Giant resonances as collective states with dissipative coupling

E. Stefanescu,[1] R. J. Liotta,[2] and A. Sandulescu[3]
[1]*National Research Institute for Micro Technology, 72996 Bucharest, Romania*
[2]*Royal Institute of Technology, Physics Department Frescati, Frescativagen 24, S-10405 Stockholm, Sweden*
[3]*Department of Theoretical Physics, Institute of Atomic Physics, Bucharest-Magurele, P.O. Box MG-6, Romania*
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We describe giant resonances as the collective coordinates of a harmonic oscillator with dissipative coupling. Using a quantum master equation, the energy and width of the first two levels are obtained as functions of temperature and of the dissipative coupling strength. On this basis, we evaluate the ratio of the two spectral linewidths. The result agrees with available experimental data. We evaluate transition matrix elements among particle excitations belonging to the environment and find a weak dependence on temperature. [S0556-2813(98)01602-1]

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I. INTRODUCTION

Giant resonances manifest themselves as broad bumps in the nuclear spectra above the neutron emission threshold. It has been established that the structure of these states consists of a large number of relevant particle-hole configurations, all of them contributing in phase to the excitation process. Because of this, giant resonances are the most collective vibrational states in nuclei.

The width of a giant resonance is composed of the ''escape'' width, which represents the coupling to the continuum, the ''spreading'' width, which represents the coupling to two-particle–two-hole and even more complex configurations, and the ''Landau damping,'' representing the breaking of the collective state in different pieces. Since giant resonances are excitations embedded in the continuum part of nuclear spectra, one would expect that the continuum has to be taken into account explicitly in their description. Yet the gross properties of these resonances can be studied suitably by means of bound (e.g., harmonic oscillator) representations [1,2]. This can be understood, since the particles in the particle-hole excitations move in high angular momentum orbits. Therefore, the corresponding centrifugal and, for protons, Coulomb barriers tend to trap this particle within the nuclear volume, hindering the particle decay of the giant resonance. The effective potential thus felt by the particle is well approximated by a harmonic oscillator potential. This also explains the small value of the escape width of the giant resonances [3].

The strong collectivity of giant resonances indicates that their double excitations are more likely to occur than the double excitation of low energy vibrational states like, e.g., the yrast quadrupole states in tin isotopes. This is why the study of double giant resonances (GR2) is a subject that is at present being pursued intensively, both experimentally [4–10] and theoretically [10–14]. These calculations evaluate properties of the GR2. A microscopic calculation of the GR2 should include as basis elements all important two-particle–two-hole excitations as well as the particle-hole configurations of the constituent giant resonances. One would thus obtain at the same time the spreading width of the giant resonances and quantities corresponding to the GR2. But such a calculation would require the diagonalization of matrices of large dimensions. This would even be more difficult if one includes the temperature as a variable in the formalism.

Alternatively, given the strong collectivity of the giant resonances and the many degrees of freedom that influence their properties, one may consider that a statistical modeling of the formation and decaying processes including concepts like mean values and variances is an appropriate path to attact this difficult problem. In this paper, we adopt such a point of view. We consider a collective coordinate q and its vibrational modes determined by a harmonic oscillator potential. That is, we consider that the giant resonance corresponds to the first excitation of the harmonic oscillator, with energy $\hbar\omega_0$, and the GR2 to the second excitation, with two quantum oscillations and energy $2\hbar\omega_0$. Besides these excitations, we distinguish an ''environment'' of all the other excitations.

In Sec. II we derive Lindblad's quantum master equation using physical concepts, since the original derivation [15] is rather abstract. We also derive a form of this equation which includes friction and diffusion [16,17]. It has been shown that this equation gives very accurate descriptions of processes in optics [18] as well as in nuclear physics [19,20]. In Sec. III we obtain the energy mean value and width of the giant resonance and of the GR2. The corresponding equations turn out to be analytically integrable. The decay spectrum of the giant resonances is presented in Sec. IV and a summary and conclusions are in Sec. V.

II. QUANTUM MASTER EQUATION AND DECAY

In our formalism the decay of the giant resonance is a time-dependent process with energy loss due to the coupling of the system to a dissipative environment. Since this is a subject which is not very familiar in nuclear physics, we will give the formalism in some detail. We therefore consider a system S with Hamiltonian H and an environment E with Hamiltonian H^E. Following Carmichael [21], we introduce the interaction V as

$$V=\hbar\sum_i s_i\Gamma_i, \tag{1}$$

where s_i are operators of the system S and Γ_i are the conjugate operators of the environment. Therefore, the Hamiltonian of the total system $T=S\oplus E$ is

$$H^T=H+H^E+V \tag{2}$$

and the corresponding equation of motion reads

$$\dot{\chi}(t)=-\frac{i}{\hbar}[H^T,\chi(t)], \tag{3}$$

where $\chi(t)=R(t)\rho(t)$ is the density operator of the total system, $\rho(t)$ is the density operator of the system S, and $R(t)$ the density operator of the environment E. When the Hamiltonian evolution of the total system T is projected on the space of the system of interest S (which is a non-Hamiltonian evolution) one obtains for the reduced density matrix $\rho(t)=\mathrm{Tr}_E\chi(t)$ the expression

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]-\frac{1}{\hbar^2}\int_0^t e^{-(i/\hbar)Ht}\times\mathrm{Tr}_E[\widetilde{V}(t),[\widetilde{V}(t'),\widetilde{R}(t')\widetilde{\rho}(t')]]e^{(i/\hbar)Ht}dt', \tag{4}$$

where the tilde labels operators in the interaction picture, e.g., $\widetilde{V}(t)=e^{(i/\hbar)(H+H^E)t}Ve^{-(i/\hbar)(H+H^E)t}$.

