Spin structure functions for three-nucleon systems: Neutrons and protons

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The relativistic quark model of nucleon and the quark-exchange formalism is used to calculate the spin structure functions (SSF) of ^{3}He, ^{3}H, neutrons and protons. We consider the quarks to be exchanged at most between two nucleons. The up and down quarks treated separately and a well behaved polarized distribution is found by considering energy-momentum conservation properly. The SSF of ^{3}He and ^{3}H and convolution approximation are used to find the SSF of protons and neutrons and the validity of the Bjorken sum rule was tested. Finally it is shown that the result of our calculation agrees qualitatively well with the available experimental data, i.e., E142, E143, SMC, and recent E154 experiments. [S0556-2813(98)03002-7]

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I. INTRODUCTION

In recent years, there have been a large number of experiments on both the polarized deep-inelastic lepton scattering by nucleons and nuclei [1–4]. In the light of these experiments the understanding of ''nucleon'' structure has become a hot topic in particle and nuclear physics.

These reports have revealed the following striking results: (1) There is a significant difference between the structure of free nucleons and bound nucleons (EMC effect) [4]. (2) Quarks carry collectively only a fraction of the nucleon spin, and the fraction which is carried by the s quark is negative and quite large [5]. (3) The Bjorken sum rule may have been violated [5].

In our previous works [6–8] and [9] we have studied points (1) and (2), respectively. But in this article we would like to examine point number three in more detail theoretically. In this context besides the spin structure function of the proton, one should calculate the spin structure function (SSF) of the neutron as well. Recently, this was obtained from deep-inelastic scattering of polarized lepton off polarized deuteron (SMC [1]), and ^{3}He (E142 and E154 [3]), targets. These results together with previous data of EMC group [4] on the proton SSF are being currently used to test the Bjorken sum rule [5].

Of particular relevance are the experiments of the ^{3}He target, which we intend to consider in this work, since the ^{3}He nucleus can be viewed as an effective neutron target. The proton pair in this nucleus are mainly in $^1S_0$ state, so the proton contribution will be averaged out [3] and the SSF of ^{3}He is mostly due to neutron rather than protons. In the same way one can argue about ^{3}H nucleus and consider it as a proton.

Up to now, the theoretical description of three nucleon (^{3}He and ^{3}H) SSF have been mainly given in terms of plane-waves impulse approximation and convolution approaches by introducing the spin-dependent spectral function [10–12].

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Several years ago a formalism was developed by Hoodbhoy and Jaffe (HJ) [13] to investigate the multiquark exchange in the nuclear system. This method, which was based on the nonrelativistic quark model, was later applied to the light nuclei [14] and nuclear matter [6,15] to calculate the quark distribution function in nucleons and nuclei (EMC effect). The result was encouraging.

In this article we intend to use the same formalism to calculate the SSF of ^{3}He and ^{3}H nuclei, as well as the SSF of neutron and proton by using the HJ quark-exchange formalism and the convolution model.

So the paper is organized as following. We begin Sec. II by introducing various definitions such as the polarized deep-inelastic cross section, the spin structure function, the sum rules, etc. In Sec. III we develop the quark-exchange formalism to calculate the spin dependent quark momentum distribution in three nucleon systems. The momentum distribution will be written as a sum of direct and exchange parts with new indices for different flavors and spin polarizations. Section IV is concerned with the relation of the distribution function to the probability of removing quarks from the target and the explicit calculation of the polarized spin structure function for various quarks flavor. Finally the numerical results and the conclusion are presented in Sec. V.

II. POLARIZED NUCLEON STRUCTURE FUNCTION

The spin structure function of nucleon g_1 and g_2, which are determined experimentally, are related to the antisymmetric part of the hadronic tensor,

$$W^A_{\mu\nu}(p,s,q)=\frac{i}{p\cdot q}\varepsilon_{\mu\nu\alpha\beta}q^\alpha\left[Ms^\beta G_1(\nu,Q^2)+\frac{1}{M}\left(s^\beta-\frac{s\cdot q}{p\cdot q}p^\beta\right)G_2(\nu,Q^2)\right], \quad (1)$$

through the spin-dependent inelastic form factor, i.e.,

$$g_1(x,Q^2)=M^2\nu G_1(x,Q^2), \quad g_2(x,Q^2)=M\nu^2G_2(x,Q^2). \quad (2)$$

In the scaling limit ($Q^2 \to \infty, \nu \to \infty$) they will reduce simply to $g_1(x)$ and $g_2(x)$ where x is the Bjorken variable which is defined as

$$x = \frac{Q^2}{2M\nu},$$

where M is the mass of nucleon.

In 1966 a sum rule was derived by Bjorken [16] which relates the difference of the first moments of protons and neutrons to the weak coupling constants for neutron decay, i.e., g_A and g_V. By including the first order perturbative quantum chromodynamics (QCD) corrections [17] this sum rule is written as

$$\int_0^1 (g_1^p(x) - g_1^n(x))dx = \frac{1}{6}\left|\frac{g_A}{g_V}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\right|, \tag{3}$$

where $\alpha_s(Q^2)$ is the QCD coupling constant. There are also separate sum rules for the proton and neutron which were derived by Ellis and Jaffe [18]. By using this assumption that we have the SU(3) symmetry and the unpolarized strange sea, they are given by the following equations:

$$\int_0^1 g_1^p(x)dx \simeq \frac{(9F-D)}{18\pi}(1-\alpha_s(Q^2)),$$
$$\int_0^1 g_1^n(x)dx \simeq \frac{(9F-4D)}{18\pi}(1-\alpha_s(Q^2)) \tag{4}$$

where F and D are the invariant matrix elements of the axial vector current [18]. The above two integrals have other interpretations in the quark-parton model as well, i.e.,

$$\Gamma_1^p = \int_0^1 g_1^p(x)dx \simeq \frac{1}{2\pi}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)(1-\alpha_s(Q^2)),$$
$$\Gamma_1^n = \int_0^1 g_1^n(x)dx \simeq \frac{1}{2\pi}\left(\frac{1}{9}\Delta u + \frac{4}{9}\Delta d + \frac{1}{9}\Delta s\right)(1-\alpha_s(Q^2)), \tag{5}$$

where

$$\Delta q_j = \int_0^1 [q_j^{\uparrow}(x) - q_j^{\downarrow}(x)]dx \tag{6}$$

which gives the ''spin-measures'' up, down, and strange quarks in the nucleons. Usually the neutron beta decay and the hypron decay relations, $\Delta u - \Delta d = F + D$ and $\Delta d - \Delta s = F - D$, are taken to extract Δu, Δd, and Δs [and are used in Eq. (5)].

