Vector mesons in the nuclear medium with a finite three-momentum

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We formulate a QCD sum rule to find the three-momentum ($\mathbf{q}$) dependence of the peak position of the vector meson spectral density in the nuclear medium. For both the longitudinal and transverse polarization direction of the vector meson with respect to $\mathbf{q}$, we find less than a 2% (0.1%) shift of the peak position in nuclear matter density and at $\mathbf{q}=0.5$ GeV/c for the ρ,ω (ϕ) mesons. This justifies neglecting the three-momentum dependence and the polarization effect when implementing the universal scaling laws of the vector mesons in understanding the dilepton spectrum in the A-A and p-A reactions. [S0556-2813(98)05602-7]

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The properties of vector mesons in the nuclear medium have attracted a lot of interest because of the potential to experimentally observe a nonperturbative aspect of QCD, namely, the restoration of spontaneously broken chiral symmetry at finite temperature or density, through dileptons from A-A or p-A reactions [1]. Many model calculations have been performed to calculate the vector meson mass shift at finite density. By now, there seems to be a consensus that the average peak position of the vector meson spectral density at zero three-momentum ($\mathbf{q}=0$) will shift down at finite density [2–6].

Indeed, there already exist dilepton data from the CERES Collaboration, which report an enhancement of low mass dileptons below the ρ meson invariant mass in S+Au and recently from Pb+Au collisions at CERN [7]. So far, all conventional collision models failed to explain the enhancement except when the ρ meson mass is allowed to decrease in the medium as predicted by theoretical calculations [8]. However, before coming to any definite conclusions, it is necessary to consider all possible conventional mechanisms.

In the nuclear medium, in addition to a possible change in the vector meson mass, there will be a breaking of Lorentz invariance and hence two independent polarization directions of the vector mesons. Each polarization will have a different dispersion relation, which to leading order in the three-momentum would be modified to $\omega^2-(1+a)\mathbf{q}^2-m_V^2=0$ with $a\neq 0$. Suppose, experimentally, that one detects a dilepton with energy ω and three-momentum $\mathbf{q}$, then the average peak due to the vector meson will appear at $M^2=(m_V^2+a\mathbf{q}^2)$, so that if $a<0$, the strength will be shifted downwards for dilepton pairs with $\mathbf{q}\neq 0$. Hence, it is important to estimate the finite $\mathbf{q}$ effect.

In this paper, we formulate a QCD sum rule to find the finite $\mathbf{q}$ effect to leading order in density for the transverse and longitudinal direction of the ρ, ω, and ϕ mesons. As we will see, for both polarization direction, we find less than 2% (0.1%) shift of the peak position at nuclear matter density and at ($\mathbf{q}=0.5$ GeV/c) for the ρ, ω, (ϕ) mesons. This is about a 10% effect of the expected scalar mass shift in medium and justifies neglecting the three-momentum dependence and the polarization effect when implementing the universal scaling laws of the vector mesons in the medium [2]. This formalism also provides the first attempt to estimate the leading $\mathbf{q}$ dependence of the V-N T matrix, when it has a small off-shell dependence.

Let us consider the correlation function between the vector current with ρ, ω, and ϕ meson quantum numbers, $J_\mu^{\rho,\omega}=\frac{1}{2}(\overline{u}\gamma_\mu u\mp\overline{d}\gamma_\mu d)$ and $J_\mu^{\phi}=\overline{s}\gamma_\mu s$ in the nuclear medium:

$$\Pi_{\mu\nu}(\omega,\mathbf{q})=i\int d^4x e^{iqx}\langle T[J_\mu(x)J_\nu(0)]\rangle_{\mathrm{NM}}. \qquad (1)$$

Here $\langle\rangle_{\mathrm{NM}}$ denotes the nuclear matter expectation value. In general, because the vector current is conserved, the correlation function in Eq. (1) will have two invariant functions [9]:

$$\Pi_{\mu\nu}(\omega,\mathbf{q})=\Pi_T q^2 \mathrm{P}^T_{\mu\nu}+\Pi_L q^2 \mathrm{P}^L_{\mu\nu}, \qquad (2)$$

where for $q=(\omega,\mathbf{q})$ and the medium at rest, we have $\mathrm{P}^T_{00}=\mathrm{P}^T_{0i}=\mathrm{P}^T_{i0}=0$, $\mathrm{P}^T_{ij}=\delta_{ij}-\mathbf{q}_i\mathbf{q}_j/\mathbf{q}^2$, and $\mathrm{P}^L_{\mu\nu}=(q_\mu q_\nu/q^2-g_{\mu\nu}-\mathrm{P}^T_{\mu\nu})$. In the limit when $\mathbf{q}\to 0$, there is only one invariant function $\Pi_L(\omega,0)=\Pi_T(\omega,0)$. In this work, we will formulate QCD sum rules for both $\Pi_L(\omega,\mathbf{q})$ and $\Pi_T(\omega,\mathbf{q})$ at finite $\mathbf{q}$.