With Eq. (1), Eq. (4) takes the form of a general master equation:

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]+\sum_{ij}\int_0^t dt' e^{-(i/\hbar)Ht}\{\langle\widetilde{\Gamma}_i(t')\widetilde{\Gamma}_j(t)\rangle\times[\widetilde{s}_i(t')\widetilde{\rho}(t'),\widetilde{s}_j(t)]+\langle\widetilde{\Gamma}_i(t)\widetilde{\Gamma}_j(t')\rangle\times[\widetilde{s}_i(t),\widetilde{\rho}(t')\widetilde{s}_j(t')]\}e^{(i/\hbar)Ht}dt', \tag{5}$$

where

$$\langle\widetilde{\Gamma}_i(t')\widetilde{\Gamma}_j(t)\rangle=\mathrm{Tr}_E[\widetilde{\Gamma}_i(t')\widetilde{R}(t')\widetilde{\Gamma}_j(t)] \tag{6}$$

are correlation functions of the environment operators and $\widetilde{s}_i(t)=e^{(i/\hbar)Ht}s_ie^{-(i/\hbar)Ht}$. In the following we consider that the state $R(t')$ of the environment does not depend on the state $\rho(t')$ of the system (Born approximation) [21,22]. For decay processes much slower than the time evolution of the environment operators (which we assume to be the valid in our case) one gets

$$\widetilde{\Gamma}_i=e^{(i/\hbar)(H+H^E)t}\Gamma_ie^{-(i/\hbar)(H+H^E)t}=e^{(i/\hbar)H^Et}\Gamma_ie^{-(i/\hbar)H^Et}, \tag{7}$$

these operators are δ correlated (the Markoff approximation) [21],

$$\langle\widetilde{\Gamma}_i(t')\widetilde{\Gamma}_j(t)\rangle=\langle\widetilde{\Gamma}_i(t)\widetilde{\Gamma}_j(t')\rangle=a_{ij}\delta(t-t'), \tag{8}$$

and Eq. (5) takes the form

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]+\sum_{ij}a_{ij}\{[s_i\rho(t),s_j]+[s_i,\rho(t)s_j]\}. \tag{9}$$

One notices that the dissipation matrix a_{ij} is positive, $a_{ii}\geqslant 0$, and Hermitian: $a_{ij}=a_{ji}^*$.

Considering the real and the imaginary parts of the dissipation parameters a_{ij} of the form

$$a_{ij}=\frac{1}{\hbar^2}D_{ij}+\frac{i}{2\hbar}\lambda_{ij},\quad \lambda_{ij}=-\lambda_{ji},\quad D_{ij}=D_{ji}, \tag{10}$$

one obtains, from Eq. (9), the Markoffian quantum master equation

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]+\frac{i}{2\hbar}\sum_{ij}\lambda_{ij}[s_i,s_j\rho(t)+\rho(t)s_j]-\frac{1}{\hbar^2}\sum_{ij}D_{ij}[s_i,[s_j,\rho(t)]]. \tag{11}$$

From the positivity of the matrix a_{ij} and the Schwartz inequality for the vectors $(b_{i1},b_{i2},\ldots,b_{in})$, one gets the fundamental constraints

$$D_{ii}\geqslant 0,\quad D_{ii}D_{jj}-D_{ij}^2\geqslant\frac{\hbar^2\lambda_{ij}^2}{4}. \tag{12}$$

For a one-dimensional system with coordinate q and momentum p, Eq. (11) takes the form of the quantum master equation depending on a friction parameter λ and three diffusion parameters D_{qq}, D_{pp}, and D_{qp}, i.e. [16],

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]-\frac{i}{2\hbar}\lambda\{[q,p\rho(t)+\rho(t)p]-[p,q\rho(t)+\rho(t)q]\}-\frac{D_{pp}}{\hbar^2}[q,[q,\rho(t)]]-\frac{D_{qq}}{\hbar^2}[p,[p,\rho(t)]]+\frac{D_{qp}}{\hbar^2}\{[q,[p,\rho(t)]]+[p,[q,\rho(t)]]\}. \tag{13}$$

From Eqs. (8) and (10) the diffusion coefficients D_{ij} and the friction coefficients λ_{ij} can be interpreted as the real and imaginary parts of the time integrals of the correlation functions, i.e.,

$$D_{ij}=\hbar^2\mathrm{Re}\left\{\int_{-\infty}^{\infty}\langle\widetilde{\Gamma}_i(t)\widetilde{\Gamma}_j(t')\rangle dt'\right\},\qquad \lambda_{ij}=2\hbar\,\mathrm{Im}\left\{\int_{-\infty}^{\infty}\langle\widetilde{\Gamma}_i(t)\widetilde{\Gamma}_j(t')\rangle dt'\right\}. \tag{14}$$

These coefficients can be related to microscopical quantities. We will show here an example of this relation by using the independent-oscillator model of Refs. [23,24]. In this case a harmonic oscillator with the Hamiltonian

$$H=\frac{p^2}{2m}+m\frac{\omega_0^2q^2}{2} \tag{15}$$

is coupled to an environment of harmonic oscillators with coordinates $\{q_i,p_i\}$ and frequencies $\{\omega_i\}$. The total Hamiltonian then is

$$H^T=H+\sum_j\left(\frac{p_j^2}{2m_j}+\frac{1}{2}m_j\omega_j^2(q_j-q)^2\right). \tag{16}$$

