Higher resonance contamination of πNN couplings obtained via the three-point function method in QCD sum rules

Kim Maltman
Department of Mathematics and Statistics, York University, 4700 Keele Street, North York, Ontario, Canada M3J 1P3
and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, Adelaide, Australia 5005
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We investigate the size of potential higher pseudoscalar resonance contaminations of the estimates of isospin-conserving and isospin-violating πNN couplings obtained using the three-point function method in QCD sum rules. For the isospin-conserving case it is shown that conventional models of the isovector pseudoscalar spectral function imply resonance decay constants large enough to create significant contaminations, and that assuming these models are incorrect, and that the decay constants are actually much smaller, implies physically implausible values for the flavor-breaking quark condensate ratios. For the isospin-violating case it is shown explicitly that such resonance contamination is unavoidably present and precludes using the three-point function method as a means of estimating the at present unmeasured isospin-violating πNN couplings. [S0556-2813(98)00401-4]

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I. INTRODUCTION

The general framework of QCD sum rules [1–3] has recently proven popular as an approach to the computation of observable of relevance to problems in nuclear and few-body physics which has, in contrast to many effective hadronic models, a rather direct connection with QCD itself. The method is attractive, first, because it is based on rather general properties of the underlying field theory [the operator product expansion (OPE), analyticity, unitarity, and the existence of appropriately subtracted dispersion relations] and, second, because it provides a means of relating integrals over physical spectral densities to the behavior of correlators at large spacelike momenta (obtained via the OPE) which, via the Borel transformation of the original dispersion relation, simultaneously exponentially suppresses higher energy contributions to the physical spectral integrals and factorially suppresses the contributions to the OPE associated with higher dimension operators. This means that one has the quite reasonable hope of, in favorable circumstances, constructing a sum rule relating observable parameters (such as masses, couplings and decay constants) occurring on the phenomenological side of the sum rule to a small number of vacuum condensates (which parametrize nonperturbative effects in QCD) appearing on the OPE side of the sum rule.

In the case of two-point functions, it is rather easy to see what ''circumstances'' are favorable to such an analysis. First, one should know that one is considering a channel where the relevant spectral function consists of a lowest lying single resonance contribution well-separated from higher resonance and/or continuum pieces. This allows one to choose a Borel mass to strongly suppress the contributions to the weighted spectral integral from the more complicated part of the physical spectral function, and hence obtain an expression for the phenomenological side of the sum rule that is dominated by a few observable parameters associated with the lowest lying physical state. Second, for such Borel masses, it must simultaneously be the case that the OPE side of the sum rule is well converged at operators of low enough dimension that the corresponding vacuum values are already known from other analyses. Leinweber [4] has provided a very clear discussion of the criteria to be satisfied for the applicability of the sum rule method, and also a procedure for sensibly checking the validity of these criteria in a given case.

In the case of observables such as the isospin-conserving and isospin-breaking πNN couplings, the situation is slightly more complicated. In the past, two different approaches have been taken to estimating these couplings in the QCD sum rule framework.

The first of these approaches is the three-point function method [2,5–7]. In this approach one considers the three-point vacuum correlator

$$G_{\pi^a NN}(p_1,p_2,q)=\int d^4x_1 d^4x_2 \exp(ip_1x_1-ip_2x_2) \times\langle 0|\mathrm{T}[\chi_N(x_1)P^a_{I=1}(0)\overline{\chi_N(x_2)}]|0\rangle, \tag{1}$$

where a labels the isospin of the pion ($\pm$ or 0), N stands for either proton or neutron, $P^a_{I=1}\equiv P^a_1$ is a pseudoscalar interpolating field for the pion, and χ_N is the Ioffe interpolating field for the nucleon. In what follows we will use the following notation for the pseudoscalar currents:

$$P_f=\overline{q_f}i\gamma_5 q_f,$$

$$P^+=\sqrt{2}\,\overline{u}\,i\gamma_5 d, \tag{2}$$

where f is a flavor label $f=u,d,s$. The $a=0$ component of the isovector current multiplet is, as usual,