The starting point is the energy dispersion relation at finite $\mathbf{q}$. For small $\mathbf{q}^2<\omega^2$, we can make a Taylor expansion of the correlation function such that

$$\begin{aligned}\mathrm{Re}\ \Pi_{L,T}(\omega^2,\mathbf{q}^2)&=\mathrm{Re}[\Pi^0_{L,T}(\omega^2,0)+\Pi^1_{L,T}(\omega^2,0)\mathbf{q}^2+\cdots]\\&=\int_0^\infty du^2\left(\frac{\rho(u,0)^0_{L,T}}{(u^2-\omega^2)}+\frac{\rho(u,0)^1_{L,T}}{(u^2-\omega^2)}\mathbf{q}^2+\cdots\right),\end{aligned} \qquad (3)$$

here $\rho(u,\mathbf{q})=1/\pi\ \mathrm{Im}\ \Pi^R(u,\mathbf{q})$, and R denotes the retarded correlation function. As we will discuss below, the left-hand side (Re Π) is known only to leading order in $\mathbf{q}^2$ and since we are interested in the leading behavior anyway, we will look at the dispersion relation for Π^1 in Eq. (3).

The real part of Eq. (3) is calculated via the operator product expansion (OPE) at large $-\omega^2\to\infty$ with finite $\mathbf{q}$. The full polarization tensor will have the following form:

$$\Pi_{\mu\nu}(\omega,\mathbf{q})=(q_\mu q_\nu-g_{\mu\nu})\Bigg[-c_0\ \ln|Q^2| + \sum_d \frac{c_{d,d}}{Q^d}A^{d,d}(\mathrm{NM})\Bigg]+\sum_{s,\tau=2}\frac{1}{Q^{s+\tau-2}} \times[c_{d,\tau}g_{\mu\nu}q^{\mu_1}\cdots q^{\mu_s}A^{d,\tau}_{\mu_1\cdots\mu_s}(\mathrm{NM})+\cdots], \tag{4}$$

where $Q^2=\mathbf{q}^2-\omega^2$. Here, $A^{d,\tau}(\mathrm{NM})$ represents the nuclear matter expectation value of an operator of dimension d and twist $\tau=d-s$, where s is the number of spin index. These operators are defined at the scale Q^2 and the c's are the dimensionless Wilson coefficients with the running coupling constant. This way of including the density effect is consistent at low energy [10]. The first set of terms in Eq. (4) come from the OPE of scalar operators, the second set from operators with nonzero spin s.

To linear order in density, the matrix elements are related to the nucleon expectation values of the operator via

$$A(\mathrm{NM})=A_0+n_n A_N, \tag{5}$$

where A_0 is the corresponding vacuum expectation value, A_N the nucleon expectation value, and n_n the nuclear density.

As in the vacuum, we will truncate our OPE up to dimension 6 operators. This implies that in our OPE in Eq. (4), we will have contributions from $(\tau,s)=(2,2),(2,4),(4,2)$. The nucleon matrix elements of the $\tau=2$ operators are well known. The $\tau=4$ matrix element appearing in the ρ, ω sum rules are similar to those appearing in electron DIS [11] and have been estimated [12,13] up to about $\pm30\%$ uncertainty from available DIS data from CERN and Slac.

The $\mathbf{q}$ dependence coming from the first line of Eq. (4), namely, the contribution from the scalar operators, come from the $\mathbf{q}$ dependence in Q^2. These form the so called ''trivial'' $\mathbf{q}$ dependence, and comes from replacing $\omega^2\to\omega^2-\mathbf{q}^2$ when going from zero to finite three-momentum. Here, we are not interested in these trivial dependence and also not in the possible change in the scalar mass m_V. Consequently we do not need the nucleon expectation value of the scalar operators. Operators with spin also partly contribute to the change in the scalar mass and hence also to the trivial changes. However, these spin parts also give the nontrivial $\mathbf{q}$ dependence. The nontrivial $\mathbf{q}^2$ dependence in the OPE is obtained by first calculating the total $\mathbf{q}^2$ term in Π^1 and then subtracting out the trivial dependence; $1/\omega^n[1+d(\mathbf{q}^2/\omega^2)]\to[d-(n/2)](\mathbf{q}^2/\omega^{n+2})$. Using this, we find the following contributions from the $\tau=2,4$ operators:

$$\Pi^1_{L,T}(\omega)/\rho_n=\frac{b_2}{\omega^6}+\frac{b_3}{\omega^8}. \tag{6}$$