III. POLARIZED QUARK-EXCHANGE FORMALISM

As in the previous works [6,8,13–15] we start by making our nucleon from three quarks:

$$|\alpha\rangle = \mathcal{N}^{\alpha^\dagger}|0\rangle = \frac{1}{\sqrt{3!}}\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3}|0\rangle, \tag{7}$$

where α stands for $\{\vec{P}, M_S, M_T\}$ nucleon states and μ denotes the quark states $\{\vec{k}, m_s, m_t, c\}$. There is a summation on the repeated indices, i.e., summation over all values of the coordinates including integration over momenta. $q^\dagger$ (q) are the creation (annihilation) operators for quarks and $\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}$ is the totally antisymmetric nucleon wave function:

$$\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} = \frac{1}{\sqrt{3!}}\epsilon_{c_1c_2c_3}\frac{1}{\sqrt{2}} \times \sum_{s,t=0,1} C^{(1/2)s(1/2)}_{m_{s_1}m_s M_S} C^{(1/2)(1/2)s}_{m_{s_2}m_{s_3}m_s} C^{(1/2)t(1/2)}_{m_{t_1}m_t M_T} C^{(1/2)(1/2)t}_{m_{t_1}m_{t_2}m_t} \times \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{P})\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P}), \tag{8}$$

where $\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P})$ is the nucleon wave function and it is approximated by a Gaussian distribution, i.e.,

$$\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P}) = \left(\frac{3b^4}{\pi^2}\right)^{(3/4)} \times \exp\left[-b^2\frac{(k_1^2 + k_2^2 + k_3^2)}{2} + \frac{b^2p^2}{6}\right]. \tag{9}$$

$C^{j_1j_2j}_{m_1m_2m}$ are the familiar Clebsch-Gordon coefficients and $\epsilon_{c_1c_2c_3}$ is the color factor. The normalization of ϕ is chosen such that

$$\langle \vec{P} M_S M_T | \vec{P}' M'_S M'_T \rangle = \delta(\vec{P} - \vec{P}')\,\delta_{M_S M'_S}\delta_{M_T M'_T} \tag{10}$$

and the overall antisymmetrization is provided by $(1/\sqrt{3!})\epsilon_{c_1c_2c_3}$.

For quark creation and annihilation operators we have the usual fermions anticommutation relations, i.e.,

$$\{q_\mu, q^\dagger_\nu\} = \delta_{\mu\nu}, \tag{11}$$

while the nucleon composite operators $\mathcal{N}^\alpha$ and $\mathcal{N}^{\alpha^\dagger}$ obey the following anticommutation relations:

$$\{\mathcal{N}^\alpha, \mathcal{N}^{\beta^\dagger}\} = \delta^{\alpha\beta} - \mathcal{N}^{\alpha\beta}, \tag{12}$$

where

$$\mathcal{N}^{\alpha\beta} = 3\mathcal{N}^{\alpha}_{\nu_1\nu_2\nu_3}\mathcal{N}^{\beta}_{\mu_1\mu_2\mu_3}\delta_{\mu_3\nu_3} \times \left(\delta_{\mu_2\nu_2} q^\dagger_{\mu_1} q_{\nu_1} - \frac{1}{2} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q_{\nu_2} q_{\nu_1}\right). \tag{13}$$

($\{q_\mu, q_\nu\}$ and $\{\mathcal{N}^\alpha, \mathcal{N}^\beta\}$ as well as their complex conjugates are zero, as usual.)

The full calculation of quark-exchange effects become very tedious if all of the three nucleons are allowed to overlap simultaneously. But since in the ^{3}He and ^{3}H nuclei (small nuclear size) the nuclear density is on average low, then it should be a good approximation to ignore the quark-exchange among three nucleons simultaneously.

Before we consider the above approximation let us define the nucleus model state as follows:

$$|\mathcal{A}_i=3\rangle=(3!)^{-1/2}\chi^{\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1\dagger}\mathcal{N}^{\alpha_2\dagger}\mathcal{N}^{\alpha_3\dagger}|0\rangle, \qquad (14)$$

where the nuclear wave function $\chi^{\alpha_1\alpha_2\alpha_3}$, which is completely antisymmetric in the nuclear coordinates, is just the conventional three-nucleon wave function and will be discussed in more detail later on. Now we can define the momentum distribution of a quark with a given flavor and spin polarizations in the three-nucleon systems as

$$\rho_{\overline{\mu}}(\vec{k};\mathcal{A}_i)=\frac{\langle\mathcal{A}_i=3|q^{\dagger}_{\overline{\mu}}q_{\overline{\mu}}|\mathcal{A}_i=3\rangle}{\langle\mathcal{A}_i=3|\mathcal{A}_i=3\rangle}, \qquad (15)$$

where by the bar sign over μ we mean omission of the summation on m_s, m_t and integration over $\vec{k}$ in the μ indices. In order to calculate $\rho_{\overline{\mu}}(\vec{k};\mathcal{A}_i)$ it is enough to evaluate

$$\langle\mathcal{A}_i=3|q^{\dagger}_{\overline{\mu}}q_{\overline{\mu}}|\mathcal{A}_i=3\rangle. \qquad (16)$$

Then, the calculation of $\langle\mathcal{A}_i=3|\mathcal{A}_i=3\rangle$ would become straightforward, i.e., just a summation over $\overline{\mu}$,

$$\begin{aligned}\langle\mathcal{A}_i=3|\mathcal{A}_i=3\rangle&=\frac{1}{9}\langle\mathcal{A}_i=3|q^{\dagger}_{\mu}q_{\mu}|\mathcal{A}_i=3\rangle\\&=\chi^{*\alpha_1\alpha_2\alpha_3}(\delta^{\alpha_1\beta_1}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}\\&\quad-\mathcal{E}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\mu\mu})\chi^{\beta_1\beta_2\beta_3},\end{aligned}$$

where

$$\mathcal{E}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\mu\mu}=\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\mu_2\mu_3\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}. \qquad (17)$$

After doing some algebra, which would be long but not difficult, and ignoring the three-body exchanged diagram (it will be discussed later on), one would find the following equation for the above expectation value [Eq. (16)]:

$$\begin{aligned}&\langle\mathcal{A}_i=3|q^{\dagger}_{\overline{\mu}}q_{\overline{\mu}}|\mathcal{A}_i=3\rangle\\&\quad=9\chi^{*\alpha_1\alpha_2\alpha_3}(\mathcal{U}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\overline{\mu\mu}}-\mathcal{V}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\overline{\mu\mu}})\chi^{\beta_1\beta_2\beta_3},\end{aligned}$$

where

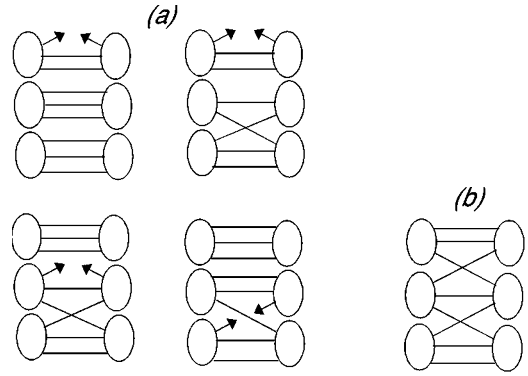

FIG. 1. (a) The graphical representation of quark one-body operator up to two-nucleon correlation. (b) The omitted three-nucleon correlation term.