$$P^0_1=\overline{u}\,i\gamma_5 u-\overline{d}\,i\gamma_5 d. \tag{3}$$

In what follows we will also require the flavor neutral isoscalar currents. For these we can use either the strange and light quark combinations

$$P_0^{\not s} = \overline{u}\, i\gamma_5 u + \overline{d}\, i\gamma_5 d,$$

$$P_0^s = \overline{s}\, i\gamma_5 s, \tag{4}$$

or the singlet and octet combinations

$$P_0^0 = [\overline{u}\, i\gamma_5 u + \overline{d}\, i\gamma_5 d + \overline{s}\, i\gamma_5 s]/\sqrt{3},$$

$$P_0^8 = [\overline{u}\, i\gamma_5 u + \overline{d}\, i\gamma_5 d - 2\,\overline{s}\, i\gamma_5 s]/\sqrt{3} \tag{5}$$

as a basis. The Ioffe currents for the nucleons are defined by

$$\chi_p = \epsilon_{abc}[([u^a]^T \mathcal{C}\gamma_\mu u^b)\,\gamma_5\gamma^\mu d^c], \tag{6}$$

$$\chi_n = \epsilon_{abc}[([d^a]^T \mathcal{C}\gamma_\mu d^b)\,\gamma_5\gamma^\mu u^c]. \tag{7}$$

Schematically, one then analyses the correlator using both a hadronic model (which involves the relevant πNN couplings as parameters) and the OPE, and matches the two representations, appropriately transformed, to extract the πNN couplings. In order to perform this analysis, it is necessary that the momentum q on the external pion leg of the correlator be large and spacelike, otherwise the OPE of the correlator will not be valid truncated to the low-dimension operators for which the analysis is practical. This condition, however, means that one is rather far from the pion pole. This problem is dealt with by looking for terms in the OPE which have the same Lorentz structure as the pion contribution and in addition have a pole of the form $1/q^2$. However, as stressed by Birse and Krippa [8], this proceedure is inherently rather dangerous, since it is not *a priori* possible to separate contributions from the pion and from higher resonances to the coefficient of $1/q^2$. Since, moreover, one must certainly work at $Q^2 \equiv -q^2 > 1$ GeV^2, i.e., rather far from the pion pole, one may no longer reasonably count on the proximity of this pole to conclude that the pion contribution is dominant, as it would be for small Q^2. The reliability of the three-point function method thus rests crucially on the assumption that the higher resonance contributions, in the region of Q^2 values under consideration, are small. The plausibility of this assumption has not previously, to our knowledge, been investigated.

The second approach to the πNN couplings is the two-point function method [2,6,8,9]. In this approach, one considers the two-point correlator

$$\Pi(p,q) = i\int d^4x\, \exp(ipx)\langle 0|\mathrm{T}[\chi_N(x)\overline{\chi_N}(0)]|\pi^a(q)\rangle. \tag{8}$$

For large spacelike values of p, the OPE for the product of the two nucleon interpolating fields can presumably be truncated at operators of relatively low dimension. One may then look at the vacuum-to-pion matrix elements of these operators in order to estimate the πNN couplings. As has been stressed by many authors, in order to be able to remove contributions associated with $N \to N^*$ transitions, one should look not at the γ_5 term in the OPE, but rather the structure $\not q\gamma_5$ [8,11–14]. The advantage of the two-point method is, of course, that it completely avoids the problem of potential contamination from higher resonance contributions, which is unavoidable in the three-point function method. The disadvantage of the method is that, although the vacuum-to-pion matrix elements of the lowest dimension operators can all be evaluated with good accuracy using chiral perturbation theory (ChPT) [10], those for the higher dimension operators are less certain. Thus, for example, the uncertainty in the value of the matrix element

$$g_s\langle 0|\,\overline{d}\widetilde{G}^{\mu\nu}\gamma_\nu u|\,\pi^+(q)\rangle \tag{9}$$

leads to uncertainties of $\sim 15\%$ in the prediction for the isospin-conserving πNN coupling using the two-point function method [8].

