Do phase-shift analyses and nucleon-nucleon potential models yield the wrong ${}^{3}P$ _i phase shifts **at low energies?**

W. Tornow

Department of Physics, Duke University, Durham, North Carolina 27708 and Triangle Universities Nuclear Laboratory, Duke Station, Durham, North Carolina 27708

H. Witała

Institute of Physics, Jagellonian University, PL-30059 Cracow, Poland

A. Kievsky

Istituto Nazionale di Fisica Nucleare, Sezione di Pisa and Dipartimento di Pisa, Universita' di Pisa, I-56100 Pisa, Italy (Received 27 November 1996; revised manuscript received 15 May 1997)

The ⁴*PJ* waves in nucleon-deuteron scattering were analyzed using proton-deuteron and neutron-deuteron data at $E_N=3$ MeV. New sets of nucleon-nucleon ${}^{3}P_j$ phase shifts were obtained that may lead to a better understanding of the long-standing $A_y(\theta)$ puzzle in nucleon-deuteron elastic scattering. However, these sets of 3P_i phase shifts are quite different from the ones determined from both global phase-shift analyses of n nucleon data and nucleon-nucleon potential models. [S0556-2813(98)02202-X]

PACS number(s): $25.10.+s$, 13.75.Cs, $24.70.+s$, 25.40.Cm

I. INTRODUCTION

Since its discovery some ten years ago, the neutrondeuteron $(n-d)$ analyzing power $A_{y}(\theta)$ puzzle [1] remains the most elusive problem in low-energy three-nucleon $(3*N*)$ elastic scattering. Although several groups $[1-4]$ have performed extensive studies with modern nucleon-nucleon (*NN*) potential models, including various types of threenucleon forces $(3NF)$, it turned out to be impossible to account for the 25–30 % discrepancy between rigorous 3*N* calculations and experimental data [see Fig. 1(a), short dash and solid curve in comparison to experimental data]. At the present time, it is not clear whether the problem is caused by on-shell or/and off-shell deficiencies in the underlying *NN* potentials used in the calculations or by some other yet unknown phenomenon. Sensitivity studies $[5-7]$ have clearly shown that $A_y(\theta)$ in *n-d* elastic scattering is governed by a complicated interplay between the ${}^{3}P_0$, ${}^{3}P_1$, and ${}^{3}P_2$ *NN* interactions. As has been established more than forty years ago, these interactions are also responsible for the *NN* analyzing power $A_y(\theta)$ [8]. However, the *n*-*d* $A_y(\theta)$ is a factor of 10 larger than the $n-p A_y(\theta)$, thus providing a greatly enhanced sensitivity to the ${}^{3}P_{j}$ *NN* interactions. It should also be noted that the p - $p A_y(\theta)$ is governed by the Coulomb interaction. In fact, the proton-deuteron $(p-d) A_y(\theta)$ is two orders of magnitude larger than the p - $p A_y(\theta)$. Therefore, *nd* and *p*-*d* data may provide a magnifying glass for determining the ${}^{3}P_{i}$ *NN* interactions.

Witała and Glöckle [5] studied the on-shell aspect of the $A_y(\theta)$ puzzle. They found a combination of ³P_i NN interactions that describes both the *n*-*d* and *NN* $A_v(\theta)$ and crosssection $\sigma(\theta)$ data. In addition, the *p*-*d* $A_y(\theta)$ data are also well described, although the calculations did not include the Coulomb interaction. The small difference in the absolute magnitude between the *n*-*d* and *p*-*d* $A_y(\theta)$ data [9] was interpreted as charge-symmetry breaking (CSB) in the ³ $P_j NN$ interactions. However, the ${}^{3}P_{j}$ interactions obtained by Witała and Glöckle exhibit a large breaking of charge independence and charge symmetry. In addition, the sign of the charge-independence breaking (CIB) and CSB is inconsistent with theoretical expectations based on the mesonexchange theory of the *NN* interaction.

After more than thirty years of intensive work the 3*N* scattering problem is now solvable with the Coulomb interaction taken into account in a rigorous way $[10]$. Unfortunately, the solution of the charged 3*N* scattering problem is currently restricted to energies below the deuteron breakup threshold (i.e., $E_p = 3.3$ MeV for *p*-*d* scattering and $E_d = 6.6$ MeV for $d-p$ scattering). Nevertheless, this new development not only has the potential of solving the $A_y(\theta)$ puzzle, but it has provided already some important additional information. First, it confirmed speculations that an $A_y(\theta)$ puzzle exists also for p -*d* elastic scattering [3] (a 20 standard deviation effect!). Second, it showed that a similiar problem exists for $iT_{11}(\theta)$ in \dot{d} -*p* scattering [11]. Third, the small difference between *n*-*d* and *p*-*d* at the maximum of the $A_y(\theta)$ angular distribution is not due to CSB as previously assumed by Witała and Glöckle [5], Takemiya [12], and Soldi *et al.* [13] but is largely caused by the Coulomb interaction $[14]$, as was argued already earlier by Tornow *et al.* [9].

