Amplitudes and resonances from an energy-dependent analysis of $\bar{p}+p\rightarrow\pi+\pi$

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The amplitudes at a series of discrete energies obtained from a previous analysis of $\bar{p}p\rightarrow\pi\pi$ have been used as input to a global energy-dependent analysis of data in the momentum range 360–1550 MeV/c. The results confirm the previous analysis and yield refined values for meson resonance parameters in this energy region. [S0556-2813(98)04906-1]

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I. DATA ANALYSIS

In a previous paper [1] we presented the results of an analysis of data on the reaction $\bar{p}p\rightarrow\pi\pi$ carried out at a series of discrete energies in the center-of-mass range 1.91–2.27 MeV. The data consisted of differential cross sections (DCS's) for both the $\pi^-\pi^+$ and $\pi^0\pi^0$ channels and angular asymmetry distributions (polarizations) for the $\pi^-\pi^+$ channel alone. These were supplemented by invariant amplitudes at each energy obtained in an earlier analysis using hyperbolic dispersion relations [2]. The latter allowed analyticity and crossing symmetry to be imposed on the solutions and thus ensured that they were consistent with the wealth of data in the $\pi N\rightarrow\pi N$ channel. The amplitude constraints, used for the first time in Ref. [1], enabled other published solutions to be ruled out and produced a set of resonance parameters more reliable than those of earlier analyses [3]. Single-energy analyses, however, do not include the correlations between amplitudes at different energies; nor in our method of extracting resonance parameters did we include correlations between different partial waves. In this paper we have therefore used the output amplitudes from Ref. [1] as starting values for an energy-dependent analysis in the same energy region.

Initially we used the same data set as was used in Ref. [1], i.e., DCS and polarization values at 20 momenta for the $\pi^-\pi^+$ channel from Ref. [4] (1973 data points) and DCS values at 14 momenta for the $\pi^0\pi^0$ channel from Ref. [5] (551 data points), giving a total of 2524 data points. In the present case, since we are making an energy-dependent analysis, the latter did not have to be interpolated to the same momenta as the measured $\pi^-\pi^+$ cross sections. As in Ref. [1], these experimental data were supplemented by values of the invariant amplitudes for the annihilation channel obtained via hyperbolic dispersion relations in Ref. [2] (3304 data points). We have also explored the compatibility of earlier data with this data set by including DCS values for the $\pi^-\pi^+$ channel from Ref. [6] (998 data points).

The parametrization used was the same as that used in Ref. [1] —i.e., we work in the JL basis—and each partial-wave helicity amplitude for a given J and L was written as

$$h_{J\pm}(W)=\frac{\alpha_{J\pm}}{M_R-W-i\Gamma/2}+k^{L+1/2}\sum_{n=1}^{n_{JL}}\beta_{J\pm}^{(n)}x^{n-1}, \qquad (1)$$

where $h_{J\pm}\equiv h_{J,L=J\pm1}$. Here W is the center-of-mass energy and

$$x\equiv\frac{2W-W_{\min}-W_{\max}}{W_{\max}-W_{\min}}. \qquad (2)$$

In the second (background) term the coefficients $\beta_{J\pm}^{(n)}$ are complex parameters and to ensure the correct behavior at the $\bar{N}N$ threshold we set $k=p/p_B$ where p_B is the momentum corresponding to $W=2.1$ GeV. In the resonance term, the parameters are the mass M_R, the width Γ, and the complex residues $\alpha_{J\pm}$. To ensure the correct threshold behavior at the $\bar{N}N$ threshold, we set

$$\alpha_{J\pm}=\gamma_{J\pm}\left(\frac{p}{p_R}\right)^{L+1/2} \quad (p\leqslant p_R) \qquad (3)$$

$$=\gamma_{J\pm} \quad (p>p_R) \qquad (4)$$

where p_R is the value of p at $W=M_R$ and $\gamma_\pm$ is a complex constant. From the values of $\gamma_\pm$ and Γ one can calculate the product of branching ratios $B_J\equiv B(R\rightarrow\pi\pi)B(R\rightarrow\bar{N}N)$. The amplitudes $h_{J\pm}$ are dimensionless and are normalized so that the integrated cross section for a given isospin is given by