To date, the isospin-conserving πNN coupling has been considered using both the two-point [2,6,8,9] and three-point [2,5,6] function methods, and the isospin-violating πNN couplings using the three-point function method [7]. The isospin-conserving coupling is, of course, known experimentally, so the point of computing it using QCD sum rules is primarily to test the plausibility of the assumptions and truncations that go into the evaluation. The hope is that success in computing the coupling in the isospin-conserving case might serve as an indication of the reliability of the approach employed and hence make the analogous calculation of the unmeasured isospin-violating couplings also plausibly reliable. Since both the two-point and three-point function treatments are successful in this regard, albeit it with significant theoretical errors, it would seem reasonable to attempt to proceed to the isospin-violating case using either method. A first attempt in this direction was made by Meissner and Henley [7], employing the three-point function method. Since the isospin violating coupling has not yet been experimentally determined, this estimate of is considerable potential interest, particularly in view of the recent revival of interest in the question of isospin-breaking in few-body systems (see for example the review of Ref. [15] for an extensive discussion of the situation up to 1990, and Ref. [16] for a list of more recent papers on the subject). We would, therefore, like to understand whether, given the potential problems of the three-point function method, this estimate is reliable or not.

In the present paper, therefore, we will investigate the question of higher resonance contaminations in the three-point function method, which is the biggest potential roadblock to using the method in the isospin-violating case. We will show that existing (albeit model dependent) understanding of the spectral density in the isovector pseudoscalar channel implies that significant contamination is present, even in the isospin-conserving case, and that requiring this understanding to be incorrect, and the contamination to be small, is equivalent to rather strong (and physically implausible) statements about the values of flavor-breaking ratios of quark condensates. We will then proceed to show that certain features of the isospin-violating analysis itself also clearly indicate the presence of significant higher resonance contamination, implying that the three-point function method cannot be reliably employed to extract the isospin-violating πNN couplings.

II. THE ISOSPIN-CONSERVING ANALYSIS

One does not know, *a priori*, the size of the couplings of the excited isovector pseudoscalar mesons, $\pi(1300)$, $\pi(1800)$, ..., to the nucleon. Thus, in order to be certain that the three-point function method for extracting $g_{\pi NN}$ is not contaminated by contributions from these resonances it is necessary that the conditions

$$\frac{f_{\pi'} m_{\pi'}^2}{(Q^2+m_{\pi'}^2)} \ll \frac{f_\pi m_\pi^2}{(Q^2+m_\pi^2)} \tag{10}$$

be satisfied, where f_M is the decay constant for meson M and π' stands for any of the excited $I=1$ pseudoscalar mesons.

The excited state pseudoscalar decay constants are not known experimentally and, owing to the fact that they have a chiral suppression, [1] will be very difficult to measure. (The $\pi(1300)$ decay constant could, in principle, be measured by separating the small pseudoscalar contribution from the large overlapping a_1 contribution in τ decays via a detailed spin-parity analysis [17].) However, they *are* related to the light current quark mass combination m_u+m_d via a series of finite energy sum rules [18]. The most recent analysis of this mass combination [18] models the continuum part of the spectral function in the isovector pseudoscalar channel in terms of a sum of $\pi(1300)$ and $\pi(1800)$ resonances. The relative contribution of the two resonances in the model continuum spectral function is constrained by duality, and the overall normalization is set by assuming the threshold value of the continuum spectral function, which can be estimated using ChPT at tree level, is saturated by the tails of the resonance contributions. If we assume the model spectral functions so constructed (which are tied to the usually quoted values of the $\overline{\text{MS}}$ masses) are reasonable, then we can read off the corresponding values of the decay constants for the $\pi(1300)$ and $\pi(1800)$. Taking the model with the best duality fit from Ref. [18], we find

$$f_{\pi(1300)} = 2.2 \text{ MeV},$$
$$f_{\pi(1800)} = 1.0 \text{ MeV}. \tag{11}$$

Forming the products of the couplings $f_M m_M^2$ to the pseudoscalar $I=1$ current, and the propagators $1/(Q^2+m_M^2)$ evaluated at $Q^2 \sim 1$ GeV^2, we then find

$$\left[\frac{f_M m_M^2}{(Q^2+m_M^2)}\right]_{Q^2=1\ \text{GeV}^2} = 1.8,\ 2.7,\ \text{and } 0.76 \text{ MeV}, \tag{12}$$

for $M=\pi$, $\pi(1300)$, and $\pi(1800)$, respectively. For such values of the excited pseudoscalar decay constants, therefore, one would be forced to conclude that the contamination from, certainly the $\pi(1300)$, and most likely also the $\pi(1800)$, would be far too large to make the method reliable.