For ρ, ω, the transverse (T) and longitudinal (L) parts give

$$b_2^T=\left(\frac{1}{2}C^q_{2,2}-\frac{1}{2}C^q_{L,2}\right)mA_2^{u+d}+(C^G_{2,2}-C^G_{L,2})mA_2^G, \tag{7}$$

$$b_3^T=\left(\frac{9}{4}C^q_{2,4}-\frac{5}{2}C^q_{L,4}\right)m^3A_4^{u+d}+\left(\frac{9}{2}C^G_{2,4}-5C^G_{L,4}\right)m^3A_4^G + \frac{1}{2}m\Bigg[-(1+\beta)\left(K^1+\frac{3}{8}K^2+\frac{7}{16}K^g\right)+K^1_{ud}(1\pm1)\Bigg], \tag{8}$$

$$b_2^L=-\frac{1}{2}C^q_{L,2}mA_2^{u+d}-C^G_{L,2}mA_2^G, \tag{9}$$

$$b_3^L=\left(\frac{1}{2}C^q_{2,4}-\frac{5}{2}C^q_{L,4}\right)m^3A_4^{u+d}+(C^G_{2,4}-5C^G_{L,4})m^3A_4^G + \frac{m}{8}(1+\beta)\left(K^2-\frac{3}{2}K^g\right), \tag{10}$$

where $\pm$ refers to the ρ and ω case. Here, m is the nucleon mass and for even n,

$$A_n^q=2\int_0^1 dx x^{n-1}[q(x,Q^2)+\overline{q(}x,Q^2)],$$

$$A_n^G=2\int_0^1 dx x^{n-1}G(x,Q^2), \tag{11}$$

where $q(x,Q^2)$ and $G(x,Q^2)$ are the quark and gluon distribution functions. We will use the HO parametrization for these obtained in Ref. [14] which should be used with the Wilson coefficients $C's$ in the $\overline{\mathrm{MS}}$ scheme. The $C^{q(G)}_{2,n}$ denotes the Wilson coefficient in the F_2 direction of the quarks' (gluon) nth moments, the $C^{q(G)}_{L,n}$ denotes that in the longitudinal directions, which are all given in [15]. Terms proportional to $K's$ come from $\tau=4$, $s=2$. We will use the set of K values obtained in Ref. [12] $(K^1,K^2,K^1_{ud},K^g)=(-0.173,0.203,-0.083,-0.238\ \mathrm{GeV}^2)$ and we take $\beta=0.5$. For the $\tau=4$ operators, we neglect the Q^2 dependence.

For the ϕ meson,

$$b_2^T=(2C^q_{2,2}-2C^q_{L,2})mA_2^s+(2C^G_{2,2}-2C^G_{L,2})mA_2^G, \tag{12}$$

$$b_3^T=(9C^q_{2,4}-10C^q_{L,4})m^3A_4^s+(9C^G_{2,4}-10C^G_{L,4})m^3A_4^G - \frac{5}{2}mC, \tag{13}$$

$$b_2^L=-2C^q_{L,2}mA_2^s-2C^G_{L,2}mA_2^G, \tag{14}$$

$$b_3^L=(2C^q_{2,4}-10C^q_{L,4})m^3A_4^s+(2C^G_{2,4}-10C^G_{L,4})m^3A_4^G-9mC, \tag{15}$$

where C is defined by $\langle N(p)|\overline{s}D_\mu D_\nu m_s s|N(p)\rangle=(p_\mu p_\nu-\frac{1}{4}m^2g_{\mu\nu})C$. We let $C=-x_s\langle N|\overline{s}m_s s|N\rangle=-0.1548x_s\ \mathrm{GeV}^2$, where we used the same number for the strange content of the nucleon as in Ref. [16] with the normalization of $\langle p|p\rangle=2\omega(2\pi)^3\delta^3(p-p)$. x_s is an unknown number, however, assuming its order to be similar to the ratio A_4^s/A_2^s, we will take $x_s=0.04$.