II. ⁴*PJ***-WAVES ANALYSIS IN NUCLEON-DEUTERON SCATTERING**

As has been shown by the Bochum-Cracow group $[15]$, a strong correlation exists between the $n-d$ ⁴ P_J and the *NN* $3P_i$ phase shifts. A change of the $3P_0$ *NN* interaction induced by multiplying its momentum space matrix element by a factor λ_0 influences ${}^4P_{1/2}$, and similar relations exist for the $({}^3P_1, {}^4P_{3/2})$ and $({}^3P_2, {}^4P_{5/2})$ pairs. Although such relations cannot be calculated at the present time for *p*-*d* scattering, we expect very similar relations to exist between the

FIG. 1. Comparison of $n-d A_y(\theta)$ data to rigorous calculations and phase-shift predictions. In panels (a) – (d) the $A_y(\theta)$ data (from Ref. [27] at E_n =3.0 MeV and from [6] at E_n = 5.0, 6.5, and 8.5 MeV) are compared to calculations (solid curve) using the AV18 potential [18,19]. The dashed curves were obtained using our modified ${}^4P_{1/2}$, ${}^4P_{3/2}$, ${}^4P_{5/2}$, $\epsilon_{3/2^-}$, and $\epsilon_{1/2^-}$ phase-shift parameters as described in the text. The dash-dotted curve in (a) was calculated from a version of AV18 where the ${}^{3}P_{i}$ *NN* interactions were modified to produce phase shifts of $0.976 \times \frac{3}{2}P_0$, $0.912 \times \frac{3}{2}P_1$, and 1.16 \times ³ P_2 , where ³ P_j are the original phase shifts associated with the AV18 *NN* potential.

 ^{4}P *J p*-*d* phase shifts and the ^{3}P *j NN* interactions. Following the Bochum-Cracow approach we found that the only other *n*-*d* phase-shift parameter that is influenced by modifications of the ${}^{3}P_0$ force component and simultaneously has an effect

TABLE I. The $p-d$ phase-shift analysis results (fourth column) of Ref. [11] at $E_p=3$ MeV are transformed to *n-d* results (last column) using the ratio obtained from the theoretical calculations for *p*-*d* (second column) and *n*-*d* (third column) scattering at $E_N=3$ MeV [11].

Parameter	$AV18+3NF$		PSA		
	$p-d$	$n-d$	$p-d$	$n-d$	
$^{4}P_{1/2}$	22.3	24.5	21.77	23.9	
$^{4}P_{3/2}$	24.2	26.0	24.30	26.1	
$^{4}P_{5/2}$	24.1	26.3	24.26	26.4	
$\epsilon_{1/2-}$	5.83	6.82	5.70	6.67	
$\epsilon_{3/2-}$	-2.23	-2.66	-2.46	-2.91	

on the description of $A_y(\theta)$, $iT_{11}(\theta)$, $T_{20}(\theta)$, $T_{21}(\theta)$, and $T_{22}(\theta)$ is the ² $P_{1/2}$ ⁻⁴ $P_{1/2}$ mixing parameter $\epsilon_{1/2}$. The magnitude of $\epsilon_{1/2}$ increases by strengthening the ³ P_0 *NN* force. Similarily, changes of the ${}^{3}P_{1}$ or ${}^{3}P_{2}$ - ${}^{3}F_{2}$ interactions affect only the ² $P_{3/2}$ ⁻⁴ $P_{3/2}$ mixing parameter $\epsilon_{3/2}$ significantly, which in turn influences the description of $A_y(\theta)$ and $iT_{11}(\theta)$.

