Isoscalar *S*-wave π-*N* scattering length a^+ from π-deuteron scattering

S. R. Beane,[1,*] V. Bernard,[2,†] T.-S. H. Lee,[3,‡] and Ulf-G. Meißner[4,§]
[1]*Department of Physics, University of Maryland, College Park, Maryland 20742*
[2]*Laboratoire de Physique Théorique, Université Louis Pasteur, F-67037 Strasbourg Cedex 2, France*
[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439*
[4]*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*
(Received 19 August 1997)

We consider constraints on the isoscalar *S*-wave π-*N* scattering length a^+ from π-deuteron scattering, to third order in small momenta and pion masses in chiral perturbation theory. To this order, the π-deuteron scattering length is determined by a^+ together with three-body corrections that involve no undetermined parameters. We extract a novel value for a combination of dimension two low-energy constants which is in agreement with previous determinations. [S0556-2813(98)01301-6]

PACS number(s): 13.75.Gx, 12.39.Fe, 25.80.Dj

Chiral perturbation theory allows one to relate distinct scattering processes in a systematic manner. Recently a methodology has been developed which relates scattering processes involving a single nucleon to nuclear scattering processes [1]. For instance, one can relate π-*N* scattering to π-nucleus scattering. The nonperturbative effects responsible for nuclear binding are accounted for using phenomenological nuclear wave functions. Although this clearly introduces an inevitable model dependence, one can compute matrix elements using a variety of wave functions in order to ascertain the theoretical error induced by the off-shell behavior of different wave functions.

Weinberg showed that to third order [$O(q^3)$, where q denotes a small momentum or a pion mass] in chiral perturbation theory the π-*d* scattering length is given by [1]

$$a_{\pi d}=\frac{(1+\mu)}{(1+\mu/2)}(a_{\pi n}+a_{\pi p})+a^{1(\mathrm{b})}+a^{1(\mathrm{c}),1(\mathrm{d})}, \qquad (1)$$

where $\mu \equiv M_\pi/m$ is the ratio of the pion and the nucleon mass. The various diagrammatic contributions to $a_{\pi d}$ are illustrated in Fig. 1. The three-body corrections are (in momentum space)

$$a^{1(\mathrm{b})}=-\frac{M_\pi^2}{32\pi^4 f_\pi^4(1+\mu/2)}\left\langle \frac{1}{\vec{q}^{\,2}}\right\rangle_{\mathrm{wf}}, \qquad (2)$$

$$a^{1(\mathrm{c}),1(\mathrm{d})}=\frac{g_A^2 M_\pi^2}{128\pi^4 f_\pi^4(1+\mu/2)}\left\langle \frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{(\vec{q}^{\,2}+M_\pi^2)^2}\right\rangle_{\mathrm{wf}}. \qquad (3)$$

$\langle \vartheta \rangle_{\mathrm{wf}}$ indicates that ϑ is sandwiched between deuteron wave functions. These matrix elements have been evaluated using a cornucopia of wave functions; results are in Table I. Clearly $a^{1(\mathrm{b})}$ dominates the three-body corrections. This is the result of the shorter range nature of $a^{1(\mathrm{c}),1(\mathrm{d})}$ as can be seen from the *r*-space expressions of Eqs. (2) and (3). It is important to stress that the dominant three-body correction turns out to be quite independent of the wave function used. This implies that the chiral perturbation theory approach, which relies on the dominance of the pion exchange, is useful in this context.

The π-*N* scattering lengths have the decomposition

$$a_{\pi n}+a_{\pi p}=2a^+=2(a_1+2a_3)/3, \qquad (4)$$

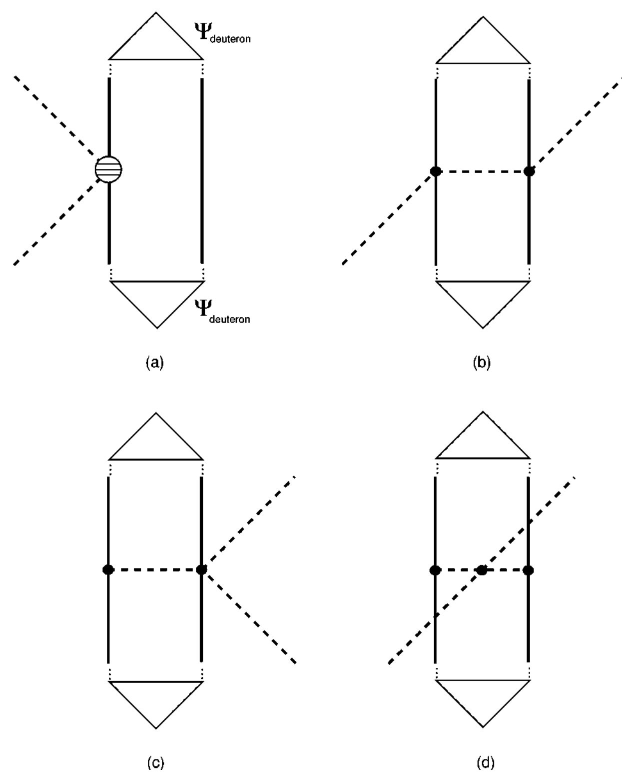

FIG. 1. Feynman graphs contributing to the π-*d* scattering length at order q^3 in chiral perturbation theory. Graph (a) is the single-scattering contribution and contains undetermined parameters. Graphs (b), (c), and (d) are three-body interactions which involve no undetermined parameters.