Assuming a Gibbs distribution for the environment oscillators, the correlation functions acquire the simple form

$$\langle q_jq_k\rangle=\frac{\hbar\delta_{jk}}{2m\omega_j}\coth\frac{\hbar\omega_j}{2kT}. \tag{17}$$

With these correlation functions, and after some algebra, one obtains

$$\begin{aligned}\dot\rho(t)=&-\frac{i}{\hbar}[\overline{H},\rho(t)]-\frac{i}{2\hbar}\frac{m_0\overline{\omega_0^2}}{4m}\delta(\overline{\omega_0})\{[q,p\rho(t)+\rho(t)p]\\&-[p,q\rho(t)+\rho(t)q]\}-\frac{m_0\overline{\omega_0^3}\delta(\overline{\omega_0})}{8\hbar}\coth\frac{\hbar\overline{\omega_0}}{2kT}\\&\times\left([q,[q,\rho(t)]]+\frac{1}{m^2\overline{\omega_0^2}}[p,[p,\rho(t)]]\right),\end{aligned} \tag{18}$$

where $\overline{\omega_0}$ is a renormalized resonance frequency corresponding to the Hamiltonian $\overline{H}=H_0+\Sigma_j\frac{1}{2}m_j\omega_j^2q^2$, $\delta(\overline{\omega})$ is the density of states at frequency $\overline{\omega}$, and m_0 is the mass of the environment oscillators in resonance with the system.

Comparing Eq. (18) with Eq. (13) one finds that the friction coefficient can be written in terms of microscopic quantities as $\lambda=(m_0\overline{\omega_0^2}/4m)\delta(\overline{\omega_0})$.

We will now show that the complicated evolution of the giant resonance can be described only by two parameters: the temperature T and the strength of the dissipative coupling λ.

III. GIANT RESONANCES AND THE DECAY OF A QUANTUM HARMONIC OSCILLATOR IN THE MARKOFF APPROXIMATION

We consider a nucleus described in a space of collective coordinates. For simplicity, we adopt a one-dimensional model, with a dominant coordinate q of a harmonic oscillator (HO) for the most probable state of the nucleus. The evolution on the other coordinates is assumed to proceed through dissipation and, therefore, the system is described by the master equation (13) with the Hamiltonian given by [see Eq. (2)]

$$H=\frac{p^2}{2m}+\frac{m\omega_0^2q^2}{2}, \tag{19}$$

where p is the momentum, m is the effective mass, and ω_0 is the frequency of the oscillator. We will derive equations for the expectation values of the observables q and p. From the master equation, by multiplying by q and p and taking the traces, one gets

$$\begin{aligned}\frac{d\langle q\rangle}{dt}&=-\lambda\langle q\rangle+\frac{\langle p\rangle}{m},\\\frac{d\langle p\rangle}{dt}&=-\lambda\langle p\rangle-m\omega_0^2\langle q\rangle.\end{aligned} \tag{20}$$

In the same fashion one obtains the expressions for the second- and fourth-order moments, like, e.g., $d\langle q^2\rangle/dt$ and $d\langle q^2p^2+p^2q^2\rangle/dt$.

Equations (20) describe the time evolution of the coordinate and momentum. The corresponding solutions are given by

$$\begin{aligned}\langle q(t)\rangle&=e^{-\lambda t}\left(\langle q(0)\rangle\cos\omega_0t+\langle p(0)\rangle\frac{\sin\omega_0t}{m\omega_0}\right),\\\langle p(t)\rangle&=e^{-\lambda t}[-m\omega_0\langle q(0)\rangle\sin\omega_0t+\langle p(0)\rangle\cos\omega_0t].\end{aligned} \tag{21}$$

At equilibrium ($t\to\infty$), the mean value of the coordinate takes the value corresponding to the minimum of the potential, which is zero in our case, while the momentum tends to zero.

The energy mean value

$$\langle H\rangle=\frac{\langle p^2\rangle}{2m}+\frac{m\omega_0^2\langle q^2\rangle}{2}, \tag{22}$$

as well as the energy width, defined as

$$\begin{aligned}\Gamma^2&=\langle H^2\rangle-\langle H\rangle^2\\&=\frac{\langle p^4\rangle}{4m^2}+\frac{m^2\omega_0^4\langle q^4\rangle}{4}+\frac{\omega_0^2\langle q^2p^2+p^2q^2\rangle}{4}-\langle H\rangle^2,\end{aligned} \tag{23}$$

can be derived from the second- and fourth-order moments. One thus gets

$$\frac{d\langle H\rangle}{dt}=-2\lambda\langle H\rangle+\frac{D_{pp}}{m}+m\omega_0^2D_{qq},$$

$$\begin{aligned}\frac{d\langle H^2\rangle}{dt}=&-4\lambda\langle H^2\rangle-\lambda\hbar^2\omega_0^2+3\frac{D_{pp}\langle p^2\rangle}{m^2}+3m^2\omega_0^4D_{qq}\langle q^2\rangle\\&+\omega_0^2(D_{pp}\langle q^2\rangle+D_{qq}\langle p^2\rangle+2D_{qp}\langle qp+pq\rangle).\end{aligned} \tag{24}$$