Table I summarizes each contribution to the b coefficients at $Q^2=1\ \mathrm{GeV}^2$. As can be seen from the table, for the trans-

TABLE I. $b_2(q)$ $[b_2(G)]$ represents the contribution of quark (gluon) operators to b_2. b_{32} (b_{34}) represents the contribution of the $\tau=2$ ($\tau=4$) operators to b_3. The units for b_2, b_3 are GeV and GeV^3, respectively

	$b_2^L(q)$	$b_2^L(G)$	b_{32}^L	b_{34}^L	$b_2^T(q)$	$b_2^T(G)$	b_{32}^T	b_{34}^T
ρ	-0.021	-0.014	0.047	0.096	0.417	-0.025	0.242	0.060
ω	-0.021	-0.014	0.047	0.096	0.417	-0.025	0.242	0.138
ϕ	-0.004	-0.028	-0.003	0.052	0.074	-0.049	-0.002	0.015

verse part of the ρ,ω, the well known twist-2 quark contribution dominates both the b_2, b_3. However, for the longitudinal part, both the quark and gluon twist-2 part contributes at order α_s and the twist-4 part becomes important, which has a larger uncertainty. This is also roughly true for the ϕ meson. At higher Q^2 values, the values of b's decrease in general.

In the vacuum, the spectral density appearing in the left-hand side of Eq. (3) is modeled with a pole and a continuum: $8\pi^2\rho(u)=F\delta(u^2-m_V^2)+c_0\theta(u^2-S_0)$. In our case, we allow the three parameters to vary nontrivially by a term proportional to $\mathbf{q}^2$ and implicitly the nuclear density n_n, such as, $F\to F+f\cdot\mathbf{q}^2$, $m_V^2\to m_V^2+a\cdot\mathbf{q}^2$, $S_0\to S_0+s\cdot\mathbf{q}^2$. Also, when using the dispersion relation in Eq. (3), there is a possible ambiguity in the subtraction constants and we have to know if there exist singularities in the spectral density of the form $\delta(u^2)$ or $\delta'(u^2)$. The contribution proportional to $\delta(u^2)$ is not known. However, the other singularity can be unambiguously calculated from the nucleon-hole contribution. This term is called the scattering term [3] and gives a nontrivial contribution to the longitudinal direction. Hence, the spectral density of $\Pi^1(\omega)$ is

$$8\pi^2\rho^1(u)=f\delta(u^2-m_V^2)-aF\delta'(u^2-m_V^2) \tag{16}$$

$$-sc_0\delta(u^2-S_0)+8\pi^2 n_n b_{\rm scatt}\delta'(u^2), \tag{17}$$

where $b_{\rm scatt}=1/(4m)$ for the longitudinal ρ,ω and zero otherwise. For conventional reason, $4\pi^2$ will be used for ϕ meson instead of $8\pi^2$.

The Borel sum rule with the $\delta(u^2)$ ambiguity subtracted out is obtained by taking the Borel transform of $\omega^2\Pi^1(\omega)$. This gives

$$\left[m_\rho^2 f+F\left(1-\frac{m_\rho^2}{M^2}\right)a\right]e^{-m_\rho^2/M^2}-c_0S_0e^{-S_0/M^2}s$$

$$=8\pi^2 n_n\left(\frac{b_2}{M^2}-\frac{b_3}{2M^4}+b_{\rm scatt}\right). \tag{18}$$

The Q^2 dependence in the b's coming from the twist-2 operators changes to the M^2 dependence [17]. The vacuum parameters are first determined from the vacuum sum rules with parameters given as in Ref. [3]. This gives $(m_V^2,F,S_0)=(0.77^2,1.48,1.43)$ GeV^2 for ρ,ω and $(m_\phi^2,F,S_0)=(1.02^2,2.19,2.1)$ GeV^2 for ϕ. Then the parameters are determined by least square fit method. The Borel interval for the transverse ρ meson is determined by requiring that the contribution from dim 6 operators are less than 35% of the dim 4 contributions, which determines the $M_{\rm min}^2\sim 1$ GeV^2. The maximum Borel mass is determined by requiring that the continuum contribution is again less than 35% of the first term in the OPE, which gives $M_{\rm max}^2\sim 2.3$ GeV^2. But it turns out that even if we choose higher $M_{\rm max}^2$, the result differ by less than 10%. The uncertainty here comes from the less reliable number of the $b_3^t(\tau=4)$, which gives less than 10% uncertainty in the final answer. Overall, combining the uncertainty coming from the continuum, we expect 40% uncertainty for the numbers for the transverse ρ and also for the transverse ω,ϕ, which can be analyzed in a similar fashion. For the longitudinal directions of ρ,ω, the dim 4 operators are suppressed by α_s and the dim 6 operators contribute at tree level. So here we choose the $M_{\rm min}\sim 2.5$ GeV^2, at which the dim 4 and dim 6 contributions are of similar order and take $M_{\rm max}\sim 3.5$ to 5 GeV^2. Here, we expect 50% uncertainty. A similar analysis can be done for the longitudinal ϕ. The results are shown in Table II, the numbers are all at the nuclear matter density. For higher density, one can just multiply the numbers with the relevant ratio to the nuclear matter density. Our results are consistent with effective hadronic model calculations. First, as can be seen from Table II, the scattering term decreases the longitudinal a value. This is consistent with the known result from the nucleon-hole contribution [6]. Second, recent calculation using resonance-nucleon hole contribution for the transverse part shows an attraction [18]. This is also consistent with our a values for the transverse part of the ρ,ω.

