spurious component from the calculated RPA strength function, almost all strong sharp peaks in the IS dipole strength function of the nuclei ^{40}Ca, ^{90}Zr, and ^{208}Pb that appeared at $E \lesssim 16$ MeV in Fig. 5 of Ref. [6], are no longer present.

Whether the calculated excitation spectra with $D_{\mu}^{\lambda=1,\tau=0}$ in Eq. (3) contains an appreciable amount of the spurious component or not may be, in principle, checked by evaluating the energy-weighted sum rule (EWSR) for the operator in Eq. (3):

$$\sum_n E_n |\langle n | D_{\mu}^{\lambda=1,\tau=0} | 0\rangle|^2 = \frac{A\hbar^2}{8\pi M}\, 11\langle r^4\rangle, \tag{11}$$

where the states $|n\rangle$ also include the spurious state. A problem is that we do not exactly know the amount of the contribution to the EWSR in Eq. (11) by the spurious state. However, it can be analytically calculated in the harmonic oscillator model. For example, taking $N=Z$, the contribution is 55% for $N_F = 5-7$, assuming that all one-particle orbitals with the harmonic oscillator principal quantum number equal to or less than N_F are occupied. We may expect that for ^{208}Pb the estimate using the harmonic oscillator model works well. On the other hand, assuming that the strength of $D_{\mu}^{\lambda=1,\tau=0}$ in Eq. (3) concentrates only on the spurious state and one intrinsically excited collective state, the ratio of the contribution by the spurious state to the total EWSR is expressed as [5,6]