For *p*-*d* and *d*-*p* scattering complete data sets exist for the observables $\sigma(\theta)$, $A_y(\theta)$, $iT_{11}(\theta)$, $T_{20}(\theta)$, $T_{21}(\theta)$, and $T_{22}(\theta)$ at E_p = 3.0 MeV, E_d = 6.0 MeV ($E_{c.m.}$ = 2.0 MeV) [16,17]. Starting from the Pisa phase-shift calculations $[3]$ at $E_{\text{c.m.}}$ = 2.0 MeV based on the Argonne AV18 *NN* potential $[18] +$ Urbana 3NF [19], results of a phase-shift search were published in Ref. [11]. In this work the high-accuracy data of the Kyushu group [17] are described with χ^2 per datum of about 1. In Table I we give the results for the parameters of interest. Surprisingly, the modifications required by *p*-*d* phase-shift analyses to remove the 30% $A_y(\theta)$ and $iT_{11}(\theta)$ discrepancies are fairly small. The largest changes are found in the mixing parameter $\epsilon_{3/2}$ (10% increase) and in the $^{4}P_{1/2}$ phase shift (2.5% reduction). The changes in $^{4}P_{3/2}$ and ${}^{4}P_{5/2}$ are less than 1%.

III. THREE-NUCLEON FORCES

It is important to point out that the *N*-*d* parameters discussed so far correspond to peripheral waves at the energies considered in the present work. Therefore, at a first glance, one would not resort to three-nucleon forces (3NFs) as a mean to fix the $A_y(\theta)$ puzzle, because 3NFs are of shorter range than two-nucleon forces. Nevertheless, present day 3NFs cause small changes in these phase-shift parameters. However, they do not resolve the $A_y(\theta)$ puzzle [1–4]. The 2π -exchange Tucson-Melbourne [20] and Brazil [21] 3NFs lower the calculated $A_y(\theta)$ slightly [1,2], i.e., the discrepancy between data and calculations is increased. The same statement holds for the effective 3NF obtained in a twobaryon coupled-channel potential approach which allows for Δ -isobar excitation [4]. Only the Urbana 3NF gives a small increase [3] in the calculated $A_y(\theta)$ [see Fig. 1(a), short dash and solid curve].

Due to the rather small changes needed to solve the $A_y(\theta)$ puzzle, we present in Table II the average values of the three ${}^{4}P_J$ phase shifts, their splittings, and the mixing parameters $\epsilon_{1/2}$ and $\epsilon_{3/2}$ as calculated from the *NN* potential models $AV14 [22]$ and AV18 with and without 3NFs. For comparison, the last row of Table II represents the phase-shift analy-

TABLE II. Calculated *p*-*d* phase-shift parameters in comparison with the phase-shift analysis (PSA) results of Ref. $[11]$.

Potential	^{4}P		${}^{4}P_{5/2} {}^{4}P_{3/2} {}^{4}P_{3/2} {}^{4}P_{1/2}$	$\epsilon_{1/2}$	$\epsilon_{3/2}$
AV14	23.83	-0.48	1.77	6.22	-2.29
$AV14+BR$	24.05	-0.37	1.60	6.40	-2.36
AV18	23.41	-0.27	2.10	5.71	-2.20
$AV18+UR$	23.54	-0.15	1.95	5.83	-2.23
$AV18+UR*$	23.69	-0.02	1.97	5.86	-2.28
PSA	23.44	-0.04	2.53	5.70	-2.46

sis results of Ref. [11]. Let us compare $AV14$ and $AV18$ first. The AV14 potential underestimates the magnitude of the experimental $A_y(\theta)$ data in *N*-*d* scattering by about a factor of 2. The discrepancy is drastically reduced in the case of the AV18 potential. This improvement is due to the smaller value obtained with AV18 for the ${}^{4}P_{1/2}$ phase shift $(22.1°$ as opposed to $22.8°$ with AV14 for *p-d*), which results in larger ${}^{4}P_{3/2}{}^{4}P_{1/2}$ and ${}^{4}P_{5/2}{}^{4}P_{1/2}$ splittings [3]. As stated earlier, smaller values for the ${}^{4}P_{1/2}$ phase shift are supported by phase-shift analyses. When the Tucson-Melbourne, Brazil (BR), or Urbana (UR) 3NFs are included in the Hamiltonian of the three-nucleon system, all three calculated ${}^{4}P_J$ phase shifts increase slightly, resulting in a reduced splitting. As has been well known, increases of ${}^{4}P_{1/2}$ and ${}^{4}P_{5/2}$ affect the calculated $A_{y}(\theta)$ in opposite ways: an increase in ${}^{4}P_{1/2}$ lowers the magnitude of $A_y(\theta)$ while an increase of ⁴ $P_{5/2}$ yields larger $A_y(\theta)$ values. For the Tucson-Melbourne 3NF the increase in ${}^{4}P_{1/2}$ is not completely canceled by the associated increase in the ${}^{4}P_{5/2}$ phase shift, thus resulting in a slightly smaller overall $A_y(\theta)$. For the Brazil and Urbana 3NFs the situation is reversed. Here, the overall effect gives a small increase in the calculated $A_y(\theta)$. The Urbana 3NF contains a short-range repulsive term (without any spin and isospin dependence). In order to check on the importance of such a term for the description of $A_y(\theta)$, we turned this term off and found basically no increase in the splittings of interest. The magnitude of the mixing paramter $\epsilon_{3/2}$ increased slightly, and the ⁴ $P_{5/2}$ ⁴ $P_{3/2}$ splitting decreased, but overall the description of $A_y(\theta)$ did not improve. This sensitivity test is denoted in Table II with AV18 $+ \mathrm{UR}^*$.

