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Nuclear symmetry energy

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To study the nuclear symmetry energy, we extend the Dirac-Brueckner approach with a Bonn one-boson-exchange nucleon-nucleon interaction to the general case of asymmetric nuclear matter. We extract the symmetry energy coefficient at the saturation to be about 31 MeV, which is in good agreement with the empirical value of 30±4 MeV. The symmetry energy is found to increase almost linearly with the density, which differs considerably from the results of nonrelativistic approaches. This finding also supports the linear parametrization of Prakash, Ainsworth, and Lattimer. We find, furthermore, that the higher-order dependence of the nuclear equation of state on the asymmetry parameter is unimportant. [S0556-2813(98)04606-8]

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Although the fact that the equation of state of nuclear matter contains a symmetry energy term has been known since the early days of nuclear physics, the experimental and theoretical study of the symmetry energy and its density dependence is becoming an increasingly interesting topic, mainly because of the recent development of radioactive ion beam facilities that allow one to study the structure and reactions of neutron-rich nuclei [1,2], in which the symmetry energy plays an important role. The recognization that the symmetry energy, especially its density dependence, has a profound effect on the properties of neutron stars [3–5] also makes the experimental and theoretical determination of this quantity very relevant and useful.

Experimentally, the symmetry energy coefficient $S_2(\rho_0)$ in nuclear matter at the saturation density ρ_0 can be extracted from a systematic study of the masses of atomic nuclei, based on, e.g., the liquid droplet model [6] or the macroscopic-microscopic model [7]. This, however, determines the symmetry energy only for a small asymmetry parameter α [$\alpha=(N-Z)/A$] and for densities around ρ_0. The situation changes with the recent advances in the development of various radioactive ion beam facilities around the world that will produce nuclei with a large neutron excess near and beyond the drip line. The study of the structure of these neutron-rich nuclei allows us to determine the symmetry energy for a large asymmetry parameter and extract a possible higher-order dependence on α. Furthermore, the collisions of neutron-rich nuclei at relativistic energies, during which nuclear matter with densities up to $(2–3)\rho_0$ is created, make it possible to study experimentally the density dependence of the symmetry energy [8,9].

Phenomenologically, different approaches have been used to study the symmetry energy of nuclear matter. Hartree-Fock [10] and Thomas-Fermi [11] calculations with Skyrme-type effective nucleon-nucleon interactions lead to a symmetry energy coefficient $S_2(\rho_0)$ in the range of 27–38 MeV, which is in agreement with the empirical value of 30±4 MeV [12]. Another phenomenological approach that has been used extensively in the study of nuclear properties is quantum hadrodynamics (QHD) which is based on the relativistic field theory [13]. The symmetry energy in this approach ranges from about 35 to 40 MeV [14–16], somewhat larger than the empirical value of 30±4 MeV.

For the density dependence of nuclear symmetry energy that is needed for the study of neutron star properties, Prakash, Ainsworth, and Lattimer [4] have proposed a number of phenomenological parametrizations. These different parametrizations have quite different consequences for properties of neutron stars and for the onset of possible kaon condensation in dense matter [17]. Very recently, Li, Ko, and Ren [9] applied these parametrizations to the study of the collisions of neutron-rich heavy ions at intermediate energies. They have found significant differences in the preequilibrium neutron/proton ratio using different parametrizations.

It is thus of great interest and importance to examine these parametrizations as well as other phenomenological approaches in a microscopic way. There are a number of microscopic studies on the symmetry energy of nuclear matter. In Refs. [18,19], variational calculations were carried out using Argonne v_{14} (AV14) or Urbana v_{14} (UV14) two-body interactions together with some phenomenological three-nucleon force. The symmetry energy coefficient obtained in the variational calculations is about 30 MeV [18,19], in good agreement with the empirical value. The symmetry energy was found to increase rather slowly with density. In Refs. [20,21], the nonrelativistic Brueckner-Hartree-Fock (BHF) approach was appied to the study of asymmetric nuclear matter. The symmetry energy coefficient obtained in these studies is again in good agreement with the empirical value. The density dependence of the symmetry energy was found to be modest [20]. It is, however, well known that the BHF approach with realistic two-nucleon interactions such as Bonn and Paris potentials does not provide a good description of nuclear matter properties [20]. Relativistic effects are known to play an important role in nuclear matter saturation [22–26] and are expected to be important for the symmetry energy as well.