$$\frac{\frac{25}{3}\langle r^2\rangle^2}{11\langle r^4\rangle}, \tag{12}$$

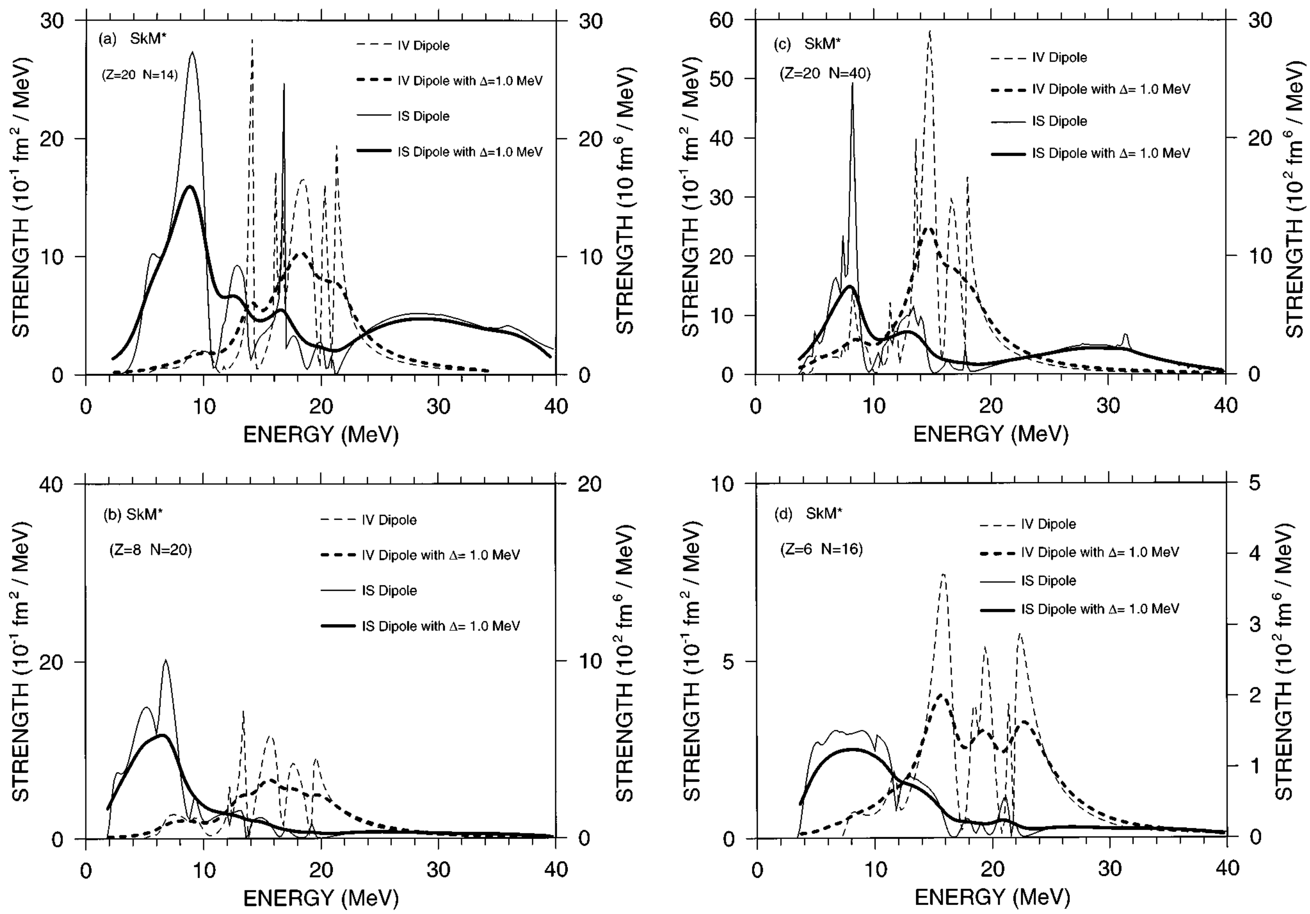

FIG. 2. The RPA IS and IV dipole strength as a function of excitation energy in (a) the proton drip line nucleus $^{34}_{20}\mathrm{Ca}_{14}$ and (b) the neutron drip line nucleus $^{28}_{8}\mathrm{O}_{20}$, (c) $^{60}_{20}\mathrm{Ca}_{40}$ and (d) $^{22}_{6}\mathrm{C}_{16}$. See the caption to Fig. 1 for details.

which is equal to 59% for the HF ground state of ^{208}Pb using the SkM* interaction. The ratio becomes smaller in lighter nuclei and in drip line nuclei. We note that even in the harmonic oscillator model ''one collective state'' is an approximation, since not all strength with the $1\hbar\omega_0$ excitations for the operator $D_{\mu}^{\lambda=1,\tau=0}$ belongs to the spurious excitation. Our present numerical calculation indicates that in ^{208}Pb the spurious state consumes up to 68% of the EWSR in Eq. (11). We have summed up the contributions from the calculated intrinsic excitations for $E_x \leqslant 90$ MeV and obtained only 32% of the EWSR.

Since our calculated RPA strength function does not contain the spreading width, namely the coupling to $2p$-$2h$ (or more complicated) configurations relative to the ground states, the calculated width of each peak cannot be directly compared with experiments. In order to simulate the coupling to those configurations, in Figs. 1 and 2 we show also the averaged strength functions,

$$\overline{S(E)} = \int S(E_0)\rho(E-E_0)dE_0 \tag{13}$$

with the weight function

$$\rho(E-E_0) = \frac{1}{\pi}\frac{\Delta}{(E-E_0)^2+\Delta^2}. \tag{14}$$

In plotting the figures we have used the value of $\Delta = 1$ MeV, which is somewhat arbitrarily chosen. The strength, which appears in the energy region below the threshold due to the averaging procedure, has no meaning.