$$\mathcal{U}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\overline{\mu\mu}}=\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3} \qquad (18)$$

and

$$\begin{aligned}&\mathcal{V}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\overline{\mu\mu}}\\&\quad=3\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\rho_1\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\\&\qquad+4\mathcal{N}^{\alpha_2}_{\overline{\mu}\mu_1\mu_2}\mathcal{N}^{\beta_2}_{\overline{\mu}\mu_2\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}\\&\qquad+2\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\overline{\mu}\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\overline{\mu}\rho_2\rho_3}\mathcal{N}^{\beta_2}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}. \qquad (19)\end{aligned}$$

The diagrammatic representation of the above equations is given in Fig. 1(a). The omitted three-body exchanged contribution is displayed in Fig. 1(b). As we mentioned before, the summation over all indices will simply raise to the multiplication factor $9\langle\mathcal{A}_i=3|\mathcal{A}_i=3\rangle$. By using the following definition in Eq. (8), for Clebsch-Gordon coefficients, i.e.,

$$\begin{aligned}D(\sigma,\mu,\nu;\alpha_i)=\frac{1}{\sqrt{3!}}\epsilon_{c_1c_2c_3}\frac{1}{\sqrt{2}}\sum_{s,t=0,1}C^{(1/2)s(1/2)}_{m_{s_\sigma}m_sM_{S_{\alpha_i}}}\\\times C^{(1/2)(1/2)s}_{m_{s_\mu}m_{s_\nu}m_s}C^{(1/2)t(1/2)}_{m_{t_\sigma}m_tM_{T_{\alpha_i}}}C^{(1/2)(1/2)t}_{m_{t_\mu}m_{t_\nu}m_t} \qquad (20)\end{aligned}$$

we can explicitly write the five terms in Eqs. (17), (18), and (19) as

$$\begin{aligned}\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}&=\delta(\vec{P}_{\alpha_1}-\vec{P}_{\beta_1})|\delta(\vec{P}_{\alpha_2}-\vec{P}_{\beta_2})\delta(\vec{P}_{\alpha_3}-\vec{P}_{\beta_3})\left(\frac{3b^2}{2\pi^2}\right)^{3/2}\\&\quad\times\exp\left[-\frac{3}{2}b^2\left(\vec{k}+\frac{\vec{q}}{3}\right)^2\right]D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_2,\sigma_3;\beta_1)\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}, \qquad (21)\end{aligned}$$

$$\begin{aligned}\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\rho_1\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}&=\Delta\left(\frac{9b^4}{8\pi^2}\right)^{3/2}\exp\left[-\frac{3}{2}b^2\left(\vec{k}+\frac{\vec{q}}{3}\right)^2\right]\exp\left[-b^2\frac{\vec{u}^2}{3}\right]\exp[-b^2\vec{v}^2]\\&\quad\times D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_3;\beta_1)D(\mu_1,\mu_2,\mu_3;\alpha_2)\\&\quad\times D(\rho_1,\mu_2,\mu_3;\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3), \qquad (22)\end{aligned}$$

$$\mathcal{N}^{\alpha_2}_{\overline{\mu}\mu_1\mu_2}\mathcal{N}^{\beta_2}_{\overline{\mu}\mu_2\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}=\Delta\left(\frac{9b^4}{7\pi^2}\right)^{3/2}\exp\left[-\frac{12}{7}b^2\left(\vec{k}-\frac{\vec{q}}{6}-\frac{\vec{u}}{2}\right)^2\right]\exp\left[-b^2\frac{\vec{u}^2}{3}\right]\exp[-b^2\vec{v}^2]D(\mu_1,\mu_2,\overline{\mu};\alpha_2)$$
$$\times D(\rho_1,\mu_2,\overline{\mu};\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3)\delta^{\alpha_1\beta_1}, \tag{23}$$

$$\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\overline{\mu}\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\overline{\mu}\rho_2\rho_3}\mathcal{N}^{\beta_2}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}=\Delta\left(\frac{9b^4}{4\pi^2}\right)^{3/2}\exp\left[-3b^2\left(\vec{k}-\frac{\vec{q}}{6}+\frac{\vec{v}}{2}\right)^2\right]\exp\left[-b^2\frac{\vec{u}^2}{3}\right]\exp[-b^2\vec{v}^2]D(\mu_1,\mu_2,\mu_3;\alpha_2)$$
$$\times D(\mu_2,\mu_3,\overline{\mu};\beta_2)D(\overline{\mu},\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3)\delta^{\alpha_1\beta_1}, \tag{24}$$

$$\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\mu_2\mu_3\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}=\Delta\left(\frac{3b^2}{4\pi^2}\right)^{3/2}\exp\left[-b^2\frac{\vec{u}^2}{3}\right]\exp[-b^2\vec{v}^2]D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\rho_1,\mu_2,\mu_3;\beta_2)$$
$$\times D(\rho_1,\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3)\delta^{\alpha_1\beta_1}, \tag{25}$$

with

$$\vec{u}=\frac{(\vec{p}_\alpha+\vec{p}_\beta)}{2},\quad \vec{v}=(\vec{p}_\beta-\vec{p}_\alpha)$$

and

$$\Delta=\delta(\vec{p}_{\alpha_1}-\vec{p}_{\beta_1})\,\delta(\vec{p}_{\alpha_2}+\vec{p}_{\alpha_3}-\vec{p}_{\beta_2}-\vec{p}_{\beta_3}).$$

For three nucleons, we use the Jacobi coordinates and the same definition as the one we did for C-G coefficients in Eq. (20), i.e,

$$D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)=\frac{1}{\sqrt{2}}\sum_{S,T=0,1}C^{(1/2)S(1/2)}_{M_{S_{\alpha_1}}M_SM_{S_i}}C^{(1/2)(1/2)S}_{M_{S_{\alpha_2}}M_{S_{\alpha_3}}\mathcal{M}_S}C^{(1/2)T(1/2)}_{M_{T_{\alpha_1}}M_TM_{T_i}}C^{(1/2)(1/2)T}_{M_{T_{\alpha_2}}M_{T_{\alpha_3}}\mathcal{M}_T}, \tag{26}$$

to write the nuclear wave function as follows:

$$\chi^{\alpha_1\alpha_2\alpha_3}=\chi(\vec{P},\vec{q})D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i). \tag{27}$$