*Electronic address: sbeane@fermi.umd.edu
†Electronic address: bernard@sbghp4.in2p3.fr
‡Electronic address: lee@anlphy.phy.anl.gov
§Electronic address: Ulf-G.Meissner@fz-juelich.de

TABLE I. Three-body corrections for various deuteron wave functions in units of M_{π}^{-1}. Note that the Bonn wave function contains some retardation effects. We use $F_{\pi}=92.4$ MeV, $g_A=1.32$, and $M_{\pi^+}=139.6$ MeV.

Wave function	$a^{1(b)}$	$a^{1(c),1(d)}$
Bonn [11]	−0.02021	−0.0005754
ANL-V18 [12]	−0.01960	−0.0007919
Reid-SC [13]	−0.01941	−0.0008499
SSC [14]	−0.01920	−0.0006987

where a^+ is the isoscalar S-wave scattering length, and a_1 and a_3 are the isospin 1/2 and 3/2 contributions, respectively. Weinberg took a^+ from experimental data and argued that $a^{1(b)}$, which dominates the three-body corrections, should be accounted for with corrections to the vertices, which he estimated using a simple model [2]. He then found a result for $a_{\pi d}$ in agreement with the then current experimental value [3]. Since Weinberg's paper, there is new experimental information about both the π-N and π-d scattering lengths that is at variance with the old data [4,5]. Moreover, since Eq. (1) is a perfectly sensible expression to $O(q^3)$ in chiral perturbation theory, we choose to take it seriously by using realistic deuteron wave functions to evaluate both Eqs. (2) and (3) in order to see what it reveals.

We can express Eq. (1) as

$$a^+=\frac{(1+\mu/2)}{2(1+\mu)}\{a_{\pi d}-(a^{1(b)}+a^{1(c),1(d)})\}, \tag{5}$$

and use experimental information about π-d scattering to predict a^+; the recent Neuchatel-PSI-ETHZ (NPE) pionic deuterium measurement [4] gives

$$a_{\pi d}=-0.0259\pm 0.0011 M_{\pi}^{-1}. \tag{6}$$

For the three-body corrections, we take the average of the $a^{1(b)}$ and $a^{1(c),1(d)}$ values in Table I:

$$a^{1(b)+1(c)+1(d)}=-0.0203 M_{\pi}^{-1}. \tag{7}$$

We then find

$$a^+=-(2.6\pm 0.5)\times 10^{-3} M_{\pi}^{-1}. \tag{8}$$

Note that although $a^{1(c),1(d)}$ is small, there is a strong cancellation between $a^{1(b)}$ and $a_{\pi d}$ which leads to a sensitivity to $a^{1(c),1(d)}$. Our value for a^+ is not consistent with the Karlsruhe-Helsinki value [6],

$$a^+=-(8.3\pm 3.8)\times 10^{-3} M_{\pi}^{-1}, \tag{9}$$

or the new NPE value deduced from the strong interaction shifts in pionic hydrogen and deuterium, which is small and positive [5]:[1]

$$a^+=(0,\dots,5)\times 10^{-3} M_{\pi}^{-1}. \tag{10}$$

The result, Eq. (8), agrees, however, with the value obtained in the SM95 partial-wave analysis, $a^+=-3.0\times 10^{-3} M_{\pi}^{-1}$ [7]. Given the ambiguous experimental situation regarding a^+, it seems most profitable to turn our formula around and use the π-d scattering data and three-body corrections to constrain undetermined parameters that appear in a^+, which has been calculated to $O(q^3)$ in chiral perturbation theory [8]:

$$4\pi(1+\mu)a^+=\frac{M_{\pi}^2}{F_{\pi}^2}\left(-4c_1+2c_2-\frac{g_A^2}{4m}+2c_3\right)+\frac{3g_A^2 M_{\pi}^3}{64\pi F_{\pi}^4}. \tag{11}$$