Summarizing, 3NFs have some influence on the individual phase shifts of interest, but, due to cancellations, the overall influence on $A_y(\theta)$ and $iT_{11}(\theta)$ is rather small. However, the present 3NF studies do not rule out the possibility that new forms of 3NFs, for example, the one proposed in Ref. [23] may eventually explain the $A_y(\theta)$ puzzle. Crucial for any new form of 3NFs must be a feature that affects the ^{4}P _{*I*} 3*N* phase shifts in a different way as do present-day 3NFs, i.e., the ${}^{4}P_{1/2}$ phase shift must be reduced and not increased. Furthermore, restricting our comments to scattering, one has to keep in mind that any 3NF is allowed to act appreciably only on very specific 3*N* observables, otherwise the good agreement found between a large body of *NN* predictions and $3N$ observables would be destroyed [24]. In addition, one should keep in mind that there is strong experimental evidence that their influence must decrease with increasing energy in the case of $A_y(\theta)$, a condition which appears quite puzzling, too. At $E_p = 650 \text{ keV}$ the calculated $A_{y}(\theta)$ using AV18 underestimates the experimental data by almost 40% [25], at $E_n = 6.5$ MeV using Bonn B [26] by 25% [6], at $E_n = 30$ MeV the *n-d* calculations with AV18 almost agree with the $n-d$ and $p-d$ data [24], and finally at $E_n = 50$ and 65 MeV the *n*-*d* calculations using again AV18 agree very well with both the $n-d$ and $p-d$ data [24].

IV. AN ALTERNATIVE APPROACH

The latter comments clearly call for detailed studies to make sure that possible alternative solutions of the $A_y(\theta)$ puzzle will not be overlooked. Witała and Glöckle $[5]$ were the first who seriously treated the $A_y(\theta)$ puzzle as an onshell effect. However, as stated earlier, their resulting ${}^{3}P_{i}$ *NN* phase shifts seem to deviate too much from what is currently accepted for CSB and CIB of the ${}^{3}P_{i}$ *NN* interactions. As mentioned above, the method employed by the Pisa group to calculate *p*-*d* scattering observables using the AV18 *NN* potential does not provide relations between individual *p*-*d* phase-shift parameters and the underlying *NN* force components. However, such relations can be obtained for $n-d$ scattering using the approach of Ref. [5]. Consequently, in the first step, we converted the $p-d \binom{4}{J}$ phaseshifts parameters into $n-d^{-4}P_J$ phase shifts and mixing parameters (given in parenthesis in Table I) using the information published in Ref. $[11]$, i.e., we multiplied the p *d* parameters *X* with the ratio $R_I = (X_I)^{n-d}/(X_I)^{p-d}$. Here, we used the average between the AV18 and $AV18+3NF$ results for X_J [11]. In the second step we used the approach of Ref. [5] and determined the λ factors by which the ³*P_i* AV18 interaction matrix elements have to be multiplied in order to reproduce our new $n-d^{-4}P_J$ phase shifts. The λ factors obtained in the present work for ${}^{3}P_{0}$, ${}^{3}P_{1}$, and ${}^{3}P_{2}$ are 0.96, 0.98, and 1.06, respectively. They differ from the λ_{eff} factors $(\lambda_{\text{eff}} = \frac{2}{3} \lambda_{nn} + \frac{1}{3} \lambda_{np})$ reported by Witała and Glöckle $[5]$ (0.92, 1.06, 1.02). In the latter work, the Bonn B [26] *NN* potential was used and *N*-*d* $A_y(\theta)$ and $\sigma(\theta)$ data were considered over an extended energy range in a trial and error approach. As an important observation we notice that the *n*-*d* $A_v(\theta)$ data of Refs. [7,27] are very well reproduced [see Fig. 1(a), dashed curve] when the theoretical phase-shift parameter of Ref. [11] are replaced by our modified parameters given in the last column of Table I. This fact clearly shows that our approach does not require any breaking of charge symmetry in addition to the small amount already contained in the AV18 *NN* potential.