The first of these equations has the solution

$$\langle H(t)\rangle=E_\infty+(E_i-E_\infty)e^{-2\lambda t}, \tag{25}$$

where E_i is the initial value of the energy and

$$E_\infty=\frac{1}{\lambda}\left(\frac{D_{pp}}{2m}+\frac{m\omega_0^2D_{qq}}{2}\right) \tag{26}$$

is the equilibrium value. Assuming a Gibbs distribution of the system at equilibrium, i.e., $P_n=(1/Q)e^{-\beta E_n}$, $Q=\Sigma_ne^{-\beta E_n}$, $\beta=(1/kT)$, and the eigenenergies $E_n=(n+\frac{1}{2})\hbar\omega_0$ of the HO we obtain

$$E_\infty=\langle H(\infty)\rangle=\frac{1}{Q}\sum_n\left(n+\frac{1}{2}\right)\hbar\omega_0e^{-(n+1/2)\beta\hbar\omega_0}, \tag{27}$$

where

$$Q=\sum_n e^{-(n+1/2)\beta\hbar\omega_0}=\frac{1}{2\sinh(\beta\hbar\omega_0/2)}. \tag{28}$$

From these expressions, we obtain a much more transparent form of the equilibrium energy (26), i.e.,

$$E_\infty=-\frac{1}{Q}\frac{\partial Q}{\partial\beta}=\frac{\hbar\omega_0}{2}\coth\frac{\hbar\omega_0}{2kT}, \tag{29}$$

depending on the oscillator ground state energy $\hbar\omega_0/2$ and on the temperature T. Considering an equipartition of the energy on the coordinates q and p, from Eqs. (26) and (29) one obtains

$$\frac{D_{pp}}{2m\lambda}=\frac{m\omega_0^2 D_{qq}}{2\lambda}=\frac{\hbar\omega_0}{4}\coth\frac{\hbar\omega_0}{2kT}, \tag{30}$$

and, consequently, the diffusion coefficients take the form [16]

$$D_{qq}=\frac{\hbar\lambda}{2m\omega_0}\coth\frac{\hbar\omega_0}{2kT},\qquad D_{pp}=\frac{\hbar\lambda m\omega_0}{2}\coth\frac{\hbar\omega_0}{2kT}, \tag{31}$$

depending on the friction coefficient λ and on the temperature T. With these expressions, the second fundamental constraint (12) takes the form

$$\frac{\hbar^2\lambda^2}{4}\coth^2\frac{\hbar\omega_0}{2kT}-D_{qp}^2\geqslant\frac{\hbar^2\lambda^2}{4},$$

which is satisfied only when $D_{qp}=0$. One then obtains, from the second of Eqs. (24),

$$\frac{d\langle H^2\rangle}{dt}=-4\lambda\left\{\langle H^2\rangle-\hbar\omega_0\left[\langle H\rangle\coth\frac{\hbar\omega_0}{2kT}-\frac{\hbar\omega_0}{4}\right]\right\}. \tag{32}$$

With the notation $E_0=\hbar\omega_0/2$ for the ground state and $E_i^*=E_i-E_\infty=E_i-E_0\eta$ for the excitation energy, where

$$\eta=\coth\frac{\hbar\omega_0}{2kT}, \tag{33}$$

one gets for the mean value of the Hamiltonian and for the width of the nth state of the HO the expressions

$$\langle H(t)\rangle_n=E_n^* e^{-2\lambda t}+E_0\eta,\qquad \Gamma_n^2=(C_0-E_n^{*2})e^{-4\lambda t}+2E_0E_n^*\eta e^{-2\lambda t}+E_0^2(\eta^2-1), \tag{34}$$

where C_0 is the integration constant. The first term of the energy mean value describes the decay of the excitation energy E_n^*, while the second term corresponds to the equilibrium energy $E_0\eta(T)$ of the oscillator at temperature T. The first term of the energy width, containing the integration constant C_0, describes the effect of the initial state distribution of the oscillator, which is determined by the initial collision leading to the giant resonance excitation. With the initial condition $C_0=E_n^{*2}$ this term disappears. The third term represents the equilibrium value of the width $\Gamma_e^2=E_0^2[\eta^2(T)-1]$ which is different from zero only at high temperature, when $\eta(T)>1$.

Notice that the friction coefficient λ is not related to the width of the giant resonance, since the time evolution given by Eqs. (20) do not describe the full effect of the environment. This is included in the general master equation (13) and, therefore, the width is given by Γ_n in Eq. (34). The value of Γ_n in this equation depends upon the time of interaction between the system and the environment. Assuming this time to have the value Δt and averaging on it one gets for the second term a factor $[1-\exp(-2\lambda\Delta t)]/(\lambda\Delta t)\approx 1/(\lambda\Delta t)$ and the width acquires the form

$$\Gamma_n^2=\frac{2E_0E_n^*\eta(T)}{\lambda\Delta t}+E_0^2(\eta^2-1). \tag{35}$$

IV. DECAY SPECTRUM OF THE GIANT RESONANCES

In the previous section we considered a double giant resonance as the first two excited states of a dissipative HO. We calculated the time evolution of the energy mean value and of the width. To calculate also the spectrum, we consider the interaction between the HO and particle excitations belonging to the environment. We thus introduce the interaction Hamiltonian as

$$V=\hbar c(\alpha C^{(x)}xr_x+C^{(u)}ur_u). \tag{36}$$

We also define a coordinate q and a conjugate coordinate s as

$$q=C^{(x)}xr_x,\quad s=C^{(u)}ur_u. \tag{37}$$

In the expressions above, x is the collective coordinate of the oscillator. The operator $u=d/dx$ defines the momentum $p=-i\hbar d/dx=-i\hbar u$, while the operators r_x and r_u describe the particle excitations belonging to the environment. The scalars $C^{(x)}$ and $C^{(u)}$ are the coupling coefficients of the operators r_x and r_u, respectively, and $\alpha=m\omega_0/\hbar$.