It is the purpose of this paper to carry out a systematical analysis of the nuclear symmetry energy in the formalism of the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach using the Bonn one-boson-exchange (OBE) potential. We will concentrate on the density dependence of the symmetry energy that is very important for neutron star properties and heavy-ion collisions. In addition to the well-known S_2 term, we will also discuss the higher-order asymmetry parameter dependence, namely, the S_4 term, of the nuclear equation of state, which so far has not been addressed in microscopic approaches. Our results will be compared to various phenomenological parametrizations of Prakash, Ainsworth, and Lattimer [4], as well as to the results of the variational and BHF calculations.

The essential point of the DBHF approach is the use of the Dirac equation for the description of the single-particle motion in the nuclear medium. The Dirac spinor, which enters the evaluation of in-medium nucleon-nucleon potential, becomes density dependent. This additional density dependence is instructive in reproducing correctly the nuclear matter saturation density and binding energy [24]. The basic quantity in the DBHF calculation is the $\tilde{G}$ matrix which satisfies the in-medium Thompson equation,

$$\tilde{G}(q',q|P,\tilde{z})=\tilde{V}(q',q)+\int \frac{d^3k}{(2\pi)^3}\tilde{V}(q',k)\times\left(\frac{\tilde{m}(k)}{\tilde{E}(k)}\right)^2\frac{\bar{Q}(k,P)}{2\tilde{E}(q)-2\tilde{E}(k)}\tilde{G}(k,q|P,\tilde{z}), \tag{1}$$

where $\tilde{E}=\sqrt{\tilde{m}^2+(\mathbf{P}/2+\mathbf{k})^2}$ and $\tilde{m}=m+U_S$, with m being nucleon mass in free space. For asymmetric nuclear matter, the angle-averaged Pauli-blocking operator has to be modified and is given by

$$\bar{Q}(k,K)=\begin{cases}1 \quad \text{if } \beta_n>1,\\ (1+\beta_n)/2 \quad \text{if } -1<\beta_n<1 \text{ and } \beta_p>1,\\ (\beta_n+\beta_p)/2 \quad \text{if } \beta_p<1 \text{ and } 0<(\beta_n+\beta_p)/2,\\ 0 \quad \text{if } (\beta_n+\beta_p)/2<0 \text{ or } \beta_n<-1,\end{cases} \tag{2}$$

where

$$\beta_{n,p}=\frac{K^2/4+k^2-k^2_{F_{n,p}}}{Kk}, \tag{3}$$

where k_{F_n} and k_{F_p} are neutron and proton Fermi momenta, respectively, with $k_{F_n}\geqslant k_{F_p}$.

From the $\tilde{G}$ matrix we can calculate the single-particle potential

$$\Sigma(k)=\mathrm{Re}\int_0^{k_F}d^3q\left(\frac{\tilde{m}(q)}{\tilde{E}(q)}\right)\left(\frac{\tilde{m}(k)}{\tilde{E}(k)}\right)\langle kq|\tilde{G}(\tilde{z})|kq-qk\rangle. \tag{4}$$

In the case of asymmetric nuclear matter, the potential energy of a single particle is

$$E_{\mathrm{pot}}=\frac{1}{\int_0^{k_{F_n}}d^3k+\int_0^{k_{F_p}}d^3k}\left(\int_0^{k_{F_n}}d^3k\,\frac{1}{2}\,\Sigma_n(k)+\int_0^{k_{F_p}}d^3k\,\frac{1}{2}\,\Sigma_p(k)\right), \tag{5}$$