Of course, one might object that the above argument, relying as it does on the model spectral functions of Ref. [18], is model dependent and therefore not conclusive. Indeed, the validity of the method of Ref. [18] for setting the overall normalization [and hence the overall scale of the corresponding decay constants in Eq. (11)] has been questioned [19], leading to the suggestion that the normalization might actually be significantly smaller than employed in Ref. [18]. However, even if the normalization of the continuum spectral function were to be decreased by *an order of magnitude*, the corresponding decay constants would be decreased only by a factor ~ 3, leaving the product of the $\pi(1300)$ coupling to the current and propagator at the level of $\sim 50\%$ of that of the π.

It thus appears extemely unlikely that the couplings of the higher pseudoscalar resonances can be neglected in the three-point function analysis of $g_{\pi NN}$. Since, however, one does not actually have an experimental value for $f_{\pi(1300)}$, one is still free to imagine that the decay constants are, for some reason, *much* more than just an order of magnitude smaller than those corresponding to the model spectral functions of Ref. [18] (for example, a spectral function a factor of 60 times smaller than that of Ref. [18] would bring the $\pi(1300)$ coupling times propagator factor to below 20% of the corresponding product for the π). This appears a rather unlikely prospect, but is one that cannot be presently ruled out. However, it is important to realize that such an assumption is not without other nontrivial consequences.

Let us, therefore, for the moment, accept the (albeit unlikely) prospect of extremely small excited pseudoscalar decay constants and consider the consequences of such an assumption for the quark masses and condensates. The first consequence is obvious, namely, if we strongly suppress the continuum contribution to the finite energy sum rule analysis for the quark mass, then we commit ourselves to significantly lower values of m_u+m_d, of order 6 MeV (in the $\overline{\text{MS}}$ scheme, at a scale of 1 GeV^2, this value to be compared with the conventionally quoted value 12 MeV). This is not necessarily a problem, since a recent analysis of world lattice data also suggests a significantly lowered value of m_u+m_d [20], a possibility also noted in Ref. [19]. It does, however, force one to significantly larger values of the light quark condensate, which can cause problems for the stability of the sum rule for the nucleon mass [21]. Moreover, such an assumption actually corresponds to rather strong constraints on the ChPT low-energy constant (LEC) H_2^r (where we adhere to the notation of Gasser and Leutwyler [10] throughout), which LEC governs the flavor breaking of the quark condensates [10]. The reason for the existence of this constraint is that the inverse weighted (corresponding to $n=-1$ in the notation of Ref. [18]) finite energy sum rule for the correlator

$$\begin{aligned}\Psi_5(q^2) &\equiv i\int d^4x e^{iq\cdot x}\langle 0|T\{\partial^\mu A_\mu^{(-)}(x), \partial^\nu A_\nu^{(+)}(0)\}|0\rangle \\ &= (m_d+m_u)^2 i\int d^4x e^{iq\cdot x} \\ &\quad \times \langle 0|T\{P^{(-)}(x), P^{(+)}(0)\}|0\rangle \end{aligned} \tag{13}$$

with $A_\mu^{(+)}$ the charged isovector axial current and $P^{(+)}$ the corresponding charged isovector pseudoscalar current, can be rewritten as a sum rule for the continuum portion of the pseudoscalar spectral function, as follows:

$$\int_{9m_\pi^2}^{s}\frac{dt}{t}\frac{\mathrm{Im}\Psi_5(t)}{\pi}=\frac{3}{8\pi^2}[m_u(s)+m_d(s)]^2 s[1+R_0(s)]$$
$$-\frac{8m_\pi^4 f_\pi^4}{F^4}(2L_8^r-H_2^r), \tag{14}$$

where $m_{u,d}(s)$ are the $\overline{\mathrm{MS}}$ running masses at scale s, $R_0(s)$ contains perturbative corrections [18], F is a leading order ChPT LEC, equal to the pion decay constant in the chiral limit, and $2L_8^r-H_2^r$ is a scale-independent combination of fourth order ChPT LEC's. If we suppress the integral on the left-hand side (LHS) by a large factor like 60, and also the running masses by of order a factor of 2, it turns out that we drive $2L_8^r-H_2^r$ to values more than 3 times smaller than those obtained in Ref. [18]. This in turn implies that $H_2^r(m_\eta)$ must *necessarily* be positive. We now argue that such values for $H_2^r(m_\eta)$ lead to physically implausible predictions for the ratios of quark condensates.