As discussed before, a nonvanishing a will shift the average peak position by $\Delta M=\sqrt{m_V^2+a\mathbf{q}^2}-m_V$, even if there is no change in the scalar mass m_V. With the values of a obtained, we have plotted the fractional change $\Delta M/m_V$ for the ρ meson in Fig. 1(a). The solid lines denote the result at nuclear matter density, and the dashed lines that at 3 times nuclear matter density. The results for the ω meson look similar to Fig. 1(a). Figure 1(b) shows the result for the ϕ meson. As can be seen from the figures, even at $\mathbf{q}$

TABLE II. Results for the parameters at nuclear matter density. The values are from best fit of the Borel sum rule in Eq. (18). The values in the bracket are the results without the scattering term.

	a	f	s	Borel interval GeV^2
Transverse ρ	-0.065	0.137	-0.008	$1\sim 2.3$
Transverse ω	-0.040	0.120	0.009	$1.3\sim 2.5$
Longitudinal ρ,ω	0.021	0.068	0.027	$2.5\sim 3.5$
	(0.061)	(-0.042)	(0.042)	
Transverse ϕ	0.004	0.010	0.009	$0.9\sim 2.0$
Longitudinal ϕ	0.009	-0.001	0.009	$2.0\sim 3.0$

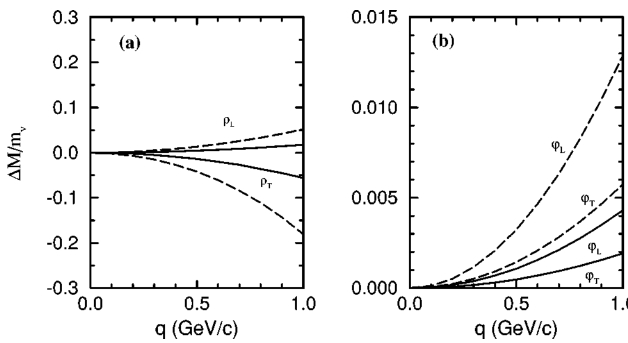

FIG. 1. (a) The fractional change $\Delta M/m_V$ of the peak position of the ρ as a function of **q**. The solid (dashed) lines show the results at nuclear matter (three times nuclear matter) density. The positive changes correspond to the longitudinal direction, the negative changes correspond the transverse directions. (b) The fractional changes of the ϕ as a function of **q**. The solid (dashed) lines show the result at nuclear matter (three times nuclear matter) density. The larger changes correspond to the longitudinal direction, the smaller changes correspond to the transverse directions.

$=1$ GeV/c, at which point our formalism breaks down, the shifts are less than 5% (0.5%) at nuclear matter density for the ρ,ω (ϕ). This is much smaller than the expected scalar mass shift of ρ,ω ($\sim$20%) and ϕ ($\sim$3%) [3] and justifies neglecting the three-momentum dependence and the polarization effect when implementing the universal scaling laws (Brown-Rho scaling) of the vector mesons in understanding the dilepton spectrum in A-A and p-A reaction [8].

The contribution of the longitudinal and transverse polarization to the dilepton spectrum depends on the angle between the sum and difference of the three-momentum of the out going dileptons [19]. However, after averaging, the contribution of the transverse polarization becomes twice that of the longitudinal polarization. Hence, to a good approximation, one can implement the finite **q** effect into model calculations by including only the transverse dispersion relation. Making a linear fit and including the scalar mass shift [3], one can parametrize the ρ meson mass in medium as follows:

$$\frac{m_V(n_n)}{m_V(0)}=1-(0.16\pm0.06)\frac{n_n}{n_0}-(0.014\pm0.005)\left(\frac{\mathbf{q}}{0.5}\right)^2\frac{n_n}{n_0}, \tag{19}$$

where **q** is in the GeV unit and n_0 is the nuclear saturation density. -0.014 changes to -0.084 (0.0005) for the ω (ϕ) meson.

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