Since rigorous *p*-*d* 3*N* calculations are not available yet above the deuteron breakup threshold, our approach cannot be extended to higher energies. Of course, rigorous *n*-*d* calculations and therefore, $n-d$ phase shifts $[15]$ are available above the *n*-*d* breakup threshold and could be used as starting values in *n*-*d* phase-shift analyses. However, our *p*-*d* phase-shift analyses clearly showed that $\sigma(\theta)$ and $A_y(\theta)$ data alone do not permit an accurate determination of the ${}^{4}P_J$ phase shifts. Data for the vector analyzing power $iT_{11}(\theta)$ or for one of the two tensor analyzing powers $T_{20}(\theta)$ or $T_{21}(\theta)$ in \tilde{d} -*n* scattering are required. Unfortunately, such data do not exist.

In order to obtain information about the energy dependence of our modification factors, we used the factors *r ^J*

FIG. 2. Prediction for the $p-p$ (top panel) and $n-p$ (bottom panel) $A_y(\theta)$ at E_N =3.0 MeV using the original AV18 *NN* potential model (solid curves) and modified AV18 matrix elements $(0.96$ \times ³ P_0 , 0.98 \times ³ P_1 , and 1.06 \times ³ P_2) obtained from a phase-shift fit to *p*-*d* data.

 $=(X_J)^{PSA}/(X_J)^{AV18+3NF}$ found at 3 MeV (Table I) and applied it to the $n-d$ AV18 phase-shift parameters X_j in the energy range from E_n = 5.0 to 8.5 MeV. As can be seen from Fig. $1(b)$ (dashed curve) our approach works also very well at $E_n = 5.0$ MeV, where the modified parameters give an excellent description of the TUNL $n-d$ data [6]. However, starting at E_n = 6.5 MeV [see Fig. 1(c)] small deviations begin to emerge at the maximum of $A_y(\theta)$ which are clearly established at E_n = 8.5 MeV [6] [see Fig. 1(d)]. Nevertheless, this fairly weak energy dependence may turn out to be important in future studies at higher energies.

V. RESULTING *NN* **FORCE COMPONENTS AND COMPARISON TO** *NN* **AND NEUTRON-DEUTERON DATA**

From the modified values found in our analysis for the $n-d$ ⁴ P_J </sup> phase shifts new *n*-*p* and *p*-*p* phase shifts were obtained for the ${}^{3}P_{i}$ interactions by applying the resulting $^{3}P_i$ λ factors (0.96, 0.98, and 1.06) to the AV18 potential matrix elements. In order to preserve the small CIB and CSB of the original AV18 potential, contrary to Ref. $[5]$, we multiplied the *pp*, *np*, and *nn* ³*P*_{*j*} interactions with the same λ_j factors. Next, we used this modified AV18 potential and calculated the *n*-*p* and *p*-*p* $A_y(\theta)$ at $E_N = 3.0$ MeV. It turned out that our new "experimental" ${}^{3}P_{j}$ phase shifts overpredicted the magnitude of the p - p [see Fig. 2(a), dashed curve] and *n*-*p* [see Fig. 2(b), dashed curve $A_y(\theta)$ by about 10 and 25 %, respectively. Since the AV18 potential is fitted to the Nijmegen NI93 *NN* phase-shift analysis (PSA) [28], our results are also inconsistent with this phase-shift analysis. The same statement holds for the VPI phase-shift analysis [29]. Unfortunately, experimental data are not available at E_N = 3.0 MeV. Our new experimental ${}^{4}P_J$ phase-shift parameters call for ${}^{3}P_{i}$ *n-p* phases that are in clear cut contradiction with existing $n-p$ low-energy $A_y(\theta)$ phase-shift data [28]. To a smaller degree this observation holds also for the $p-p^{-3}P_j$ phases and *p*-*p* phase-shift $A_y(\theta)$ data [28]. However, here the disagreement is not so dramatic due to the fact that the *p* $p A_y(\theta)$ at low energies is governed by the Coulomb force. Nevertheless, we conclude that our approach must have a sizable uncertainty.