Then by assuming the nucleus to be in the rest frame and defining the Fourier transform of $\chi(\vec{P},\vec{q})$, we can write the expectation values of Eqs. (21)–(25) between the nucleus wave function as

$$\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}\chi^{\beta_1\beta_2\beta_3}=\left(\frac{3b^2}{2\pi^2}\right)^{3/2}\exp\left[-\frac{3}{2}b^2\vec{k}^2\right]D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_2,\sigma_3;\beta_1)$$
$$\times D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}, \tag{28}$$

$$\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\overline{\mu}\sigma_2\sigma_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\rho_1\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\chi^{\beta_1\beta_2\beta_3}$$
$$=I\left(\frac{27b^2}{8\pi^2}\right)^{3/2}\exp\left[-\frac{3}{2}b^2\vec{k}^2\right]D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_3;\beta_1)D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\rho_1,\mu_2,\mu_3;\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)$$
$$\times D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i), \tag{29}$$

$$\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_2}_{\overline{\mu}\mu_1\mu_2}\mathcal{N}^{\beta_2}_{\overline{\mu}\mu_2\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}\chi^{\beta_1\beta_2\beta_3}$$
$$=I\left(\frac{27b^2}{7\pi^2}\right)^{3/2}\exp\left[-\frac{12}{7}b^2\vec{k}^2\right]D(\mu_1,\mu_2,\overline{\mu};\alpha_2)D(\rho_1,\mu_2,\overline{\mu};\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)$$
$$\times D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_1\beta_1}, \tag{30}$$

$$\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\underline{\beta_2}}_{\mu\mu_2\mu_3}\mathcal{N}^{\underline{\alpha_3}}_{\mu\rho_2\rho_3}\mathcal{N}^{\beta_2}_{\mu_1\rho_2\rho_3\delta^{\alpha}_1\beta_1}\chi^{\beta_1\beta_2\beta_3}$$
$$=I\left(\frac{27b^2}{4\pi^2}\right)^{3/2}\exp[-3b^2\vec{k}^2]D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\mu_2,\mu_3,\overline{\mu;}\beta_2)D(\overline{\mu,}\rho_2,\rho_3;\alpha_3)$$
$$\times D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_1\beta_1}, \quad (31)$$

$$\chi^{*\alpha_1\alpha_2\alpha_3}\delta^{\alpha_1\beta_1}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\mu_2\mu_3\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3\delta^{\alpha}_1\beta_1}\chi^{\beta_1\beta_2\beta_3}$$
$$=I\left(\frac{3}{2}\right)^3 D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\rho_1,\mu_2,\mu_3;\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)$$
$$\times D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_1\beta_1}, \quad (32)$$

where

$$I=8\pi^2\int_0^\infty x^2dx\int_0^\infty y^2dy\int_{-1}^1 d(\cos\,\theta)\exp\left[-\frac{3x^2}{4b^2}\right]|\chi(x,y,\cos\theta)|^2.$$

All of the above equations, i.e., (28)–(32), have been calculated with the same approximation as the one used by HJ [13] and other authors [6,7,15], i.e., a leading order expansion for $\chi(\vec{P},\vec{q})$. This means that we ignore the Fermi motion in the three-nucleon systems. But the validity of this approximation has been verified by HJ [13] and Modarres *et al.* [6]. They have found that for $b<1$ fm it is possible to ignore the variation in the nuclear wave function over the nucleon size distances which is justified for the low densities and the small nucleon radius. However, in general the Fermi motion is approximately important for $x>0.85$ [6].

By assuming $m_t=j$ and doing some angular momentum algebra [19] (in order to perform the summation over various ''m'' values and reduce the number of D coefficients to the $3j$, $6j$, and $9j$ symbols as is discussed in the Appendix and Ref. [14]), we can calculate the spin polarization momentum density for each flavor as

$$\Delta\rho_j(\vec{k};\mathcal{A}_i)=\rho_{j\uparrow}(\vec{k};\mathcal{A}_i)-\rho_{j\downarrow}(\vec{k};A_i), \quad (33)$$

where

$$\Delta\rho_j(\vec{k};\mathcal{A}_i)=\sum_{j,k} M_{jk}\exp(-a_k\vec{k}^2). \quad (34)$$

The explicit matrix representation of Eq. (34) for $\mathcal{A}_i={}^3$He and ^{3}H by using the Appendix is as follows:

$$\begin{pmatrix}\Delta\rho_u(\vec{k};{}^3\mathrm{H})\\ \Delta\rho_d(\vec{k};{}^3\mathrm{H})\\ \Delta\rho_u(\vec{k};{}^3\mathrm{He})\\ \Delta\rho_d(\vec{k};{}^3\mathrm{He})\end{pmatrix}=\frac{b^3}{1+0.552I}\begin{pmatrix}0.367 & -0.313I & 1.612I & -0.026I\\ -0.201 & 0.162I & 0.601I & 0.026I\\ -0.201 & 0.162I & 0.601I & 0.026I\\ 0.367 & -0.313I & 1.612I & -0.026I\end{pmatrix}\begin{pmatrix}\exp\left(-\frac{3}{2}b^2\vec{k}^2\right)\\ \exp\left(-\frac{3}{2}b^2\vec{k}^2\right)\\ \exp\left(-\frac{12}{7}b^2\vec{k}^2\right)\\ \exp(-3b^2\vec{k}^2)\end{pmatrix}. \quad (35)$$

IV. NUCLEUS STRUCTURE FUNCTION

The polarized momentum distribution for various flavors in each nucleus, Eq. (35), can be related to the corresponding parton distribution at the hadronic scale Q_0^2 according to the following equation [20]:

$$\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)=\frac{m}{xM}\int \Delta\rho_j(\vec{k};\mathcal{A}_i)\,\delta\!\left(x-\frac{k_+}{M}\right)d\vec{k}, \quad (36)$$

where m (M) is the quark (nucleon) mass and k_+ is the light-cone momentum of initial quarks $(k_+=k_0-k_z)$. But this is not entirely correct, since (i) $\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)$ do not vanish for $x>1$ and $x<0$, (ii) Eq. (36) is not covariant, and (iii) no final state interaction is included. In order to take into account the above requirements and the relativistic corrections we should rewrite $\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)$ as [20]

$$\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)=\frac{1}{(1-x)^2}\int \Delta\rho_j(\vec{k};\mathcal{A}_i)\,\delta\!\left(\frac{x}{(1-x)}-\frac{k_+}{M}\right)d\vec{k}. \quad (37)$$

Now by doing the angular integration we find

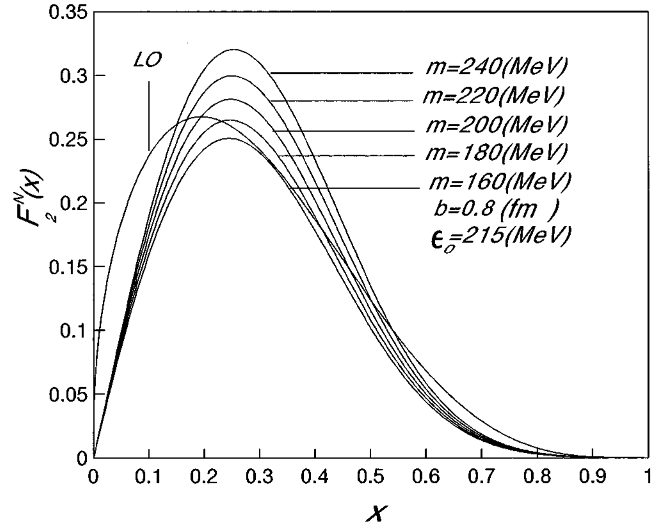

FIG. 2. Comparison of unpolarized nucleon structure function used in our calculation with corresponding NSF from Ref. [22] (LO).