Because of the interaction V, the particle excitations couple to states of the HO. The structure of those excitations is different for different HO states. For the first (ground) HO state that structure corresponds to the vacuum of excitation, for the second state (giant resonance) it is mainly two-particle–two-hole (2p2h) excitations [25,26], for the third (GR2) it is mainly 4p4h, and so on. For each HO state, the particle excitations form a band, as will be shown below. The members of a particular band mix with the corresponding collective state due to the interaction V. Therefore each state in the band carries a certain amount of collectivity through the piece of the HO state that has contributed to the mixing.

The interaction potential V takes a familiar form introducing the annihilation and creation operators A and $A^\dagger$ of the harmonic oscillator,

$$A=\sqrt{\frac{\alpha}{2}}\left(x+\frac{u}{\alpha}\right),\quad A^\dagger=\sqrt{\frac{\alpha}{2}}\left(x-\frac{u}{\alpha}\right), \tag{38}$$

and the annihilation and creation operators a and $a^\dagger$ of the particle excitation,

$$\mu a^{\dagger}=C^{(x)}r_x+C^{(u)}r_u, \quad \mu a=C^{(x)}r_x-C^{(u)}r_u. \quad (39)$$

In terms of these creation and annihilation operators the expressions (36) and (37) become

$$V=\hbar c\sqrt{\frac{\alpha}{2}}\mu(Aa^{\dagger}+A^{\dagger}a). \quad (40)$$

$$q=\mu x\frac{a^{\dagger}+a}{2}, \quad s=\mu\frac{d}{dx}\frac{a^{\dagger}-a}{2}. \quad (41)$$

Labeling by the latin letters k, l the collective states and by i_k, i_l the corresponding particle excitations in the field of the harmonic oscillator, the transition elements of the coordinates q,s take the form

$$q_{i_kj_l}=\mu\langle k|x|l\rangle\left\langle \varepsilon_{i_k}\left|\frac{a^{\dagger}+a}{2}\right|\varepsilon_{j_l}\right\rangle,$$
$$s_{i_kj_l}=\mu\left\langle k\left|\frac{d}{dx}\right|l\right\rangle\left\langle \varepsilon_{i_k}\left|\frac{a^{\dagger}-a}{2}\right|\varepsilon_{j_l}\right\rangle. \quad (42)$$

To obtain the spectrum as a function of these matrix elements, we will assume that the interaction is small and that the time t is short. With these assumptions we solve the master equation in the framework of perturbation theory [27]. For this purpose, we take the second-order expansion of the density operator in the interaction picture:

$$\rho'=\rho^{(0)}+\rho'^{(1)}+\rho'^{(2)}. \quad (43)$$

From the master equation (13), we obtain the equations

$$\frac{d\rho^{(0)}}{dt}=0,$$
$$\frac{d\rho'^{(1)}}{dt}=-\frac{i}{\hbar}[V'(t),\rho^{(0)}], \quad (44)$$
$$\frac{d\rho'^{(2)}}{dt}=-\frac{i}{\hbar}[V'(t),\rho'^{(1)}(t)]+L'(\rho^{(0)};t),$$

where we used the notation

$$L[\rho(t)]=-\frac{i}{2\hbar}\lambda\{[q,p\rho(t)+\rho(t)p]-[p,q\rho(t)+\rho(t)q]\}$$
$$-\frac{D_{pp}}{\hbar^2}[q,[q,\rho(t)]]-\frac{D_{qq}}{\hbar^2}[p,[p,\rho(t)]]$$
$$+\frac{D_{qp}}{\hbar^2}\{[q,[p,\rho(t)]]+[p,[q,\rho(t)]]\}. \quad (45)$$

We assume that the initial state, which we will denote by $|m\rangle$, corresponds to the unperturbed density operator [27] (i.e., $\rho^{(0)}=|m\rangle\langle m|$).

The diagonal matrix elements of the density are

$$\rho_{ii}(m;t)=\frac{1}{\hbar^2}|V_{mi}|^2\frac{\sin^2(\omega_i t/2)}{(\omega_i/2)^2}$$
$$+2\left(D_{qq}s_{mi}^2+\frac{D_{pp}}{\hbar^2}q_{mi}^2-\lambda q_{mi}s_{mi}\right)t. \quad (46)$$

The first term of this expression results from the Hamiltonian part of Eq. (13), i.e., from the isolated (closed) system. The corresponding width at time t is given by $\Delta_H\simeq 1/t$ according to Heisenberg's uncertainty principle [27]. One can thus interpret the parameter $\hbar\lambda$ in Eqs. (21) as the width corresponding to the isolated system at time $1/\lambda$. As mentioned above, this is not related to the with Γ_n given by Eq. (35).

The second term in Eq. (46) corresponds to the dissipative processes that takes place as the result of the interaction with the environment. This interaction is defined by the parameters λ, D_{qq}, D_{pp}, and D_{qp}.