To see this, note that, once H_2^r is fixed, the flavor breaking ratios of quark condensates are simultaneously fixed at next-to-leading order in the chiral expansion [10], for example,

$$\frac{\langle 0|\overline{s}s|0\rangle}{\langle 0|\overline{u}u|0\rangle}=1+3\mu_\pi-2\mu_K-\mu_\eta+\frac{8(m_K^2-m_\pi^2)}{F^2}(2L_8^r+H_2^r), \tag{15}$$

where $\mu_M=m_M^2\ln(m_M^2/\mu^2)/32\pi^2F^2$, with μ the ChPT renormalization scale. With $H_2^r(m_\eta)>0$ and $L_8^r(m_\eta)=(1.1\pm0.3)\times10^{-3}$ [22], we see from Eq. (15) that

$$\frac{\langle 0|\overline{s}s|0\rangle}{\langle 0|\overline{u}u|0\rangle}>1.38. \tag{16}$$

Thus, assuming that the excited resonance decay constants are sufficiently small to be able to neglect their contributions to the three-point function sum rule simultaneously commits one to the highly unnatural situation of a strange quark condensate larger in magnitude than the light quark condensate. In addition, one finds that, owing to the relation between the flavor breaking and isospin breaking condensate ratios given by Eq. (9.5) of Ref. [10], such a value for the strange to up quark condensate ratio implies, for the isospin-breaking condensate ratio γ defined by

$$\gamma\equiv\frac{\langle 0|\overline{d}d|0\rangle-\langle 0|\overline{u}u|0\rangle}{\langle 0|\overline{u}u|0\rangle}, \tag{17}$$

a value

$$\gamma>1.5\times10^{-3}, \tag{18}$$

in contradiction with extractions of γ from a variety of sources [3,23–27] all of which obtain $\gamma<0$.

In view of the results of the last paragraph, we conclude that the hypothesis that one may neglect the higher resonance contamination in the three-point function analysis of the isospin conserving coupling $g_{\pi NN}$ is a highly unpalatable one. The only way to ''save'' the three-point function treatment is to *assume* that, for some reason, the ratios of the coupling constants of all higher pseudoscalar resonances to the nucleon relative to that of the pion are tiny. This possibility is highly unnatural, but of course, not ruled out by the discussion below. We will, therefore, in the next section, turn to the case of the analysis of the isospin-breaking couplings, for which the meson-nucleon couplings are now implicitly present, and demonstrate more directly the presence of analogous higher resonance contaminations.

III. THE ISOSPIN-VIOLATING ANALYSIS

In this section we will concentrate on the three-point function analysis of the difference of π^0pp and π^0nn couplings,

$$\delta g=g_{\pi^0nn}-g_{\pi^0pp}. \tag{19}$$

In order to perform this analysis it is necessary to take into account the fact that P_1^0 is not a suitable interpolating field for the physical π^0, if one wishes to treat isospin breaking effects. This follows from the observation that

$$\langle 0|P_1^0|\eta\rangle\neq0. \tag{20}$$

As a consequence, if one were to use P_1^0 as π^0 interpolating field, then even if one could ignore higher resonance contributions, the result of the analysis would be a mixture of the isospin breaking π coupling and the isospin conserving η coupling (the latter multiplied by an isospin breaking factor describing the coupling of the η to the $I=1$, $I_z=0$ current). As noted above, there is no means to separate the contributions corresponding to different mesons in the three-point function approach. Meissner and Henley [7], who first performed the isospin violating analysis, dealt with this problem by choosing a current combination with no vacuum-to-η matrix element, namely,