$$\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)=\frac{2\pi M}{(1-x)^2}\int_{k_{\min}}^{\infty}\Delta\rho_j(\vec{k};\mathcal{A}_i)k\,dk \tag{38}$$

with

$$k_{\min}(x)=\frac{\left(\dfrac{xM}{1-x}+\epsilon_0\right)^2-m^2}{2\left(\dfrac{xM}{1-x}+\epsilon_0\right)} \tag{39}$$

and

$$k_0=(\vec{k}^2+m^2)^{1/2}-\epsilon_0, \tag{40}$$

where ϵ_0 is the quark binding energy. The calculation without the above correction has been discussed in Ref. [21]. Finally the spin polarized structure function for each nucleus can be written as follows:

$$g_1(x;\mathcal{A}_i)=\frac{1}{2}\sum_j e_j{}^2\Delta q_j^v(x,Q_0^2;\mathcal{A}_i). \tag{41}$$

V. RESULTS AND DISCUSSION

In order to do the numeric calculation for $g_1(x;\mathcal{A}_i)$, we first fix ϵ_0 and the quark mass by using the unpolarized nucleon structure function (NSF) according to Eqs. (37)–(40) [but by omitting Δ and $\mathcal{A}_i$ from these equations and considering a Gaussian approximation for $\rho_j(\vec{k})$ with parameter b]. Then we fit the above nucleon structure function to the recent NSF which have been given in Ref. [22] at $Q_0^2=4\ \mathrm{GeV}^2$ [we use the leading-order evolution formalism (LO)]. The comparison of our NSF with LO is given in Fig. 2. We find that NSF is (not) very sensitive to m (ϵ_0 and $0.7\ \mathrm{fm}\leqslant b\leqslant 1\,\mathrm{fm}$) as is seen from Fig. 2. However, we chose the values of $\epsilon_0=215\ \mathrm{MeV}$, $b=0.8\ \mathrm{fm}$, and $m=180\ \mathrm{MeV}$ (the nearest fit to LO).

We consider ^{3}He and ^{3}H wave functions (χ) to have only the s-channel partial wave and we take them from Refs. [13,23] (it has been calculated by solving Faddeev equation and the result is comparable with those of Ref. [26]) since the d-channel contribution is very small due to the centrifugal barrier (the mixed symmetry s'-channel and d-channels account for about 1–2 % and 5–9 %, respectively, as has been discussed in more detail in Refs. [13,14,23–25]) and it tends to reduce nucleon overlap in excess of the nucleon-nucleon short-range repulsion. However the contributions of the different components of ^{3}He and ^{3}H wave functions to their charge density distribution have been investigated by Friar *et al.* [23] and it indicates that it is a good approximation to ignore such components. So because our results are not very sensitive to the b parameter, we can absorb this effect by changing b.

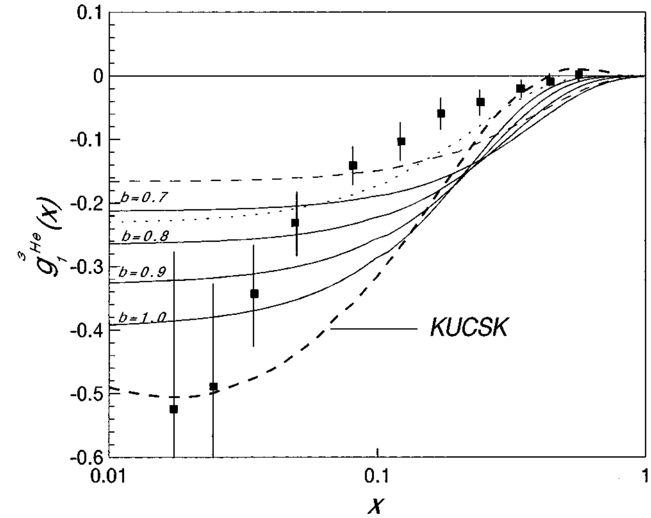

FIG. 3. $g_1^{^3\mathrm{He}}(x)$, our calculation (full curves) and E154 experiment [3] (full box). Dashed ($b=0.7$ fm) and dotted ($b=1.0$ fm) curves are without quark-exchange effect, respectively. Heavy dashed curve is Kaptari *et al.* (KUCSK) result.

Now we are in a position to calculate $g_1(x;{}^3\mathrm{He})$ and $g_1(x;{}^3\mathrm{H})$ using Eqs. (35) and (41). The results are given in Figs. 3 and 4 for various values of the b parameter. Since, to

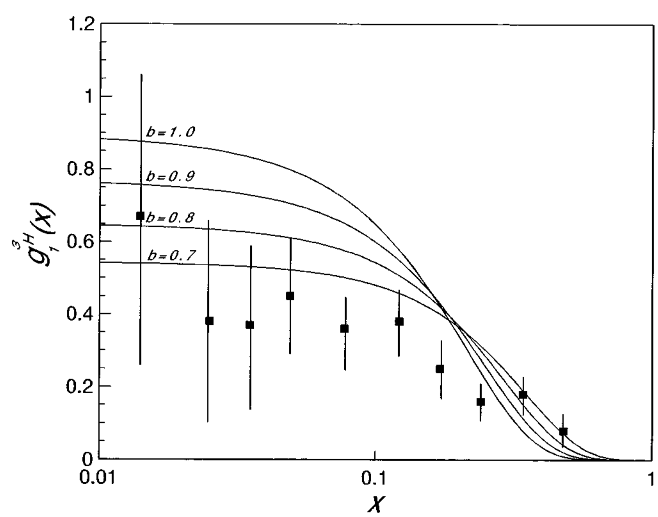

FIG. 4. $g_1^{^3\mathrm{H}}(x)$, our calculation (full curves), and SMC experiment [1] (full box).

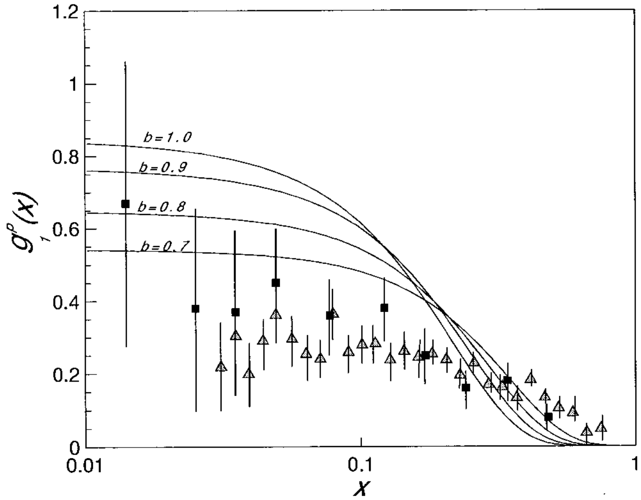

FIG. 5. $g_1^p(x)$, our calculation (full curves), SMC experiment [1] (full box), and E143 experiment [2] (full triangle).