Considering quasiequilibrium transition processes, that is, when temperature can be defined, one gets from Eqs. (31) and (46) the transition spectrum

$$\mathcal{T}_{mi}(T)=\frac{\rho_{ii}(m;t)}{t}=\frac{1}{\hbar^2}|V_{mi}|^2\frac{\sin^2(\omega_i t/2)}{(\omega_i t/2)^2}t$$
$$+\lambda\left[(\alpha q_{mi}^2+\alpha^{-1}s_{mi}^2)\coth\frac{\hbar\omega_0}{2kT}-2q_{mi}s_{mi}\right]. \quad (47)$$

Since we are interested in the decay from a state $|i_k\rangle$ of a shell $|k\rangle$ to a state $|j_l\rangle$ of another shell $|l\rangle$, the first term, representing only the width of the initial level, does not enter. Thus, from Eq. (47) we obtain

$$\mathcal{T}_{i_kj_l}(T)=\frac{\rho_{j_lj_l}(i_k;t)}{t}$$
$$=\lambda\left[(\alpha q_{i_kj_l}^2+\alpha^{-1}s_{i_kj_l}^2)\coth\frac{\hbar\omega_0}{2kT}-2q_{i_kj_l}s_{i_kj_l}\right], \quad (48)$$

depending on the matrix elements (42). For the three states $|0\rangle$ (i.e., ground state), $|1\rangle$ (i.e., the giant resonance), and $|2\rangle$ (i.e., GR2) of the harmonic oscillator, we calculate the matrix elements

$$\langle 1|x|0\rangle=\frac{1}{\sqrt{2\alpha}}, \quad \left\langle 1\left|\frac{d}{dx}\right|0\right\rangle=-\sqrt{\frac{\alpha}{2}},$$
$$\langle 2|x|1\rangle=\frac{1}{2\sqrt{2\alpha}}, \quad \left\langle 2\left|\frac{d}{dx}\right|1\right\rangle=-\frac{1}{2}\sqrt{\frac{\alpha}{2}}. \quad (49)$$

Thus, we notice that only transitions within two successive bands, $|i_2\rangle\rightarrow|j_1\rangle$ or $|i_1\rangle\rightarrow|j_0\rangle$, are allowed, while all the transitions inside a band $|i_k\rangle\rightarrow|j_k\rangle$, or over a band, $|i_2\rangle\rightarrow|j_0\rangle$, are forbidden. For the two allowed transition processes one obtains, from Eq. (48),

$$\mathcal{T}_{i_1 j_0}(T)=\frac{\mu^2\lambda}{2}\left[(Q^2_{i_1 j_0}+S^2_{i_1 j_0})\coth\frac{\hbar\omega_0}{2kT}+2Q_{i_1 j_0}S_{i_1 j_0}\right],$$

$$\mathcal{T}_{i_2 j_1}(T)=\frac{\mu^2\lambda}{8}\left[(Q^2_{i_2 j_1}+S^2_{i_2 j_1})\coth\frac{\hbar\omega_0}{2kT}+2Q_{i_2 j_1}S_{i_2 j_1}\right], \tag{50}$$

where $\mathcal{T}_{i_1 j_0}$ corresponds to the transition from the giant resonance to the ground state and $\mathcal{T}_{i_2 j_1}$ from the GR2 to the giant resonance. In these expressions we used the notation

$$Q_{i_k j_l}=\left\langle \varepsilon_{i_k}\left|\frac{a^\dagger+a}{2}\right|\varepsilon_{j_l}\right\rangle, \quad S_{i_k j_l}=\left\langle \varepsilon_{i_k}\left|\frac{a^\dagger-a}{2}\right|\varepsilon_{j_l}\right\rangle. \tag{51}$$

Notice that the state j_0 is the vacuum of excitations, as mentioned above.

In terms of the creation and annihilation operators $a^\dagger$ and a one gets

$$\mathcal{T}_{i_1 j_0}(T)=\frac{\mu^2\lambda}{4}\left[(\langle\varepsilon_{i_1}|a^+|\varepsilon_{j_0}\rangle^2+\langle\varepsilon_{i_1}|a|\varepsilon_{j_0}\rangle^2)\coth\frac{\hbar\omega_0}{2kT}+\langle\varepsilon_{i_1}|a^+|\varepsilon_{j_0}\rangle^2-\langle\varepsilon_{i_1}|a|\varepsilon_{j_0}\rangle^2\right],$$

$$\mathcal{T}_{i_2 j_1}(T)=\frac{\mu^2\lambda}{16}\left[(\langle\varepsilon_{i_2}|a^+|\varepsilon_{j_1}\rangle^2+\langle\varepsilon_{i_2}|a|\varepsilon_{j_1}\rangle^2)\coth\frac{\hbar\omega_0}{2kT}+\langle\varepsilon_{i_2}|a^+|\varepsilon_{j_1}\rangle^2-\langle\varepsilon_{i_2}|a|\varepsilon_{j_1}\rangle^2\right]. \tag{52}$$

V. COMPARISON WITH EXPERIMENTAL DATA

In this section we will analyze available experimental data on the evolution of giant resonances with temperature. This is a rather controversial subject mainly because there are several interpretations of the experimental data. However, it seems to be well established that the energy ω_0 and the percentage of the sum rule are practically independent of the temperature [28]. Instead, the experimental width Γ increases steadily with temperature, from about 4 MeV to a value that goes from 8 MeV [29] to 12 MeV [30] for nuclei which are not too ''hot,'' i.e., up to about 100 MeV of excitation energy. This increase was explained as an effect induced by the angular momentum transferred to the system through the complete fusion reactions used to populate those nuclei [31]. If this is indeed the case, then the width would be independent of the temperature. This interpretation has been supported by experiments where the influence of angular momenta has been eliminated [29].