The actual uncertainty associated with our λ factors is difficult to evaluate. One has to keep in mind that one cannot expect to obtain exact information about the *NN* interaction at a particular incident nucleon energy from studying *N*-*d* scattering at this energy, because an extended range of *NN* energies comes into play in the 3*N* system (from E_N to $-\infty$). This fact is most likely the reason why the new "experimental'' ${}^{3}P$ _i phase shifts fail to provide a better description of the *NN* $A_y(\theta)$. In addition, our approach of multiplying the matrix elements of a given *NN* potential is only a convenient, but not necessarily the only and the most adequate method for modifying the *NN* interaction at a specific energy. It should also be noted that our *NN* calculations using the AV18 matrix elements do not include the Mott-Schwinger interaction [30], which affects the *n*-*p* $A_y(\theta)$ angular distribution considerably. In addition, a change of the $3P_2 - 3F_2$ matrix element modifies not only the $3P_2$ phase shift, but also the ${}^{3}F_{2}$ and ϵ_{2} phase parameters. Regardless of the actual uncertainty, we think that our approach may turn out to be useful in more sophisticated, future investigations. Some of the flavor will be given in the next section.

VI. CONCLUSION

We clearly showed that the present *NN* potential models with and without present 3NFs cannot describe the \tilde{N} -*d* analyzing power $A_y(\theta)$ and \hat{d} -*p* vector analyzing power $iT_{11}(\theta)$ data. Present-day 3NFs have only small effects on the ${}^{4}P_J$ *N*-*d* interactions. Furthermore, these changes are in disagreement with findings based on *p*-*d* phase-shift analyses. Assuming that also more sophisticated 3NFs than presently available have only small effects on the ${}^{4}P$ _{*I}N-d* interac-</sub> tions, our studies support the conjecture that the low-energy ³P_i phase shifts obtained from the analysis of NN data are questionable [31]. As a consequence, since *NN* potentialmodel parameters are fitted to *NN* phase shifts, the present *NN* potential models may have built in the incorrect ${}^{3}P_{j}$ interactions. Due to the higher sensitivity of 3*N* data, a consistent analysis of *NN* and 3*N* data is required to obtain the "correct" *NN* ${}^{3}P_{j}$ phase shifts. However, such an approach is difficult since the treatment of the 3*N* part of our proposed two-way procedure requires an explicit use of particular *NN* potential-model parametrizations. The procedure adopted in the present work, i.e., multiplication of the ${}^{3}P_{j}$ force components of an existing *NN* potential model with energy independent λ factors, is not adequate and gives only qualitative information about the ${}^{3}P_{i}$ *NN* interactions.

In order to support our suspicion we consider again $E_N = 3$ MeV. Using a trial and error approach we found (without

FIG. 3. Left-hand side: Comparison of p - p (top panel) and n - p (bottom panel) $A_y(\theta)$ calculations at E_y =3.0 MeV using the original NI93 PSA (solid curves) and modified ${}^{3}P_j$ phase shifts $(0.976\times{}^{3}P_0, 0.912\times{}^{3}P_1,$ and $1.16\times{}^{3}P_2)$ inserted into the NI93 PSA (dashed curves). The error bars are explained in the text. Right-hand side: Same for original AV18 *NN* potential (solid curves) and modified ${}^{3}P_{j}$ matrix elements $(0.98\times^{3}P_0, 0.90\times^{3}P_1,$ and $1.13\times^{3}P_2)$ inserted into the AV18 potential (dashed curves).

exploring the entire parameter space) that the phase-shift combination $0.976 \times {}^{3}P_{0}$, $0.912 \times {}^{3}P_{1}$, and $1.16 \times {}^{3}P_{2}$ describes all NN phase-shift data [28] almost as well as does the original phase-shift combination ${}^{3}P_0$, ${}^{3}P_1$, and ${}^{3}P_2$ obtained from the NI93 phase-shift analysis (PSA). Here, "almost as well'' refers to the fact that differences between the modified and original descriptions of up to 5% were tolerated. This solution uses the same small CIB and CSB as contained in the NI93 PSA and in the AV18 *NN* potential model. In Figs. $3(a)$ and $3(b)$ the solid curves represent the p -*p* and n -*p* $A_y(\theta)$ angular distributions calculated from the NI93 PSA analysis. In comparison, the dashed curves were obtained using our modified ${}^{3}P_{j}$ phase shifts. The error bars indicate the typical uncertainties of individual experimental data points available at the energies closest to $E_N=3$ MeV, i.e., $E_p = 5.05$ MeV [32] and $E_n = 10$ MeV [33]. Not surprisingly, at E_N =10 and 25 MeV our modification factors are not as accurate anymore. In fact, at these energies we observed differences of up to 7 and 13 % for *n*-*p* and *p*-*p* scattering observables, respectively, i.e., our modified *NN* phase shifts do not describe the NN phase-shift data $[28]$ accurately. In our trial and error approach we did not find any single combination of phase-shift modification factors with $\lambda \neq 1$ that provided an accurate description of the *NN* phase-shift data in the entire energy range up to $E_N=25$ MeV and simultaneously described $p-d$ scattering data at E_p $=$ 3 MeV. Nevertheless, if one multiplies the AV18 interactions with the λ factors associated with the phase-shift modifications discussed above, i.e., 0.98 , 0.90 , and 1.13 (the phase-shift modification factors are not identical to the factors by which the interaction matrix elements have to be