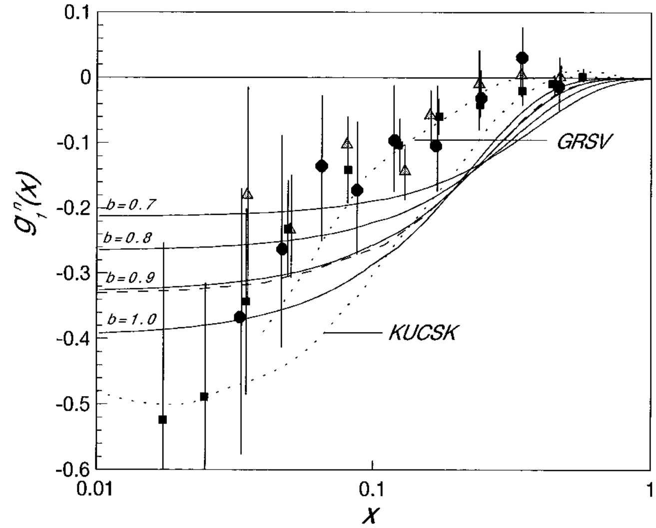

FIG. 6. $g_1^n(x)$, our calculation (full curves), HERMES [3] (full circle), E154 experiment [3] (full box), and E142 experiment [3] (full triangle). The results of Glück *et al.* [29] and Kaptari *et al.* [10] are shown by GRSV and KUCSK, respectively. Dashed curve represents ^{3}He result of Fig. 3.

a large extent ^{3}He (^{3}H) can be regarded as a neutron (proton) target, we also present the data points from E154 (SMC) experiments in Fig. 3 (4) as well. In Fig. 3 we also give SSF of ^{3}He without quark-exchange effect for $b=0.7$ and 1.0 fm. One can see that the quark-exchange effect brings the calculation closer the experimental data. The heavy dashed curve in Fig. 3 stands for the calculation of Kaptari *et al.* (KUCSK), [10]. We should mention here that the authors of Ref. [10] have used the free SSF of neutron and proton from various experiments (i.e., E142, E143, and SMC) to calculate the SSF of ^{3}He. Consequently their result depends on their spectral functions as well as the parametrization of SSF of nucleons at small x values.

However, in order to see how the SSF of proton and neutron would look like without nuclear structure effect, we calculate them by using the convolution approximation according to Ref. [27]. In this respect we write

$$\Delta q_j^v(x,Q_0^2;\mathcal{A}_i)=a\sum_N \int \Delta q_j^v\left(\frac{x}{y_{\mathcal{A}_i}},Q_0^2;N\right) f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})dy_{\mathcal{A}_i}, \tag{42}$$

where $f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})$ are the Fermi gas nucleon distributions in each nuclei and a is the nuclear asymmetry [27] (when $j=1/2$, $a=1$). Next we expand $\Delta q_j^v(x/y_{\mathcal{A}_i},Q_0^2;N)$ of Eq. (42) around $x/\langle y_{\mathcal{A}_i}\rangle$ with $\langle y_{\mathcal{A}_i}\rangle = 1+\overline{\epsilon}/M$, and by taking into account this fact that $f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})$ are narrow around $\langle y_{\mathcal{A}_i}\rangle$, we can write [28]

$$\Delta q_j^v\left(\frac{x}{\langle y_{\mathcal{A}_i}\rangle},Q_0^2;N\right)=\Delta q_j^v(x,Q_0^2;\mathcal{A}_i). \tag{43}$$

For $\overline{\epsilon}$, the average removal energy of the nucleon in the nucleus, we use value of -26 MeV corresponding to ^{3}He and ^{3}H nuclei [27].

The comparison of numeric calculation of SSF for protons and neutrons with the corresponding experimental data are given in Figs. 5 and 6. The results of Glück *et al.* [29] and Kaptari *et al.* [10] by using the NLO radiative parton model (based on fitting) and the appropriate spectral function, respectively, are also displayed. $xg_1^p(x)$ is displayed in Fig. 7, and the work of Gehrmann and Stirling [30] is also given for comparison. It is seen that our results are in good agreement with the experimental data. In order to see the effect of convolution approximation we also present the $g_1^{^3\mathrm{He}}(x)$ in Fig. 6. The up and down quark SSF functions in the proton are given in Figs. 8 and 9.

Table I shows the comparison of Δu^p, Δd^p, Γ_1^p, and Γ_1^n with the corresponding experiments. It is seen that we get an overall agreement with the various experiments. From this table it is possible to obtain a value for the Bjorken sum rule, Eq. (3). Doing so, one obtains $\Gamma_1^p-\Gamma_1^n=0.20297\pm0.0335$, which can be compared with the experimental prediction of the SMC group $\Gamma_1^p-\Gamma_1^n=0.20\pm0.05\pm0.04$.

In conclusion the spin-dependent inelastic electron scattering from polarized ^{3}He and ^{3}H were studied. This was

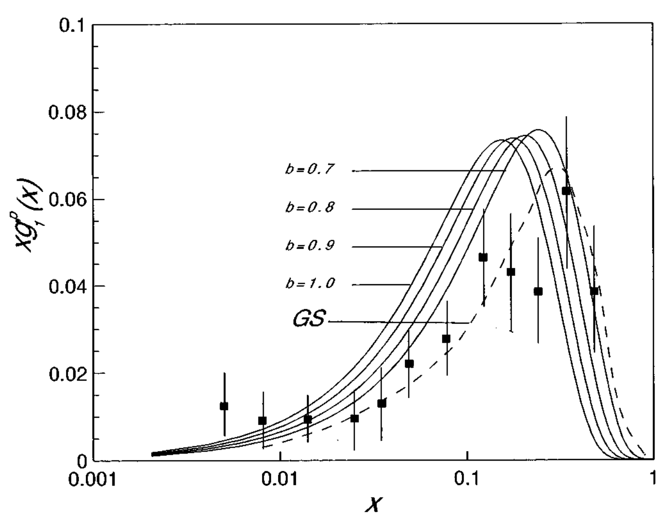

FIG. 7. $xg_1^p(x)$, our calculation (full curves), SMC experiment [1] (full box), and Gehrmann and Stirling [30] (dashed curve).

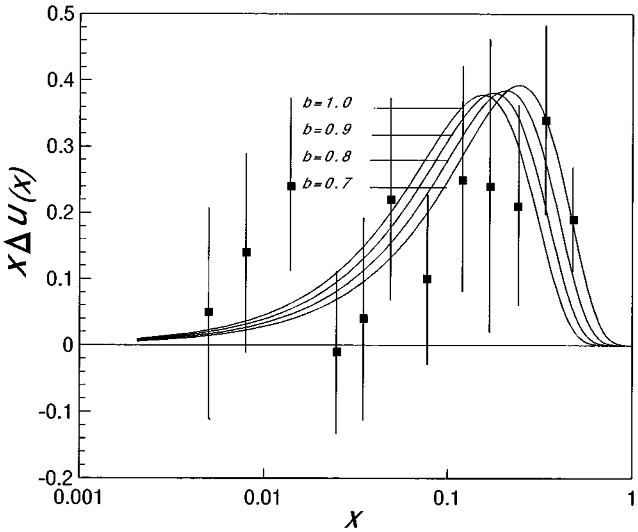

FIG. 8. As in Fig. 7 but for $x\Delta u(x)$.