In Fig. 1 the calculated RPA strength functions corresponding to the IV dipole operator $D_{\mu}^{\lambda=1,\tau=1}$ in Eq. (2) and the IS dipole operator $D_{\mu}^{\lambda=1,\tau=0}$ in Eq. (3) are shown for the β-stable nuclei, $^{208}_{82}\mathrm{Pb}_{126}$ and $^{40}_{20}\mathrm{Ca}_{20}$. Both the RPA strength functions (1) and the averaged ones with $\Delta = 1$ MeV in Eq. (13) are shown. The SkM* interaction is used both in the HF and the RPA calculation. In $^{208}_{82}\mathrm{Pb}_{126}$ the peak energy of the ISGDR in the RPA strength function (1) is 25.0 MeV, while the energy defined by the formula

$$\overline{E} = \frac{m_1}{m_0} \tag{15}$$

is equal to 23.4 MeV. In Eq. (15) the energy-weighted moments m_k are defined by

$$m_k = \int S(E)E^k dE. \tag{16}$$

The ISGDR in the energy region of 18 to 30 MeV of Fig. 1(a), which is expressed by the thin solid line, consumes 27% of the total EWSR in Eq. (11), namely, about 85% of the EWSR coming from the calculated intrinsic excitations.

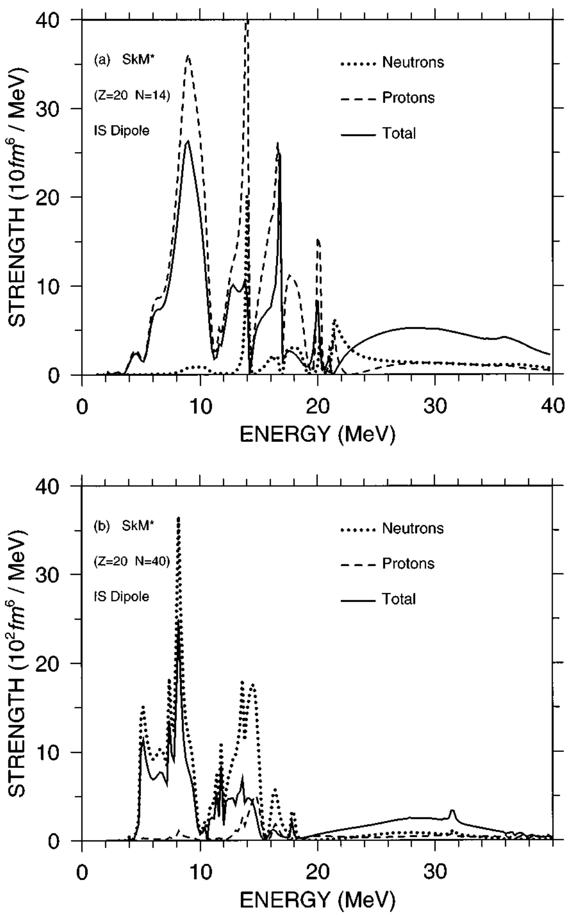

FIG. 3. The RPA dipole response of protons and that of neutrons to the IS operator (3) in (a) $^{34}_{20}\mathrm{Ca}_{14}$ and (b) $^{60}_{20}\mathrm{Ca}_{40}$, in comparison with the total RPA IS dipole response. In the estimate of the strength function the spurious component is eliminated using the method of Eqs. (8) and (9) at all energies.

It is seen from Fig. 1(a) that in $^{208}_{82}\mathrm{Pb}_{126}$ the calculated IVGDR lies energetically about 10 MeV lower than the estimated ISGDR. The measured peak energy of the IVGDR is 13.4 MeV, which is almost equal to our averaged calculated value, $\bar{E}=13.3$ MeV, while the ISGDR is reported [7] to be found around 22.5 MeV.