multiplied), respectively, one finds that the *n*-*d* $A_y(\theta)$ -puzzle at $E_n=3$ MeV is reduced considerably [see Fig. 1(a), dashdotted curve in comparison to dashed curve. The right-hand side of Fig. 3 shows the associated description of the *NN* $A_y(\theta)$ at $E_N=3$ MeV using the modified AV18 interaction. The main reason for the disagreement between Figs. $3(a)$ and $3(c)$ and Figs. $3(b)$ and $3(d)$, respectively, lies in the superior treatment of electromagnetic effects in the NI93 PSA. As pointed out earlier, the Mott-Schwinger interaction has a substantial influence on the shape and magnitude of the *n*-*p* $A_y(\theta)$ angular distribution.

Clearly, our result obtained for the *NN* $A_y(\theta)$ supports the finding of Ref. $[31]$: in the energy range up to about 15 MeV, basically only $A_y(\theta)$ is sensitive to modifications of the ${}^{3}P_{i}$ phase shifts. Therefore, the three ${}^{3}P_{i}$ phase shifts cannot be determined in an unique way: a broad band of solutions exists that equally well describe the low-energy *NN* $A_y(\theta)$ phase-shift data. Table III provides some details taken from Ref. [31]. Here, the uncertainties of the individual ${}^{3}P_{i}$ *NN* phase shifts are given at low energies. The quoted uncertainties are based on the Nijmegen *NN* phaseshift analysis [28]. The associated phase-shift predictions for the observables of interest were treated as fake data and it was determined by how much the ${}^{3}P_{j}$ phase shifts could be modified without deviating by more than 1% for *n*-*p* scattering and by more than 2% for *p*-*p* scattering from the Nijmegen phase-shift predictions. Clearly, the ${}^{3}P_{j}$ phase shifts can not be determined accurately, even if ''perfect'' *NN* data would exist. This observation raises the question about the accuracy of the extrapolation procedure used in global *NN* phase-shift analyses to obtain low-energy phase

TABLE III. Uncertainty of *NN* ³ P_j phase shifts at $E_N = 3$ and 10 MeV. The quoted uncertainties correspond to $\pm 1\%$ and $\pm 2\%$ changes in phase-shift data for $A_y(\theta)$ and $\sigma(\theta)$ for *n-p* scattering (left side) and p - p scattering (right side).

E_n (MeV)	$3P_0$	$3P_1$	$^{3}P_{2}$	E_n (MeV)	$^{3}P_{0}$	$3P_1$	$3P_2$
3	$> +100\%$ $> -200\%$	$> +240%$ $>-240%$	$>+300%$ $>-250%$		$\pm 20\%$	± 25%	$\pm 135%$
10	$+70%$ $-160%$	$+60%$ $-195%$	$+85%$ $-145%$	10	$\pm 24\%$	\pm 18%	± 33%

shifts from high-energy data where several observables are sensitive to the ${}^{3}P_{i}$ *NN* interactions. Therefore, we propose the following strategy: On the phase-shift level one has to determine smoothly varying, energy dependent multiplication factors for the ${}^{3}P_0$, ${}^{3}P_1$, and ${}^{3}P_2$ potential-model phase shifts in the energy range from 0 to about 50 MeV. These new phase shifts must connect smoothly to the existing (assumed "correct") high-energy phase shifts. On the potential level, ${}^{3}P_0$, ${}^{3}P_1$, and ${}^{3}P_2$ *NN* interactions have to be constructed that reproduce the new ${}^{3}P_{j}$ phase shifts. It may be necessary to readjust slightly other *NN* interaction components in order to describe the ''recommended'' *NN* database to which the potential used was fitted. Finally, this new potential model must be employed in 3*N* calculations and the results must be compared to *n*-*d* and *p*-*d* data. Depending on the results, several iterations of the entire procedure may be required. Only then will it be possible to answer the question whether nuclear physics has not found yet the "correct" ${}^{3}P_{i}$ *NN* interactions at low energies, or whether the mesonexchange picture of the *NN* interaction breaks down already at fairly large internucleon distances, or finally, whether sizable effects of 3NFs exist in nature at low energies. Because we are not aware of compelling arguments for the latter two scenarios, we have concentrated in the present paper on the first issue.

