Effects of multipion correlations on the source distribution in ultrarelativistic heavy-ion collisions

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The multipion correlation effect on the source distribution is studied. It is shown that multipion Bose-Einstein correlations make the average radius of the pion source become smaller. The isospin effect on the pion multiplicity distribution and the source distribution is also discussed. [S0556-2813(98)03702-9]

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Ultrarelativistic hadronic and nuclear collisions provide a unique environment to create dozens, and in some cases hundreds, of pions [1–3]. To study the pion source distributions in these processes, therefore, one must take into account the effects of multipion Bose-Einstein (BE) correlations [4–14]. Among others, Lam and Lo [4] suggested that the larger isospin fluctuations were due to the Bose nature of the emitted pions. Pratt [7] suggested that if the number of bosons in a unit value of phase space is large enough, bosons may condense into the same quantum state and a pion laser could be created. Considering isospin effects, Pratt and Zelevinsky [8] used the method in Ref. [7] to explain Centauro events [15,16]. Zajc [6] first used the Monte Carlo method to analyze multipion Bose-Einstein correlation effects on two-pion interferometry. A detailed derivation of multipion Bose-Einstein correlations can be found in Ref. [12]. The bosonic nature and isospin of the pion should affect the single-pion spectrum distribution in coordinate space. However, this issue has not yet been discussed in the literature. The purpose of this paper is to analyze the effects of multipion correlations and isospin on the source distribution in coordinate space.

We begin with write down the definition for the n-pion inclusive source distribution:

$$P_n(x_1, \ldots ,x_n)=\langle j^*(x_1)\cdots j^*(x_n)j(x_1)\cdots j(x_n)\rangle, \quad (1)$$

which can be explained as the probability of observing n pions at point $\{x_i ,i=1,n\}$ all in the same n-pion event. Here $j(x)$ is the current of pions, which can be expressed as [9,12,13]

$$j(x)=\int j(x',p)\exp\{ip(x-x')\}\gamma(x')d^4x'd^4p, \quad (2)$$

where $j(x',p)$ is the probability amplitude of finding a pion with momentum p, emitted by the emitter at x'. $\gamma(x')$ is a random phase factor which has been taken away from $j(x',p)$. All emitters are uncorrelated in coordinate space if

$$\langle \gamma^*(x)\gamma(y)\rangle=\delta^4(x-y). \quad (3)$$

Taking the phase average and using Eq. (3), one can reexpress the n-pion inclusive distribution [Eq. (1)] as

$$P_n(x_1, \ldots ,x_n)=\sum_\sigma \rho_{1,\sigma(1)}\cdots\rho_{n,\sigma(n)}, \quad (4)$$

with

$$\rho_{i,j}=\rho(x_i ,x_j)=\langle j^*(x_i)j(x_j)\rangle, \quad \int \rho(x,x)d^4x=n_0. \quad (5)$$

Here $\sigma(i)$ denotes the ith element of a permutation of the sequence $1,2,3, \ldots ,n$, and the sum over σ runs over all $n!$ permutations of this sequence. n_0 is the mean pion multiplicity without multipion Bose-Einstein correlations.

Taking into account the n-pion correlation effect, the normalized modified i-pion inclusive distribution in n-pion events, $P_i^n(x_1, \ldots ,x_i)$, can be written as

$$P_i^n(x_1, \ldots ,x_i)=\frac{\int \prod_{j=i+1}^n d^4x_jP_n(x_1, \ldots ,x_n)}{\int \prod_{j=1}^n d^4x_jP_n(x_1, \ldots ,x_n)}, \quad (6)$$

which can be explained as the probability of finding i pions at point $\{x_j ,j=1,i\}$ in n-pion events.