For higher energies the situation is even more unclear and a rather lively debate is at present taking place on the issue of the dependence of the width on T. Thus, in Refs. [31–33] one argues that there is a maximun of the angular momentum that is transferred to the nucleus at about 100 MeV. Above this energy the nucleus fissions and, therefore, the width is independent of the temperature at high energies. However, in another interpretation one finds that the experimental values can be adjusted by assuming that the width increases with T [34,35]. This interpretation is supported by very detailed calculations performed within the framework of the Landau-Vlasov transport theory [36]. In this theory, one finds that there are a number of processes contributing to the evolution of the giant resonance with temperature, like, e.g., competition between one- and two-body dissipation and a transition from pure quantum to classical sound waves. A review of this can be found in Ref. [37].

This rather confusing situation indicates that the formalism presented in this paper, which is very different to the other ones mentioned above, is perhaps timely.

We can thus evaluate the evolution of the width by using Eq. (35). From this expression one obtains for the ratio of the widths of the states $n=2$ (i.e., GR2) and $n=1$ at $T=0$ the value

$$\frac{\Gamma_2}{\Gamma_1}=\sqrt{\frac{E^*_2}{E^*_1}}, \tag{53}$$

which, from $E^*_2=2\hbar\omega_0$, $E^*_1=\hbar\omega_0$, is $\Gamma_2/\Gamma_1=\sqrt{2}$. This agrees with the corresponding experimental quantity of Ref. [4]. It also agrees with previous calculations performed within a different theoretical framework [12]. Yet, even here one finds disagreements among different publications. From the experimental side, one has to consider that there is a large spread of the data. A detailed review of these difficulties, as well as a very useful compilation of the data, can be found in Ref. [38]. A value of 2 for the ratio above is definitely not ruled out by the experiments. In fact, one may even prefer this value since it is obtained by folding two Lorentzians while the factor of $\sqrt{2}$ corresponds to the folding of two Gaussians. It is likely that the shape of the constituent giant resonances in the double giant resonance mode is more similar to a Lorentzian as compared to a Gaussian function. From the theoretical side [39] the decay of the multiphonon state formed through the excitation of many giant dipole resonances was studied within the framework of a model consisting of a dissipative quantum pendulum. One thus finds that the width of such an N-phonon state is proportional to N, which is $N=2$ for the double giant resonance.

There is also a number of microscopic calculations that include the effect of two-particle–two-hole excitations on the giant resonances. One can thus evaluate simultaneously the width of the giant resonance and the double giant resonance, as done in Ref. [40]. In this calculation the ratio (53) turns out to be 1.5.

Perhaps even more important than the ratio discussed above is to notice that the dependence upon T of the width Γ in Eq. (35) is defined by the factor $\eta(T)$ [Eq. (33)] which is $\eta\approx 1$ for reasonable values of T. For instance, assuming $\omega_0 = 15$ MeV one trivially obtains that for $T=1$ MeV it is $\coth[\hbar\omega_0/(2kT)] = 1.000\,001$, and for $T=6$ MeV it is $\coth[\hbar\omega_0/(2kT)] = 1.18$. In other words, our formalism predicts that the widths are independent of T.

The transition spectrum corresponding to the decay of the giant resonance is given by Eq. (52). In $\mathcal{T}_{i_1 j_0}$ the mixing of the state i_1 with the giant resonance decreases as the energy of this state (the environment) departs from the energy of the

giant resonance (the system). The same is valid in $\mathcal{T}_{i_2 j_1}$ with respect to the state i_2 and the GR2. The ratio between the corresponding widths is $\sqrt{2}$, as has been shown above.

One notices in Eq. (52) that, at zero temperature, the spectrum is dependent upon the matrix elements of the creation operators only, as expected in a Tamm-Dancoff approximation (TDA) (shell model) formalism. But thermal fluctuations produce excitations where the particle states are *annihilated*. This can be interpreted as the result of a transition from a sharp to a diffuse Fermi surface due to the thermal fluctuations. As an analogy, this terms reminds one of the backward-going components of the random phase approximation (RPA).

The decay rates (52) are proportional to the coefficient μ^2, i.e., to the coupling of the particle excitations to the HO field, and are also proportional to the dissipative coupling strength λ of the collective motion. They depend on the temperature T through the coth function, as was the case with the width analyzed above. Therefore, even in the processes of formation and decay of the giant resonance the dependence upon T is practically negligible.

VI. SUMMARY AND CONCLUSIONS

In this paper we have studied giant resonances as a collective motion of nucleons described by the coordinate of a harmonic oscillator (HO). This quantum open system is assumed to be embedded in a hot environment. Using a quantum master equation, the energy and width of the first level, which is the giant resonance, and of the second level, which is the double giant resonance (GR2), were obtained as functions of temperature and of the dissipative coupling strength that appears due to the energy exchange between the system and the environment. We singled out particle excitations that are part of the environment, which may include even the nuclear continuum, and introduced a first-order interaction between the HO and that part of the environment which is defined by the particle excitations. This allowed us to define creation and annihilation operators for the HO as well as for the particle excitations. Within this basis we evaluated the transition spectrum as a function of the transition matrix elements. The transition strength is proportional to the dissipative strength λ and to the coupling coefficient μ^2. Thus, in the deexcitation process a part of the energy of the collective motion is transferred to the particle excitations only when a dissipative coupling is present (i.e., $\mu^2\lambda \neq 0$).