$$\sigma=\frac{\pi}{p^2}\sum_J (2J+1)\{|h_{J+}|^2+|h_{J-}|^2\}. \qquad (5)$$

The quality of fits to data over a range of energies is always considerably worse than that obtained at a single energy. This is partly due to normalization differences between different experiments and even between different energies for the same experiment. Also, isolated discrepant points may make an anomalously large contribution to whatever measure is used to judge the quality of the fit. To reduce the latter effects, we have used robust estimation, minimizing the quantity

TABLE I. Resonance masses and widths in units of GeV, obtained from fitting DCS and polarization data [4–6], together with the values of the product of branching ratios.

J	Mass	Width	B_J
0	1.95	0.17–0.18	0.13–0.15
1	1.96	0.15–0.17	0.059–0.064
2	1.93	0.14–0.15	0.011
3	2.02	0.23	0.002–0.006 [a]
4	2.00	0.16–0.18	0.0022–0.0024
5	2.19	0.22	0.0011–0.0018

[a]There is a misprint in the corresponding entry in Table II of [1], where this number is given as 0.028 instead of 0.0028.

$$\sum_{i=1}^{N_{\rm pts}} \ln(1+0.5z_i^2), \qquad (6)$$

where $N_{\rm pts}$ is the total number of data points and where, for a given data point,

$$z_i=\frac{y_i-y_{{\rm param},i}}{\sigma_i}, \qquad (7)$$

y_i being the input data point, $y_{{\rm param},i}$ being the prediction for the same data point from the parametrization, and σ_i being the error in the data point. This reduces the influence of isolated discrepant points compared to the usual χ^2 minimization, although we will also quote the resulting χ^2 values. In each fit we used as starting values the amplitudes found in Ref. [1], with the resonance parameters loosely constrained to lie close to their initial values, typically within 50 MeV, although this was not an absolute constraint. The point here is that we are not attempting to make a systematic search of the entire parameter space, but rather to test the compatibility of the energy dependence of our previous solution with the whole data set. In practice, the resonance parameters showed no significant tendency to move away from their initial values. Starting with the data from Refs. [4] and [5], we found that a solution could be found with a χ^2 per data point averaged over experiments as follows: 2.05 ($\pi^-\pi^+$ channel), 0.61 ($\pi^0\pi^0$ channel), and 0.11 (invariant amplitudes). These values are, as expected, higher than obtained in single-energy analyses. Allowing small renormalizations (typically less than 10%) on the experimental DCS data reduced these values by 21% ($\pi^-\pi^+$) and 15% ($\pi^0\pi^0$). In addition 69 of the 1973 $\pi^-\pi^+$ data points contribute more than 10 to χ^2; removing these reduces the $\pi^-\pi^+$ average χ^2 per data point from 2.05 to 1.45. However, whether or not these various adjustments are made, the resulting solution remains essentially unchanged. We also tested the compatibility of earlier data [6] with the accurate DCS data from LEAR [4] by including the former in the fit. There was little change in either the quality of the fit, the amplitudes, or the resonance parameters and so we conclude that the newer LEAR data [4] are compatible with the older DCS data.