Now we define the function $G_i(x,y)$ as [7,12]

$$G_i(x,y)=\int \rho(x,x_1)dx_1\rho(x_1,x_2)dx_2\cdots\rho \times(x_{i-2},x_{i-1})dx_{i-1}\rho(x_{i-1},y). \quad (7)$$

From the expression of $P_n(x_1, \ldots ,x_n)$ [Eq. (4)], the single-pion inclusive distribution in coordinate space can be expressed as

$$P_1^n(x)=\frac{1}{n}\frac{1}{\omega(n)}\sum_{i=1}^n G_i(x,x)\omega(n-i), \quad (8)$$

with

$$\omega(n)=\frac{1}{n!}\int P_n(x_1, \ldots ,x_n)\prod_{k=1}^n d^4x_k. \quad (9)$$

Here $\omega(n)$ is the pion multiplicity distribution probability. Experimentally, one usually mixes all events to analyze the single-pion inclusive distribution. Thus the single-pion inclusive distribution reads

$$P_1^\phi(x)=\frac{\sum_{n=1}^\infty\omega(n)nP_1^n(x)}{\langle n\rangle\sum_n\omega(n)}, \quad (10)$$

with

$$\langle n \rangle = \frac{\sum_{n=1}^{\infty} \omega(n) n}{\sum_n \omega(n)}. \tag{11}$$

Using Eq. (8), one can write down the single-pion inclusive distribution

$$P_1^{\phi}(x) = \frac{1}{\langle n \rangle} \sum_{i=1}^{\infty} G_i(x,x). \tag{12}$$

In the following, we will employ a simple model to analyze the multipion correlation effects on the source distribution. We assume that

$$\rho(x,y) = n_0 \left(\frac{1}{\pi R_1^2} \right)^{3/2} \exp[-(\vec{x}+\vec{y})^2/4R_1^2] \times \exp[-(\vec{x}-\vec{y})^2/4R_2^2] \sqrt{\delta(x_0)\,\delta(y_0)}. \tag{13}$$

Here R_1 and R_2 are the parameters that represents the radius of the chaotic source and the correlation length of pions, respectively. $x=(x_0,\vec{x})$ are the pion's four-dimensional coordinates. Then, $G_n(x,y)$ can be expressed as

$$G_n(x,y) = n_0^n \sqrt{\delta(x_0)\,\delta(y_0)}\, \alpha_n \exp\{-a_n(\vec{x}^2+\vec{y}^2) + g_n \vec{x}\cdot\vec{y}\}, \tag{14}$$

where

$$a_{n+1} = a_1 - \frac{g_1^2}{4b_n}, \quad g_{n+1} = \frac{g_1 g_n}{2b_n}, \quad b_n = a_n + a_1, \tag{15}$$

and

$$\alpha_{n+1} = \alpha_n \left(\frac{1}{b_n} \right)^{3/2} \left(\frac{1}{R_1^2} \right)^{3/2}, \tag{16}$$

with

$$a_1 = \frac{1}{4R_1^2} + \frac{1}{4R_2^2}, \quad g_1 = \frac{1}{2R_2^2} - \frac{1}{2R_1^2}, \quad \alpha_1 = \left(\frac{1}{\pi R_1^2} \right)^{3/2}. \tag{17}$$

From the above formula, we obtain the pion source distribution in n-pion events $[P_1^n(\vec{x}) = \int P_1^n(x) dx_0]$, which we show in Fig. 1. Evidently, because of the multipion Bose-Einstein correlation effects, the pions are concentrated in the same state. The mean radius of the source becomes smaller. The larger the pion multiplicity, the larger the BE correlation effects on the source distribution. Multipion correlation effects on the source distribution $[P_1^{\phi}(\vec{x}) = \int P_1^{\phi}(x) dx_0]$ are shown in Fig. 2, where we mix the pion multiplicity. It is clear that as the mean pion multiplicity becomes larger, the multipion correlation effects on the source distribution become larger.

The pion state with isospin can be written as [10,17]

$$|\phi\rangle = \exp\left(\int \vec{j}(x)\cdot\vec{a}^{+}(x) \right) |0\rangle, \tag{18}$$