We found that through the transfer of energy mentioned above the particle excitations form bands surrounding the collective states. This produces a mixing between the collective vibrations of the HO and the particle states belonging to the corresponding band. The closer these states are to the collective vibration, the larger is the mixing between them. As a result, each particle state carries a collectivity which decreases with the distance in energy to the corresponding collective state. This generates the spreading width of the giant resonance as well as of the GR2. The structure of the bands is different for the different collective states. Thus, for the giant resonance the corresponding band consists mainly of two-particle–two-hole excitations while for the GR2 the band consists mainly of four-particle–four-hole excitations. We found that transitions within a band as well as transitions crossing a band are forbidden.

In comparison with other existing theories, the width, calculated from the second-order moments of the Hamiltonian, is a function of a term proportional to the friction strength $\hbar\lambda$, related to the Hamiltonian part of the master equation and to a non-Hamiltonian term which corresponds to quantum diffusion processes. On this basis, we evaluated the ratio of the two spectral linewidths and obtained for this ratio the value $\sqrt{2}$ at $T=0$, which agrees both with previous calculations [12] and with available experimental data [4].

We found that the energy, the width, and even the transition spectrum are practically independent of the temperature for reasonable values of T, which agrees with previous interpretations of the experimental data [28,31–33]. However, it is worthwhile to point out that this is a controversial issue and there are other interpretations of those data which predict strong dependence of the width with temperature [34–37].

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[1] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[2] K. Goeke and J. Speth, Annu. Rev. Nucl. Sci. **32**, 65 (1982), and references therein.
[3] P. Curutchet, T. Vertse, and R. J. Liotta, Phys. Rev. C **39**, 1020 (1989).
[4] J. Ritman *et al.*, Phys. Rev. Lett. **70**, 533 (1993).
[5] K. Boretzky *et al.*, Phys. Lett. B **384**, 30 (1996).
[6] R. Schmidt *et al.*, Phys. Rev. Lett. **70**, 1767 (1993).
[7] J. A. Scarpaci, *et al.*, Phys. Rev. Lett. **71**, 3766 (1993).
[8] J. Blomgren *et al.*, Phys. Lett. B **362**, 34 (1995).
[9] A. Buda *et al.*, Phys. Rev. Lett. **75**, 798 (1995).
[10] Ph. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995).
[11] F. Catara, Ph. Chomaz, and N. Van Giai, Phys. Lett. B **77**, 1 (1992).
[12] Ph. Chomaz and N. Van Giai, Phys. Lett. B **282**, 13 (1992).
[13] C. Volpe, F. Catara, Ph. Chomaz, M. V. Andrés, and E. G. Lanza, Nucl. Phys. **A589**, 521 (1995).
[14] H. Kurasawa and T. Suzuki, Nucl. Phys. **A597**, 374 (1996).
[15] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).
[16] A. Sandulescu and H. Scutaru, Ann. Phys. (N.Y.) **173**, 277 (1987).
[17] A. Isar, A. Sandulescu, H. Scutaru, E. Stefanescu, and W. Scheid, Int. J. Mod. Phys. E **3**, 635 (1994).
[18] A. Sandulescu and E. Stefanescu, Physica A **161**, 525 (1989).
[19] E. Stefanescu, A. Sandulescu, and W. Greiner, Int. J. Mod. Phys. E **2**, 233 (1993).
[20] E. Stefanescu, W. Scheid, A. Sandulescu, and W. Greiner, Phys. Rev. C **53**, 3014 (1996).
[21] H. Carmichael, *An Open System Approach to Quantum Optics*,

Lecture Notes in Physics Vol. 18, (Springer-Verlag, Berlin, 1993).
[22] Ke-Hsueh Li, Phys. Rep. **134**, 1 (1986).
[23] G. W. Ford, J. T. Lewis, R. F. O'Connell, Phys. Rev. A **37**, 4419 (1988).
[24] G. W. Ford, J. T. Lewis, and R. F. O'Connell, Ann. Phys. (N.Y.) **252**, 362 (1996).
[25] M. Baranger, Nucl. Phys. **A149**, 225 (1970).
[26] G. G. Dussel, E. C. Seva, and H. M. Sofia, Phys. Rev. C **49**, 1989 (1994).
[27] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1955).
[28] P. Piatelli *et al.*, Nucl. Phys. **A599**, 63c (1996).
[29] E. Ramakrishnan *et al.*, Nucl. Phys. **A599**, 49c (1996).
[30] J. J. Gaardhøje, Annu. Rev. Nucl. Part. Sci. **42**, 482 (1992).
[31] A. Bracco *et al.*, Phys. Rev. Lett. **62**, 2080 (1989).
[32] H. J. Hofman *et al.*, Nucl. Phys. **A571**, 301 (1994).
[33] G. Enders *et al.*, Phys. Rev. Lett. **69**, 249 (1992).
[34] K. Yoshida *et al.*, Phys. Lett. B **245**, 7 (1990).
[35] J. Kasagi *et al.*, Nucl. Phys. **A538**, 585c (1992).
[36] A. Smerzi *et al.*, Phys. Lett. B **320**, 216 (1994).
[37] V. Baran *et al.*, Nucl. Phys. **A599**, 29c (1996).
[38] H. Emling, Prog. Part. Nucl. Phys. **33**, 729 (1994).
[39] G. Baur and B. Bertulani, Nucl. Phys. **A482**, 328c (1988); in Proceedings of the International School of Heavy Ion Physics, Erice, 1986 (unpublished).
[40] V. Yu Ponomarev *et al.*, Phys. Rev. Lett. **72**, 1164 (1994).