It should be stressed, however, that to this order there appear large cancellations between the individual terms [8] which lead one to suspect that a calculation at $O(q^4)$ should be performed to obtain a more precise prediction for this anomalously small observable. This, however, goes beyond the scope of this manuscript. The sole undetermined parameter entering the $O(q^3)$ computation of $a_{\pi d}$ is therefore a combination of c_1, c_2, and c_3:

$$\Delta\equiv -4c_1+2(c_2+c_3), \tag{12}$$

where we can now write

$$a_{\pi d}=\frac{1}{2\pi(1+\mu/2)}\left\{\frac{M_{\pi}^2}{F_{\pi}^2}\left(\Delta-\frac{g_A^2}{4m}\right)+\frac{3g_A^2 M_{\pi}^3}{64\pi F_{\pi}^4}\right\} +a^{1(b)}+a^{1(c),1(d)}, \tag{13}$$

and solve for Δ,

$$\Delta=\frac{2\pi F_{\pi}^2}{M_{\pi}^2}(1+\mu/2)\{a_{\pi d}-(a^{1(b)}+a^{1(c),1(d)})\} +\frac{g_A^2}{4m}\left(1-\frac{3mM_{\pi}}{16\pi F_{\pi}^2}\right), \tag{14}$$

in order to constrain Δ using Eqs. (2), (3), and (6). We find

$$\Delta=-(0.08\pm 0.02)\ \ \mathrm{GeV}^{-1}, \tag{15}$$

[1]Note that this result might still change a bit since a more sophisticated treatment of Doppler broadening for the width of the hydrogen level has to be performed. Also, the NPE group did not yet quote a value for a^+. We rather used their figure combining the H and d results to get the band given. From the hydrogen shift and width alone one would deduce a negative value with a sizable error for a^+.

TABLE II. Values of the LEC's c_i in GeV^{-1} for $i=1,\ldots,3$. Also given are the central values (CV's) and the ranges for the c_i from resonance exchange. The asterisk denotes an input quantity. This table is adopted from [9].

i	c_i	c_i^{res} CV	c_i^{res} ranges
1	-0.93 ± 0.10	-0.9*	
2	3.34 ± 0.20	3.9	$2\ldots4$
3	-5.29 ± 0.25	-5.3	$-4.5\ldots-5.3$
Δ	-0.18 ± 0.75	0.8	$-3.0\ldots+2.6$

where we have taken into account the error in the determination of $a_{\pi d}$.

In Table II we give values of the relevant c_i's obtained from a realistic fit to low-energy pion-nucleon scattering data and subthreshold parameters [9]. Central values lead to $\sigma(0)=47.6$ MeV and $a^+=-4.7\times10^{-3}M_\pi^{-1}$. These values of the c_i's give the conservative determination:

$$\Delta=-(0.18\pm0.75)\ \ \mathrm{GeV}^{-1}. \tag{16}$$

Also shown in Table II are values of c_i's deduced from resonance saturation. It is worth mentioning that an independent fit to pion-nucleon scattering including also low-energy constants related to dimension-3 operators finds results consistent with the fit values of Table II [10].

To summarize, we have shown that the recent precise data on the π-deuteron scattering length can be used to constrain a combination of dimension-2 low-energy constants of the chiral effective pion-nucleon Lagrangian. This determination gives a result in agreement with previous determinations that use independent input [9,10]. Therefore, a consistent picture of nucleon chiral perturbation theory is emerging. Next, these calculations should be carried out one order further which would allow one to *precisely* deduce the isoscalar S-wave π-N scattering length from the accurately measured π-d scattering length. Work along these lines is in progress.

V.B. and U.G.M. are grateful to the Nuclear Theory Group at Argonne National Laboratory for hospitality while part of this work was completed. We thank D.R. Phillips for valuable conversations and A. Badertscher, J.-P. Egger, and R. Workman for useful communications. This research was supported in part by the U.S. Department of Energy, Nuclear Physics Division [Grants Nos. DE-FG02-93ER-40762 (S.R.B.), W-31-109-ENG-38 (T.S.H.L.)], by NATO Collaborative Research Grant No. 950607 (V.B., T.S.H.L., U.G.M.), and by the Deutsche Forschungsgemeinschaft [Grant No. ME 864/11-1 (U.G.M.)].

[1] S. Weinberg, Phys. Lett. B **295**, 114 (1992).
[2] T. Ericson and W. Weise, *Pion and Nucleons* (Clarendon Press, Oxford, 1988).
[3] E. Bovet *et al.*, Phys. Lett. **153B**, 231 (1985).
[4] D. Chatellard *et al.*, Phys. Rev. Lett. **74**, 4157 (1995); Nucl. Phys. (to be published).
[5] D. Sigg *et al.*, Phys. Rev. Lett. **75**, 3245 (1995); D. Sigg *et al.*, Nucl. Phys. **A609**, 269 (1996); **A617**, 526(E) (1997).
[6] R. Koch, Nucl. Phys. **A448**, 707 (1986).
[7] R.A. Arndt *et al.*, Phys. Rev. C **52**, 2120 (1995).
[8] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Phys. Lett. B **309**, 421 (1993).
[9] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Nucl. Phys. **A615**, 483 (1997).
[10] M. Mojzis, Z. Phys. C (to be published).
[11] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[12] R.B. Wiringa, V.G. Stoks, and R. Schiavilla Phys. Rev. C **51**, 38 (1995).
[13] R.V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[14] R. de Tourrell and D.W. Sprung, Nucl. Phys. **A210**, 193 (1973).