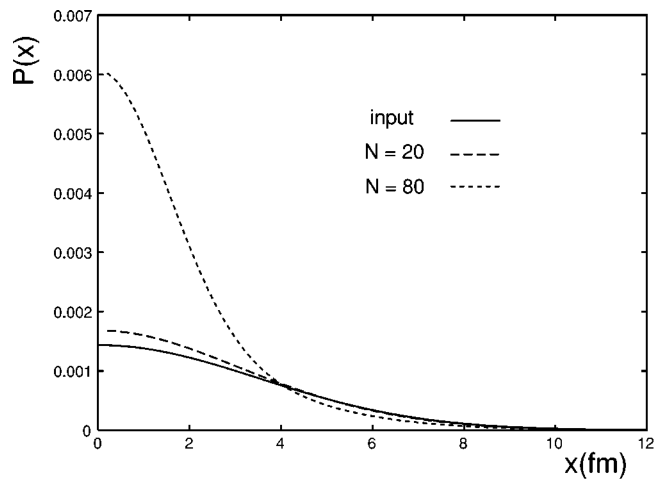

FIG. 1. Multipion correlation effects on the source distribution. The solid line corresponds to the input source distribution. The dashed and dotted lines correspond to $n=20$ and 80, respectively. The input value of R_1 and R_2 is 5 and 0.8 fm, respectively.

where $\vec{j}$ and $\vec{a}$ are the vectors in isospin space which can be expressed as

$$\vec{j}(x) = j(x) \left[\frac{1}{\sqrt{2}} \sin\theta e^{-i\phi}, \cos\theta, -\frac{1}{\sqrt{2}} \sin(\theta) e^{i\phi} \right] \tag{19}$$

and

$$\vec{a}^{+}(x) = [a_{\pi^-}^{+}(x), a_{\pi^0}^{+}, a_{\pi^+}^{+}]. \tag{20}$$

Then the state with total isospin I and z component of isospin I_z is

$$|\phi, I, I_z\rangle = \int \sin\theta d\theta d\phi Y_{I,I_z}^{*}(\theta,\phi) \exp\left(\int \vec{j}(x)\cdot\vec{a}^{+}(x) \right) |0\rangle. \tag{21}$$

$Y_{I,I_z}(\theta,\phi)$ is the spherical harmonic of angular momentum I and z component I_z. From the above formula we can calcu-

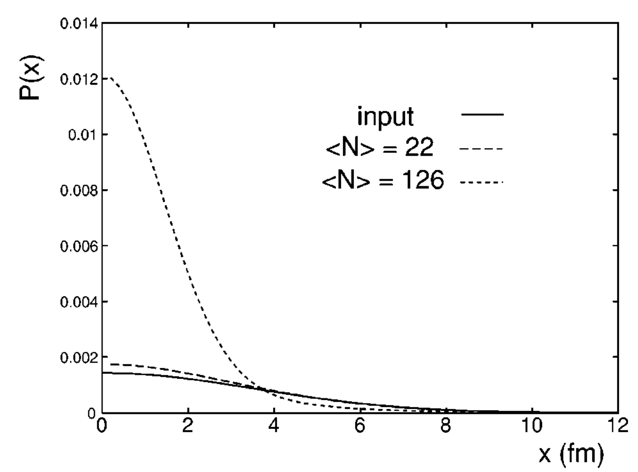

FIG. 2. Multipion correlation effects on the source distribution. The solid line corresponds to the input source distribution. The dashed and dotted lines correspond to $\langle n \rangle = 22$ and 126, respectively. The input value of R_1 and R_2 is 5 and 0.8 fm, respectively.

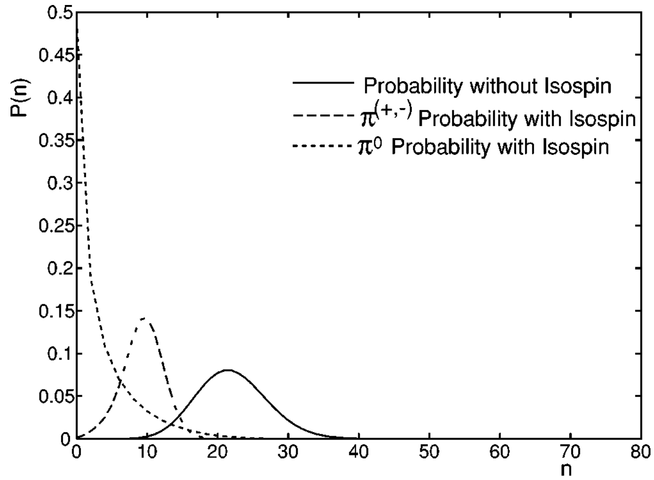

FIG. 3. Pion multiplicity distribution. The solid line corresponds to the pion probability distribution without the isospin effect. The dashed and dotted lines correspond to π^0 and $\pi^{+,-}$ probability distributions, respectively. The input value of R_1, R_2, and n_0 is 5 fm, 0.8 fm, and 20, respectively.