$$P_{\pi^0}\equiv P_1^0+\epsilon P_0^8, \tag{21}$$

where the pseudoscalar currents are as defined above and the choice

$$\epsilon=\theta_0=\frac{\sqrt{3}}{4}\frac{m_d-m_u}{m_s-\hat{m}}, \tag{22}$$

where $\hat{m}=(m_u+m_d)/2$, and θ_0 is the leading order π-η mixing angle [10], ensures that

$$\langle 0|P_{\pi^0}|\eta\rangle=0 \tag{23}$$

to leading order in the chiral expansion. The choice of interpolating field with this property is not unique [16]; in fact, for any α, defining

$$P(\alpha)=(P_u-P_d)+\epsilon(\alpha)[\alpha(P_u+P_d)+(1+\alpha)P_s]$$
$$=(P_u-P_d)+\epsilon(\alpha)\frac{1}{\sqrt{3}}[-P_8+(3\alpha+1)P_0], \tag{24}$$

one may find an $\epsilon(\alpha)$ such that

$$\langle 0|P(\alpha)|\eta\rangle = 0. \tag{25}$$

The set of solutions of Eq. (25), as a function of α, have been worked out to next-to-leading order in the chiral expansion in Ref. [16]. Among other results of this analysis, it is found that the next-to-leading corrections are significant; for example, at next-to-leading order, the Meissner-Henley field choice must be modified to

$$P'_{\pi^0} \equiv P_1^0 + 1.27\theta_0 P_0^8 \tag{26}$$

if one wishes to maintain zero vacuum-to-η matrix element.

We will now explain why the existence of the above family of potential π^0 interpolating fields is of relevance to our current discussion. Note that the choice of P_{π^0} was made by Meissner and Henley with the problem of potential higher resonance contamination in mind. Indeed, they argued that this choice of interpolating field is the one that would suppress possible η' contributions [7]. It turns out, however, that this is not the case, and the reason that it fails to be so leads us immediately into a consideration of the larger class of π^0 interpolating fields.

Let us, therefore, first outline why the interpolating field P_{π^0} necessarily has nonvanishing coupling to the η'. In the chiral limit, of course, there is no flavor breaking whatsoever, hence no isospin breaking couplings, and no flavor breaking meson decay constants. Once we introduce the quark mass matrix, with its flavor breaking $m_s - \hat{m}$ difference and isospin breaking $m_d - m_u$ difference, all flavor and isospin breaking effects are potentially present. In the case of the isospin breaking and flavor breaking decay constants of the η', one may obtain a leading estimate of the ratio of these decay constants using $\mathrm{SU}(3)_F$ arguments. Indeed, we know that the breaking is produced by the quark mass matrix, which has the following decomposition into singlet, octet isovector and octet hypercharge components

$$M = \frac{1}{3}(m_s + 2\hat{m}) - \frac{1}{2}(m_d - m_u)\lambda_3 - \frac{1}{\sqrt{3}}(m_s - \hat{m})\lambda_8. \tag{27}$$

To leading order in the isospin-breaking and flavor-breaking mass differences, therefore, the vacuum-to-η' matrix elements of P_1^0 and P_0^8 are given simply by the product of the mass-dependent coefficients of λ_3 and λ_8 in Eq. (27) with a common $8_F \times 8_F \to 1_F$ reduced matrix element. Recasting the ratio of mass factors in terms of the mixing angle θ_0 defined above, one then finds, straightforwardly, that

$$\langle 0|P_{\pi^0}|\eta'\rangle = 3\theta_0 \langle 0|P_0^8|\eta'\rangle, \tag{28}$$

where $\langle 0|P_0^8|\eta'\rangle = \mathcal{O}(m_s - \hat{m})$. The RHS is thus nonzero, of $\mathcal{O}(m_d - m_u)$, and in fact has a numerical enhancement (the factor 3) brought about by the fact that the couplings of the η' to the P_1^0 and P_0^8 components of P_{π^0} add coherently. Note that a similar argument, using a first order treatment of flavor and isospin breaking, would predict that the isospin violating and flavor violating axial current η' decay constants were of the same sign, in agreement with the results of a recent QCD sum rule analysis of the isospin violating $\langle 0|T(A_\mu^3 A_\nu^8)|0\rangle$ correlator [28].