The resonance parameters of the solutions found are

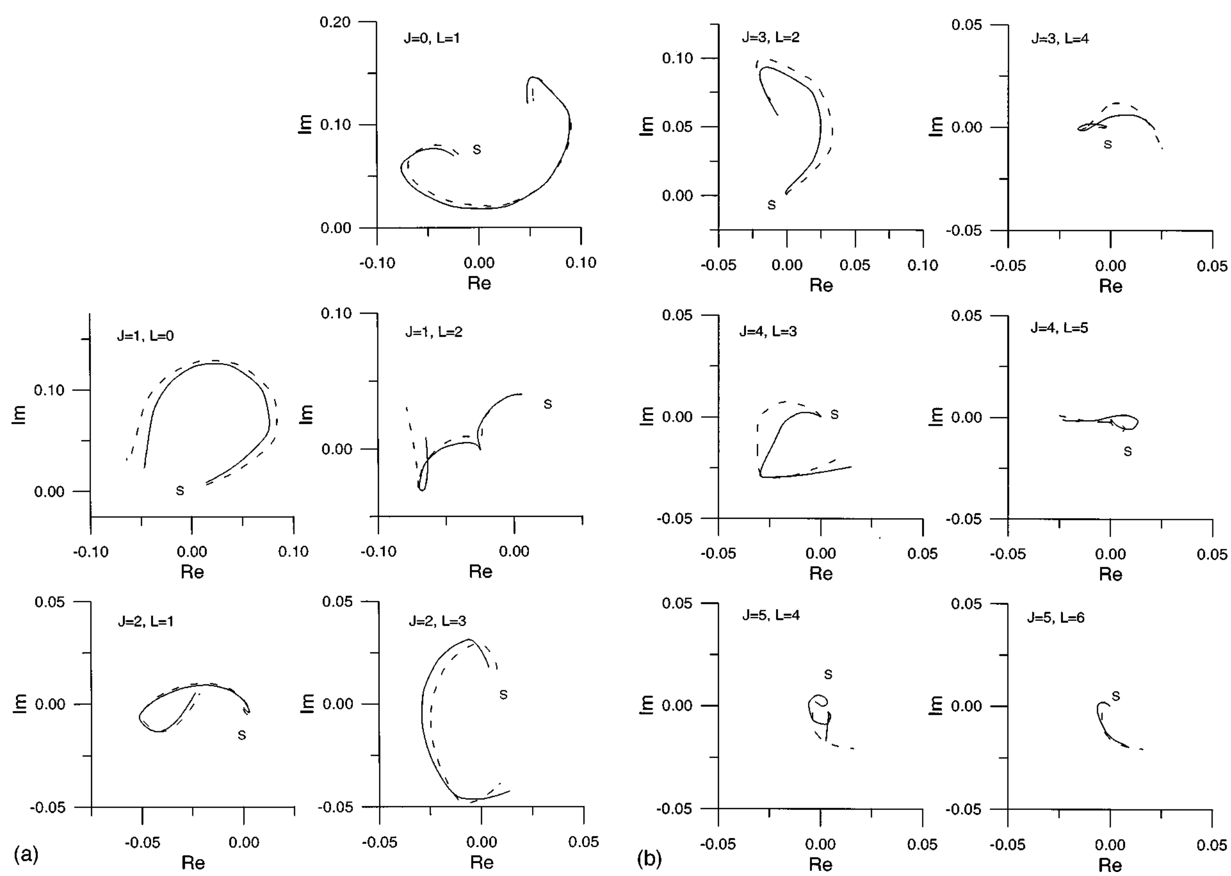

FIG. 1. Partial-wave helicity amplitudes in the JL basis obtained from an energy-dependent fit to the data from [4,5] (solid lines) and including the older data [6] (dashed lines). In each case the start of the argand diagram is indicated by S. (a) $J=0,1,2$. (b) $J=3,4,5$.

shown in Table I. The range of values spans those found in different solutions and using the two data sets. The parameters are rather similar to those found in Ref. [1], as is the pattern of their couplings to the different helicity states. This is discussed in detail in Ref. [1] and will not be repeated here. For $J=0$, the width has increased slightly and the value of B_0 is somewhat smaller, but there is still strong evidence for a state with an abnormally large coupling to the $\bar{N}N$ channel. Two places where the present solution distinguishes different possibilities found in Ref. [1] are the $J=3$ and $J=4$ waves where in both cases smaller couplings are preferred. In addition, for $J=4$ a smaller width is found, closer to the width of the established $f_4(2050)$ although somewhat smaller than the accepted value. The corresponding amplitudes are shown in Fig. 1 for solutions with and without the older $\pi^-\pi^+$ experiments [6].

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