late the isospin effect on the pion probability and pion source distribution. For the sake of simplicity, we will only consider in the following the $I=0$, $I_z=0$ case. One can easily check that $|\phi,0,0\rangle$ can be expanded in Fock space as

$$|\phi,0,0\rangle=\sum_{n_{\pi^0}}\sum_{n_c/2}\frac{1}{2^{n_c/2}}B\left[\frac{1}{2}+\frac{n_{\pi^0}}{2},\frac{n_c}{2}+1\right]|n_{\pi^0}\rangle\left|\frac{n_c}{2}\right\rangle\left|\frac{n_c}{2}\right\rangle, \tag{22}$$

with

$$|n_{\pi^0,\pi^+,\pi^-}\rangle=\frac{[\int j(x)a^+_{\pi^0,\pi^+,\pi^-}d^4x]^n}{n!}|0\rangle. \tag{23}$$

Here $|n_\pi\rangle$ denotes the n-pion state and $B(x,y)$ is the beta function. $n_c=n_{\pi^+}+n_{\pi^-}$ is the total number of charged pions. Because of isospin conservation, we have $n_{\pi^+}=n_{\pi^-}=n_c/2$ and n_{π^0} must be even. At this stage we can calculate the π^0 and $\pi^{+,-}$ probability distributions according to

$$P(n_{\pi^0})=\frac{\Sigma_{n_c}\omega(n_{\pi^0})\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}+1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\omega(n_{\pi^0})\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}+1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)} \tag{24}$$

and

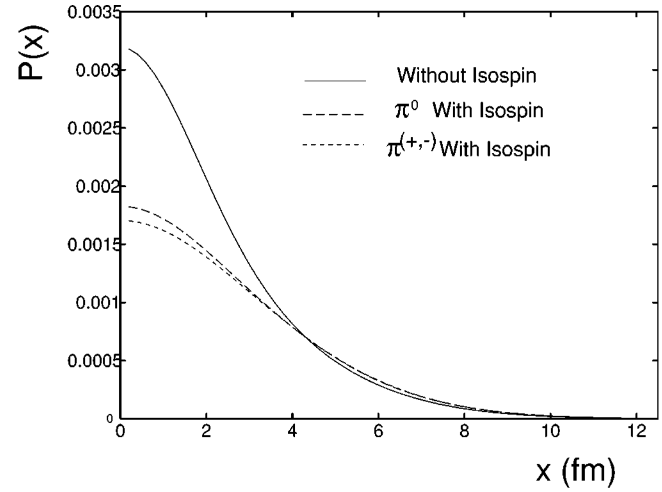

FIG. 4. Multipion correlation and isospin effects on the source distribution. The solid line corresponds to the source distribution without the isospin effect. The dashed and solid lines correspond to π^0 and $\pi^{+,-}$ source distributions, respectively. The input value of R_1 , R_2, and n_0 is 5 fm, 0.8 fm, and 40, respectively.

$$P\left(n_{\pi^+}=n_{\pi^-}=\frac{n_c}{2}\right)=\frac{\Sigma_{n_{\pi^0}}\omega(n_{\pi^0})\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}+1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\omega(n_{\pi^0})\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}+1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)}. \tag{25}$$

The π^0 and $\pi^{+,-}$ probability distributions are shown in Fig. 3. For comparison, the pion probability without isospin conservation is also shown as a solid line in Fig. 3. We notice that due to the isospin effect, the π^0 and $\pi^{+,-}$ probability distributions are quite different from each other now. This phenomenon has been noticed in Ref. [8]. But the method given here enables us to calculate the isospin effect for any isospin state, not limited to isosinglet states as discussed in Ref. [8]. From the π^0 and $\pi^{+,-}$ probability distributions, we acquire the mean multiplicities $\langle n_{\pi^0}\rangle$ and $\langle n_{\pi^{+,-}}\rangle$:

$$\langle n_{\pi^0}\rangle=\frac{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)n_{\pi^0}\omega(n_{\pi^0})}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_0}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)\omega(n_{\pi^0})} \tag{26}$$

and

$$\langle n_{\pi^{+,-}}\rangle=\frac{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)n_{\pi^{+,-}}\omega(n_{\pi^0})}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_0}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)\omega(n_{\pi^0})}. \tag{27}$$