If one considers allowing an admixture of the singlet pseudoscalar current into the interpolating field (i.e, allowing α to deviate from $-1/3$), one has, of course, in addition to the $8_F \times 8_F \to 1_F$ reduced matrix element which governs $\langle 0|P_1^0|\eta'\rangle$ and $\langle 0|P_0^8|\eta'\rangle$, the $1_F \times 1_F \to 1_F$ reduced matrix element relevant to $\langle 0|P_0^0|\eta'\rangle$. One can then certainly, in principle, find a value of α such that $\langle 0|P(\alpha)|\eta'\rangle = 0$. The problem is that, even to do so at leading order in the quark masses, one would need to know the ratio of the two reduced matrix elements above, and this information is not available. Moreover, even if it were, this would not necessarily ensure that, for such a value of α, the couplings of the higher resonances other than the η' were small for the same value of α.

Since we do not know, *a priori*, how to choose a π^0 interpolating field (i.e., a value of α) to remove even η' contamination, let alone one possible contamination associated with yet higher pseudoscalar resonances, it is necessary to look for some sort of *post facto* indication of the absence of such contributions. One obvious way of doing so is to study the extracted results for what is nominally the isospin violating πNN coupling, δg, as a function of α. If one can find a region of α values for which the results are not sensitive to α, then one might argue this was a signal that, for such values of α, the effect of couplings to all higher resonances is negligible. In contrast, if one is unable to find such a region of α values then, since the various interpolating fields differ only in their couplings to the excited pseudoscalar resonances, beginning with the η', it is clear that, in general, there are large contaminations from the higher pseudoscalar resonances and that, as a consequence, one has no reliable way of choosing a particular interpolating field for which such contaminations are small. We will see, unfortunately, that it is the latter situation which holds for the three-point function analysis. Moreover, we will demonstate that the variation of δg with α is essentially as large as the value of δg extracted in the Meissner-Henley analysis, and hence that no reliable estimate of δg can be made using the three-point function method.

In order to demonstrate the claims of the last paragraph, it is necessary to understand how the generalization from the specific Meissner-Henley interpolating field choice, $\alpha = -1/3$, to arbitrary α affects the sum rule for the isospin breaking πNN coupling. It is straightforward to show that, in the general case, the final sum rule (the analogue of Eq. (24) of Ref. [7]) becomes

$$\left[-\frac{\delta g}{g_{\pi NN}}\right] = -\frac{2}{3}\gamma + \frac{4}{3}\kappa(\alpha) + \left(\frac{\delta M_N}{M_N}\right)\frac{(2\pi)^2 m_\pi^2 f_\pi^2 M^2}{M^6 + \frac{1}{4}g_s^2\langle G^2\rangle M^2} \times \left[2\left(\frac{m_d - m_u}{m_d + m_u}\right) + \gamma - 2\frac{\delta M_N}{M_N}\frac{M_N^2}{M^2}\right], \tag{29}$$

where M is the Borel mass, $\langle G^2\rangle$ the gluon condensate, g_s the strong coupling constant, δM_N the quark-mass-difference

contribution to the nucleon mass splitting, and $\kappa(\alpha)$ the coefficient of P_u+P_d appearing in the interpolating field $P(\alpha)$ (so, for example, for the Meissner-Henley choice P_{π^0}, $\kappa=\theta_0/\sqrt{3}$). The terms in the second line of Eq. (29) arise from evaluating the isospin breaking difference of couplings of the Ioffe currents to the neutron and proton states using the chiral odd sum rule for the nucleon two-point functions [7]. The reason for the appearance of $\kappa(\alpha)$ in Eq. (29), i.e., the dependence on only the light quark $I=0$ content of the interpolating field, is that contributions of the strange quark component of the interpolating field to the OPE side of the sum rule vanish to the order considered in obtaining the sum rule.