ACKNOWLEDGMENTS

We acknowledge Dr. M. Rentmeester from the Nijmegen group for performing several calculations with our modified $3P_i$ phase shifts. This work was supported in part by the U.S. Department of Energy, Office of High Energy and Nuclear Physics, under Grant No. DEFG05-91-ER40619 and by the Maria Sklodowska-Curie II Fund under Grant No. MEN/NSF-94-161. Some of the numerical calculations were performed on the Cray T916 of the North Carolina Supercomputing Center at the Research Triangle Park, North Carolina, and on the Cray Y-MP of the Hochstleistungsrechenzentrum in Jülich, Germany.

- [1] H. Witała, D. Hüber, and W. Glöckle, Phys. Rev. C 49, R14 (1994) , and references therein.
- [2] S. Ishikawa, Y. Wu, and T. Sasakawa, in Contributed Papers, 14th International IUPAP Conference on Few Body Problems in Physics, Williamsburg, Virginia, edited by F. Gross, 1994 (unpublished), p. 789.
- [3] A. Kievsky, M. Viviani, and S. Rosati, Phys. Rev. C 52, R15 $(1995).$
- [4] K. Chmielewski, J. Haidenbauer, U. Meyer, S. Nemoto, P. U. Sauer, and N. W. Schellingerhout, Few-Body Syst., Suppl. **8**, 394 (1995).
- [5] H. Witała and W. Glöckle, Nucl. Phys. **A528**, 48 (1991).
- [6] W. Tornow *et al.*, Phys. Lett. B 257, 273 (1991).
- [7] J. E. McAninch, W. Haeberli, H. Witała, W. Glöckle, and J. Golak, Phys. Lett. B 307, 13 (1993).
- $[8]$ E. Clementel and C. Villi, Nuovo Cimento **5**, 1166 (1957) , and references therein.
- [9] W. Tornow, C. R. Howell, R. L. Walter, and I. Slaus, Phys. Rev. C 45, 459 (1992).
- @10# A. Kievsky, M. Viviani, and S. Rosati, Nucl. Phys. **A577**, 511 $(1994).$
- [11] A. Kievsky, S. Rosati, W. Tornow, and M. Viviani, Nucl. Phys. A607, 402 (1996).
- $[12]$ T. Takemiya, Prog. Theor. Phys. **86**, 975 (1991) .
- [13] A. Soldi, B. Vlahovic, and I. Slaus, J. Phys. G 22, LG5 (1996).
- [14] W. Tornow, A. Kievsky, S. Rosati, and M. Viviani, Phys. Rev. C 55, 525 (1997).
- [15] D. Hüber, J. Golak, H. Witała, W. Glöckle, and H. Kamada, Few-Body Syst. **19**, 175 (1995).
- [16] L. D. Knutson, L. O. Lamm, and J. E. McAninch, Phys. Rev. Lett. 71, 3762 (1993).
- [17] S. Shimizu et al., Phys. Rev. C 52, 1193 (1995).
- [18] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
- $[19]$ R. B. Wiringa (private communication).
- [20] S. A. Coon, M. D. Scadron, P. C. McName, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 243 (1979); S. A. Coon and W. Glöckle, Phys. Rev. C 23, 1790 $(1981).$
- [21] M. R. Robilotta and H. T. Coelho, Nucl. Phys. **A460**, 645 $(1986).$
- [22] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29.** 1207 (1984).
- [23] R. B. Wiringa, invited talk in XVth International Conference on Few-Body Problems in Physics, Groningen, The Netherlands, 1997 (unpublished).
- [24] W. Glöckle, H. Witała, D. Hüber, H. Kamada, and J. Golak, Phys. Rep. 274, 107 (1996).
- [25] M. H. Wood et al., TUNL Progress Report XXXVI, 1997.
- [26] R. Machleidt, Adv. Phys. **19**, 189 (1989).
- [27] J. E. McAninch, L. O. Lamm, and W. Haeberli, Phys. Rev. C **50**, 589 (1994).
- [28] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C 48, 792 (1993).
- [29] R. A. Arndt, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C 50, 2731 (1994).
- [30] J. Schwinger, Phys. Rev. **73**, 407 (1948).
- [31] T. Tornow and W. Tornow, TUNL Progress Report XXXVI, 1997.
- [32] M. D. Barker, P. C. Colby, W. Haeberli, and P. Signell, Phys. Rev. Lett. **49**, 1056 (1982).
- [33] D. Holslin, J. McAninch, P. A. Quin, and W. Haeberli, Phys. Rev. Lett. **61**, 1561 (1988).