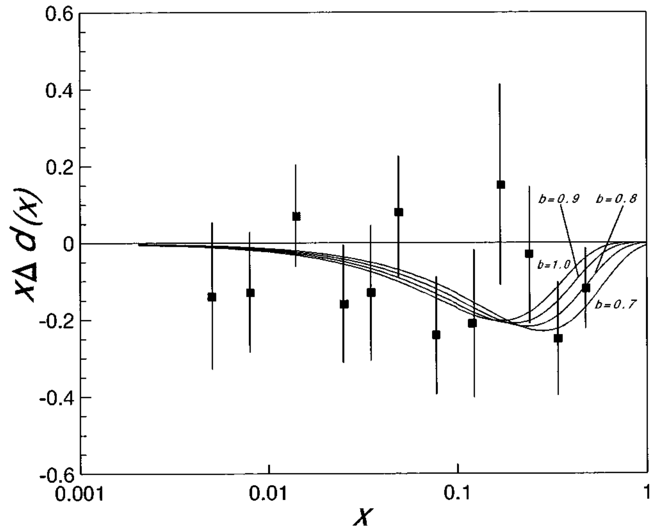

FIG. 9. As in Fig. 7 but for $x\Delta d(x)$.

done by defining a model for nuclei which takes into account the quark-exchange structure in a constituent picture. We can state that the quark-exchange effect are not negligible and they should be observable in both unpolarized and polarized deep inelastic scattering from nuclei. Our results show that ^{3}He and ^{3}H nuclei are indeed good neutron and proton spin targets (for 0.7 fm$\leqslant b \leqslant$1.0 fm). However, our result should be considered as quantitative because of the variation of parameter b introduced in our model. So we can argue that further investigation on the polarized deep-inelastic cross section can reveal more information about quark structure of neutrons and protons (both experimentally and theoretically). The model we developed here can be extended to other nuclei by taking into account the short range nucleon correlations properly. Better calculations can be done by considering the two-nucleon spectral function rather than the simple convolution model in order to calculate the SSF of protons and neutrons. The effect of possible excitations of a nucleon to a Δ can be built in above model calculations. However, a more sophisticated calculation is needed to check the magnitude of these effects. Finally we have found that the present data on $g_1^{p,n}(x,Q^2)$ are in agreement with the model developed by us. These suggest that the quarks are accountable for about half the nucleon spin.

APPENDIX

The results of the ‘‘m’’ sums for various terms in the text are as follows. For Eq. (21):

$$D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_2,\sigma_3;\beta_1)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\,\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}$$

$$=\delta_{m_{s_{\overline{\mu}}}m'_{s_{\overline{\mu}}}}\delta_{SS'}\delta_{s_1s_1'}\delta_{s_2s_2'}\delta_{s_3s_3'}\sum_{\mathcal{S}}[1/2]^2[\mathcal{S}]\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -m_{s_{\overline{\mu}}} & -M_{S_i} & m_{s_{\overline{\mu}}}+M_{S_i}\end{pmatrix}^2\begin{Bmatrix}\mathcal{S} & 1/2 & 1/2\\ 1/2 & s_1 & 1/2\\ 1/2 & 1/2 & S\end{Bmatrix}$$

$$\times\delta_{m_{t_{\overline{\mu}}}m'_{t_{\overline{\mu}}}}\delta_{TT'}\delta_{t_1t_1'}\delta_{t_2t_2'}\delta_{t_3t_3'}\sum_{\mathcal{T}}[1/2]^2[\mathcal{T}]\begin{pmatrix}1/2 & 1/2 & \mathcal{T}\\ -m_{t_{\overline{\mu}}} & -M_{T_i} & m_{t_{\overline{\mu}}}+M_{T_i}\end{pmatrix}^2\begin{Bmatrix}\mathcal{T} & 1/2 & 1/2\\ 1/2 & t_1 & 1/2\\ 1/2 & 1/2 & T\end{Bmatrix}.$$

For Eq. (22):

TABLE I. The comparison of various quantities explained in Eqs. (5) and (6) with experimental data.

	Q^2(GeV^2)	Δu^p	Δd^p	Γ_1^p	Γ_1^n
Our calculation					
	4	0.7911 ± 0.126	-0.4272 ± 0.075	0.1520 ± 0.024	-0.05097 ± 0.0095
SMC	10	$1.01\pm0.19\pm0.14$	$-0.57\pm0.22\pm0.11$	$0.136\pm0.011\pm0.11$	
E143	3			$0.127\pm0.004\pm0.010$	
E142	2				-0.022 ± 0.011
E154	5				$-0.037\pm0.004\pm0.010$
HERMES	2.5				$-0.037\pm0.013\pm0.011$

$$D(\overline{\mu},\sigma_2,\sigma_3;\alpha_1)D(\overline{\mu},\sigma_2,\sigma_3;\beta_1)D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\rho_1,\mu_2,\mu_3;\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)$$
$$\times D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)$$
$$=\delta_{m_{s_{\overline{\mu}}}m'_{s_{\overline{\mu}}}}\delta_{SS'}\delta_{s_1s_1'}\delta_{s_2s_2'}\delta_{s_3s_3'}\sum_{\mathcal{S}}[1/2]^4[\mathcal{S}]\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -m_{s_{\overline{\mu}}} & -M_{S_i} & m_{s_{\overline{\mu}}}+M_{S_i}\end{pmatrix}^2\begin{pmatrix}\mathcal{S} & 1/2 & 1/2\\ 1/2 & s_1 & 1/2\\ 1/2 & 1/2 & S\end{pmatrix}\begin{pmatrix}1/2 & 1/2 & S\\ s_3 & 1/2 & 1/2\\ 1/2 & s_2 & 1/2\end{pmatrix}$$
$$\times\delta_{m_{t_{\overline{\mu}}}m'_{t_{\overline{\mu}}}}\delta_{TT'}\delta_{t_1t_1'}\delta_{t_2t_2'}\delta_{t_3t_3'}\sum_{\mathcal{T}}[1/2]^4[\mathcal{T}]\begin{pmatrix}1/2 & 1/2 & \mathcal{T}\\ -m_{T_{\overline{\mu}}} & -M_{T_i} & m_{t_{\overline{\mu}}}+M_{T_i}\end{pmatrix}^2\begin{pmatrix}\mathcal{T} & 1/2 & 1/2\\ 1/2 & t_1 & 1/2\\ 1/2 & 1/2 & T\end{pmatrix}\begin{pmatrix}1/2 & 1/2 & T\\ t_3 & 1/2 & 1/2\\ 1/2 & t_2 & 1/2\end{pmatrix}.$$