From the expression for $P(\alpha)$ in Eq. (24), it is evident that

$$\kappa(\alpha)=\alpha\epsilon(\alpha). \tag{30}$$

In order to complete our investigation we, therefore, require only the expression for $\epsilon(\alpha)$, obtained in Ref. [16]:

$$\epsilon(\alpha)=-\sqrt{3}\theta_0\left[1+\left(\frac{-(10+9\alpha)\mu_\pi+6(1+\alpha)\mu_K+(4+3\alpha)\mu_\eta}{3F^2}\right)-\frac{32}{3F^2}(4+3\alpha)(\overline{m_K^2}-m_\pi^2)(3L_7^r+L_8^r)+\left(\frac{3m_\eta^2+m_\pi^2}{64\pi^2F^2}\right)\right.$$
$$\left.\times\left\{1+\left[\frac{m_\pi^2}{\overline{m_K^2}-m_\pi^2}\right]\ln\left(\frac{m_\pi^2}{\overline{m_K^2}}\right)\right\}+\frac{(m_\eta^2-m_\pi^2)}{64\pi^2F^2}[1+\ln(m_K^2/\mu^2)]\right], \tag{31}$$

where $\overline{m_K^2}$ is the average of the K^+ and K^0 squared masses, the chiral log terms μ_M are as defined above and the L_k^r are the usual renormalized fourth order LECs, in the notation of Gasser and Leutwyler [10]. In Eq. (31) the expression has been written so that the 1 occurring in the square brackets corresponds to the contribution obtained at leading order in the chiral expansion, while the remaining terms give the next-to-leading order corrections. The above results reproduce the Meissner-Henley field choice for $\alpha=-1/3$, if one keeps only the leading order contribution in Eq. (31).

If we take the latest evaluation of the quark mass ratios from ChPT [29], then we have

$$\theta_0=(1.1\pm0.2)\times10^{-2}. \tag{32}$$

(The upper end of the error bound would correspond to the evaluation of $r=(m_d-m_u)/(m_d+m_u)$ obtained using $\eta\to3\pi$, which corresponds to large values for the violation of Dashen's theorem advocated by a number of authors [30–35].) For the Meissner-Henley choice $\alpha=-1/3$, we then find, for the contribution to $-\delta g/g_{\pi NN}$ generated by the isoscalar component of the π^0 interpolating field (required to remove the η contamination from the final result),

$$[-\delta g/g_{\pi NN}]_{\kappa(-1/3)}=\frac{4}{3}\frac{\theta_0}{\sqrt{3}}[1+0.27]=1.1\times10^{-2}, \tag{33}$$

while, for comparison, for the choice $\alpha=-1$, which removes the strange quark content from the interpolating field, we find

$$[-\delta g/g_{\pi NN}]_{\kappa(-1)}=\frac{4}{3}\sqrt{3}\theta_0[1+0.21]=3.1\times10^{-2}. \tag{34}$$

To understand the implication of these results, one should bear in mind that the range of values for $-\delta g/g_{\pi NN}$ extracted by Meissner and Henley was $(1.7–3.0)\times10^{-2}$. (The range reflects the full range of uncertainties in all of the input parameters γ, θ_0, and δM_N.) We thus find that the corrections required to remove the η contamination are large on the scale of the result obtained. Moreover, since $\kappa(\alpha)=\alpha\epsilon(\alpha)$, and the results of Ref. [16] show $\epsilon(\alpha)$ to be slowly varying with α, and greater than $\sqrt{3}\theta_0$ in magnitude over a wide range of α values, we see also that the results for what is nominally the isospin breaking coupling is very sensitive to α, varying, for example, by an amount as large as the maximum value quoted by Meissner and Henley over the range between $\alpha=0$ and $\alpha=-1$. We thus conclude, in light of the discussion above, that the three-point function evaluation of the isospin breaking coupling is plagued by unknown higher resonance contamination, and as such cannot provide a reliable estimate of this quantity.

IV. CONCLUSIONS

We have shown that the three-point function method for the treatment of both the isospin conserving and isospin violating πNN couplings is plagued by problems with higher resonance contamination. In the course of this investigation we have also seen how information from the finite energy sum rules for the light quark masses, chiral perturbation theory, and sum rules for the chiral LEC's can sometimes be profitably employed to elucidate the physical content of other sum rules treatments.

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