In Fig. 2 we show the IV and the IS dipole strength functions for the proton drip line nucleus $^{34}_{20}\mathrm{Ca}_{14}$ and the neutron drip line nuclei $^{28}_{8}\mathrm{O}_{20}$, $^{60}_{20}\mathrm{Ca}_{40}$ and $^{22}_{6}\mathrm{C}_{16}$. It is well known that in very light nuclei it is difficult to interpret the observed IV dipole strength in terms of a single resonance (''IVGDR'') frequency, since the difference between the energies of relevant p-h excitations may be comparable with or even larger than the width of the possible giant resonance. The multiple peak structure can be seen in the IV dipole strength function in Fig. 2. Exactly in those light drip line nuclei it is seen that the major part of the IS dipole transition strength is consumed by the threshold strength and lies clearly below the ''IVGDR,'' while the higher-lying IS dipole strength is hardly observed as a single giant resonance (''ISGDR'') since it becomes so broad, with an extremely large tail. In $^{34}_{20}\mathrm{Ca}_{14}$ and $^{60}_{20}\mathrm{Ca}_{40}$ one can recognize the very broad bump of the high-energy ''ISGDR,'' though the low-energy IS dipole strengths just above the threshold appear as stronger and better defined peaks. The IVGDR in these nuclei lies energetically between the low-energy IS dipole peaks and the very broad ''ISGDR''.

In Figs. 3(a) and 3(b) the RPA dipole response of protons and that of neutrons to the operator (3) are shown for $^{34}_{20}\mathrm{Ca}_{14}$ and $^{60}_{20}\mathrm{Ca}_{40}$, respectively, in comparison with the total RPA IS dipole response. For simplicity, in the estimate of the strength function the spurious component is eliminated using the method with Eqs. (8) and (9) at all energies. The low-energy threshold strength consists predominantly of proton excitations in the proton drip line nucleus $^{34}\mathrm{Ca}$, while it comes exclusively from neutron excitations in the neutron drip line nuclei such as $^{22}\mathrm{C}$, $^{28}\mathrm{O}$ and $^{60}\mathrm{Ca}$. The unique structure of the threshold strength, which comes essentially from the uncorrelated excitations of protons or neutrons with small binding energies, is very similar to that for other noncompression multipoles studied previously. However, in Figs. 3(a) and 3(b) it is very interesting to observe that in the high-energy ''ISGDR'' region the neutron contribution interferes constructively with the proton contribution, as expected for the IS collective mode. In contrast, in the lower energy region the neutron and proton contributions almost always interfere destructively.

We have performed numerical calculations with several Skyrme interactions and confirmed that the conclusions drawn by using the SkM* interaction remain the same for all Skyrme interactions, except for numerical details. For example, the calculated frequency of the IVGDR is sensitive to the value of the symmetry energy coefficient of the Skyrme interactions used. In contrast, since the ISGDR is a compression mode, the calculated frequency is sensitive to the incompressibility of the Skyrme interaction employed. A higher frequency of the ISGDR is obtained for Skyrme interactions with a higher incompressibility. For example, the peak energy of the ISGDR calculated by using the SIII interaction is considerably higher than that estimated by employing the SkM* interaction. See, for example, Ref. [6].

After finishing the present work, we received a report [8] in which the dipole response in nuclei with large neutron excess is studied using the HF plus RPA model. In the calculation of Ref. [8] the continuum states were represented by a finite set of oscillator functions and, thus, obtained as discrete states.

In conclusion, we have studied both the IS and the IV dipole strength function of drip line nuclei in comparison with those of β-stable nuclei, using the self-consistent HF plus the RPA with Skyrme interactions. The spurious (c.m.) component, which should be degenerate in the self-consistent calculation but may be admixed into excitation spectra in practical calculations, is carefully subtracted from the calculated spectra. In lighter drip line nuclei the low-energy threshold strength consumes a considerable part of the IS dipole strength, while the high-energy ISGDR (compression mode) becomes so broad that it may be difficult to identify it experimentally in a given excitation energy region. Furthermore, those low-energy IS dipole peaks lie clearly lower than the IVGDR. In contrast, in β-stable nuclei such as $^{208}\mathrm{Pb}$ and $^{40}\mathrm{Ca}$ the frequency of the ISGDR is definitely much higher than that of the IVGDR. The presence of the low-lying IS dipole peaks in drip line nuclei may play a very important role in electron or hadron scattering experiments, though it may not have so much effect on photon scattering.

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