It is well known that in the case of the isospin singlet, if the multiparticle BE correlation in coordinate space is neglected ($R_2=0$ fm), there would be $\langle n_{\pi^0}\rangle=\langle n_{\pi^+}\rangle=\langle n_{\pi^-}\rangle$ [17]. As the multiparticle BE correlation is included in coordinate space, however, such a simple relation does not hold for the isosinglet any more. As an example, to make a quantitative sense, we set $R_1=5$ fm, $R_2=0.8$ fm, and $n_0=20$ and obtain $n_{\pi^0}=2.86$ and $n_{\pi^+}=n_{\pi^-}=9.38$.

The π^0 and $\pi^{+,-}$ source distribution reads

$$P_{\pi^0}(x)=\frac{\langle\phi,0,0|a^+_{\pi^0}(x)a_{\pi^0}(x)|\phi,0,0\rangle}{\langle\phi,0,0|\phi,0,0\rangle\langle n_{\pi^0}\rangle}=\frac{1}{\langle n_{\pi^0}\rangle}\frac{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)\Sigma_{i=1}^{n_{\pi^0}}G_i(x,x)\omega(n_{\pi^0}-i)}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)\omega(n_{\pi^0})} \quad (28)$$

and

$$P_{\pi^{+,-}}(x)=\frac{\langle\phi,0,0|a^+_{\pi^{+,-}}(x)a_{\pi^{+,-}}(x)|\phi,0,0\rangle}{\langle\phi,0,0|\phi,0,0\rangle\langle n_{\pi^{+,-}}\rangle}$$

$$=\frac{1}{\langle n_{\pi^{+,-}}\rangle}\frac{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega(n_{\pi^0})\omega\left(\frac{n_c}{2}\right)\Sigma_{i=1}^{n_{\pi^{+,-}}}G_i(x,x)\omega\left(\frac{n_c}{2}-i\right)}{\Sigma_{n_{\pi^0}}\Sigma_{n_c}\frac{1}{2^{n_c}}B^2\left(\frac{n_{\pi^0}}{2}+\frac{1}{2},\frac{n_c}{2}+1\right)\omega^2\left(\frac{n_c}{2}\right)\omega(n_{\pi^0})}. \quad (29)$$

The π^0 and $\pi^{+,-}$ inclusive distributions are shown in Fig. 4. For comparison, we also give the π inclusive distribution without the isospin effect in the same figure. Notice that there is a difference among $\pi^0,\pi^{+,-}$ and π inclusive distributions caused by multiparticle BE correlations in coordinate space.

In this paper, we have not considered energy constraint effects on the source distribution as done in Ref. [13]. Also the source model presented here is not a realistic model. As stressed in Refs. [6,8], for a more or less real model, the amount of calculational work will increase astronomically. To our best knowledge, there is so far no method which enables us to calculate quickly multiparticle BE correlations in the above process. But we think that for the purpose of illustrating the general features of multipion correlation effects on the source distribution, our toy model is good enough. In Ref. [18], Fowler *et al.* found that the restriction on the multiplicity has a great influence on BE correlations. We did not address this issue, simply because our prime purpose in this paper is to discuss the multipion BE correlation and isospin effects on the source distribution in coordinate space, which has not yet been discussed in previous publications. We leave the systematical discussion of all these factors on the source distribution for a future publication.

In conclusion, the multipion Bose-Einstein correlation effects on the pion source distribution have been discussed. We showed that multipion correlations make the average radius of the source become smaller. The larger the pion multiplicity, the larger the multipion correlation effects on the source distribution. Isospin effects on the pion probability and pion source distribution are also discussed. We observed that multipion correlations distort the relation $\langle n_{\pi^0}\rangle=\langle n_{\pi^+}\rangle=\langle n_{\pi^-}\rangle$ which exists in isospin-singlet state without multiparticle BE correlations in coordinate space.

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