For Eq. (23):

$$D(\mu_1,\mu_2,\overline{\mu};\alpha_2)D(\rho_1,\mu_2,\overline{\mu};\beta_2)D(\rho_1,\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_1\beta_1}$$
$$=\delta_{m_{s_{\overline{\mu}}}m'_{s_{\overline{\mu}}}}\delta_{s_1s_1'}\delta_{s_3s_3'}\sum_{\mathcal{S}S_kS_l}(-1)^{s_2+s_2'+S_k+S_l+m_{s_{\overline{\mu}}}-M_{S_i}}[1/2]^3[S]^{1/2}[S']^{1/2}[s_2]^{1/2}[s_2']^{1/2}[S_k][S_l][\mathcal{S}]$$
$$\times W(1/2,S,1/2,S';1/2,\mathcal{S})W(S_k,1/2,1/2,s_3;1/2,1/2)W(\mathcal{S},s_2',1/2,1/2;s_2,1/2)\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -M_{S_i} & M_{S_i} & 0\end{pmatrix}$$
$$\times\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -m_{s_{\overline{\mu}}} & m_{s_{\overline{\mu}}} & 0\end{pmatrix}\begin{pmatrix}\mathcal{S} & S_k & S_l\\ S & 1/2 & 1/2\\ S' & 1/2 & 1/2\end{pmatrix}$$
$$\times\delta_{m_{t_{\overline{\mu}}}m'_{t_{\overline{\mu}}}}\delta_{t_1t_1'}\delta_{t_3t_3'}\sum_{\mathcal{T}T_kT_l}(-1)^{t_2+t_2'+T_k+T_l+m_{t_{\overline{\mu}}}-M_{T_i}}[1/2]^3[T]^{1/2}[T']^{1/2}[t_2]^{1/2}[t_2']^{1/2}[T_k][T_l][\mathcal{T}]$$
$$\times W(1/2,T,1/2,T';1/2,\mathcal{T})W(T_k,1/2,1/2,t_3;1/2,1/2)W(\mathcal{T},t_2',1/2,1/2;t_2,1/2)\begin{pmatrix}1/2 & 1/2 & \mathcal{T}\\ -M_{T_i} & M_{T_i} & 0\end{pmatrix}$$
$$\times\begin{pmatrix}1/2 & 1/2 & \mathcal{T}\\ -m_{t_{\overline{\mu}}} & m_{t_{\overline{\mu}}} & 0\end{pmatrix}\begin{pmatrix}\mathcal{T} & T_k & T_l\\ T & 1/2 & 1/2\\ T' & 1/2 & 1/2\end{pmatrix}\begin{pmatrix}T_l & \mathcal{T} & T_k\\ 1/2 & t_2' & 1/2\\ 1/2 & t_2 & 1/2\end{pmatrix}.$$

For Eq. (24):

$$D(\mu_1,\mu_2,\mu_3;\alpha_2)D(\mu_2,\mu_3,\overline{\mu};\beta_2)D(\overline{\mu},\rho_2,\rho_3;\alpha_3)D(\mu_1,\rho_2,\rho_3;\beta_3)D(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_i)D(\beta_1,\beta_2,\beta_3;\mathcal{A}_i)\delta^{\alpha_1\beta_1}$$
$$=\delta_{m_{s_{\overline{\mu}}}m'_{s_{\overline{\mu}}}}\delta_{s_1s_1'}\delta_{s_3s_3'}\sum_{\mathcal{S}S_kS_l}(-1)^{S_k+S_l-m_{s_{\overline{\mu}}}+M_{S_i}}[1/2]^3[S]^{1/2}[S']^{1/2}[S_k][S_l][\mathcal{S}]W(1/2,S,1/2,S';1/2,\mathcal{S})W$$
$$\times(S_k,1/2,1/2,s_3;1/2,1/2)W(S_l,1/2,1/2,s_2;1/2,1/2)W(\mathcal{S},S_k,1/2,1/2;S_l,1/2)\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -M_{S_i} & M_{S_i} & 0\end{pmatrix}\begin{pmatrix}1/2 & 1/2 & \mathcal{S}\\ -m_{s_{\overline{\mu}}} & m_{s_{\overline{\mu}}} & 0\end{pmatrix}$$
$$\times\begin{pmatrix}\mathcal{S} & S_k & S_l\\ S & 1/2 & 1/2\\ S' & 1/2 & 1/2\end{pmatrix}\delta_{m_{t_{\overline{\mu}}}m'_{t_{\overline{\mu}}}}\delta_{t_1t_1'}\delta_{t_3t_3'}\sum_{\mathcal{T}T_kT_l}(-1)^{T_k+T_l-m_{t_{\overline{\mu}}}+M_{T_i}}[1/2]^3[T]^{1/2}[T']^{1/2}[T_k][T_l][\mathcal{T}]$$
$$\times W(1/2,S,1/2,S';1/2,\mathcal{S})W(S_k,1/2,1/2,s_3;1/2,1/2)W(S_l,1/2,1/2,s_2;1/2,1/2)W(\mathcal{S},S_k,1/2,1/2;S_l,1/2)$$

$$\times\begin{pmatrix} 1/2 & 1/2 & \mathcal{S} \\ -M_{S_i} & M_{S_i} & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 & \mathcal{S} \\ -m_{s_{\overline{\mu}}} & m_{s_{\overline{\mu}}} & 0 \end{pmatrix}\begin{Bmatrix} \mathcal{S} & S_k & S_l \\ \mathcal{S} & 1/2 & 1/2 \\ S' & 1/2 & 1/2 \end{Bmatrix}\delta_{m_{t_{\overline{\mu}}} m'_{t_{\overline{\mu}}}}\delta_{t_1 t'_1}\delta_{t_3 t'_3}$$

$$\times\sum_{\mathcal{T} T_k T_l}(-1)^{T_k+T_l-m_{t_{\overline{\mu}}}+M_{T_i}}[1/2]^3[T]^{1/2}[T']^{1/2}[T_k][T_l][\mathcal{T}]$$

$$\times W(1/2,T,1/2,T';1/2,\mathcal{T})W(T_k,1/2,1/2,t_3;1/2,1/2)W(T_l,1/2,1/2,t_2;1/2,1/2)W(\mathcal{T},T_k,1/2,1/2;T_l,1/2)$$

$$\times\begin{pmatrix} 1/2 & 1/2 & \mathcal{T} \\ -M_{T_i} & M_{T_i} & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 & \mathcal{T} \\ -m_{t_{\overline{\mu}}} & m_{t_{\overline{\mu}}} & 0 \end{pmatrix}\begin{Bmatrix} \mathcal{T} & T_k & T_l \\ T & 1/2 & 1/2 \\ T' & 1/2 & 1/2 \end{Bmatrix}.$$

In the above equations, we set $M_{S_i}=1/2$, $M_{T_i}=1/2$ or $-1/2$ (for ^{3}He and ^{3}H nuclei), $m_{s_{\overline{\mu}}}=\pm 1/2$ (for two quark polarizations), and $m_{t_{\overline{\mu}}}=1/2$ or $-1/2$ (for up and down quarks).

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