while the kinetic energy is given by

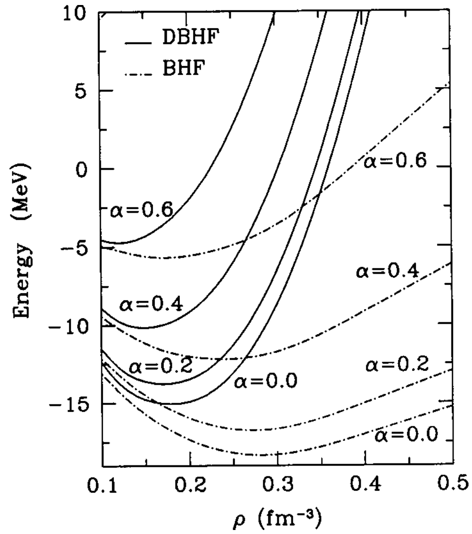

FIG. 1. Equation of state of nuclear matter for a number of asymmetry parameters.

$$E_{\mathrm{kin}}=\frac{1}{\int_0^{k_{F_n}}d^3k+\int_0^{k_{F_p}}d^3k}\left(\int_0^{k_{F_n}}d^3k\,\frac{mm^*(k)+k^2}{E^*(k)}+\int_0^{k_{F_p}}d^3k\,\frac{mm^*(k)+k^2}{E^*(k)}\right). \tag{6}$$

The energy per nucleon, or nuclear equation of state, is then given by

$$E=E_{\mathrm{pot}}+E_{\mathrm{kin}}-m. \tag{7}$$

In Fig. 1, we show the nuclear equation of state for a number of asymmetry parameters. We compare our results with those of Ref. [20] obtained in the BHF approach. As is well known, the BHF approach saturates nuclear matter at a much too high density. In the DBHF calculation, the nuclear matter saturation properties are better reproduced. The binding energy and the saturation density become progressively smaller as α increases.

Let us introduce ΔE as the energy difference between symmetric and asymmetric nuclear matter,

$$\Delta E=E(\rho,\alpha)-E(\rho,0). \tag{8}$$

We find that at all densities considered here, ΔE increases almost linearly with α^2, indicating that the α^4 and higher-order terms are not important. To a good extent we can express the equation of state of asymmetric nuclear matter as

$$E(\rho,\alpha)=E(\rho,0)+S_2(\rho)\alpha^2+S_4(\rho)\alpha^4. \tag{9}$$

The usual symmetry energy S_2 is thus defined as

$$S_2(\rho)=\frac{1}{2}\left.\frac{\partial^2E(\rho,\alpha)_{\mathrm{bin}}}{\partial\alpha^2}\right|_{\alpha=0}, \tag{10}$$

and similarly,

$$S_4(\rho)=\frac{1}{24}\left.\frac{\partial^4E(\rho,\alpha)_{\mathrm{bin}}}{\partial\alpha^4}\right|_{\alpha=0}. \tag{11}$$

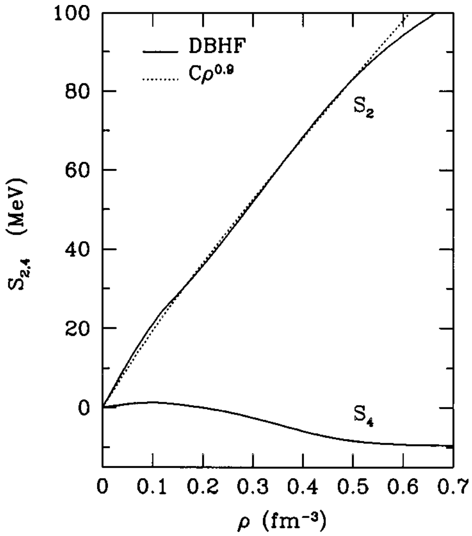

FIG. 2. Density dependence of symmetry parameters.

The density dependences of S_2 and S_4 obtained in our calculation are shown in Fig. 2 by the solid curve. S_2 increases almost linearly with density. Actually, a parametrization in terms of $(\rho/\rho_0)^{0.9}$ fits the theoretical curve reasonably well, as shown in the figure by the dotted curve. At nuclear matter saturation density, our calculation gives a symmetry energy coefficient $S_2(\rho_0)$ of about 31 MeV, which is in good agreement with the empirical value of about 30 ± 4 MeV [12]. The BHF and the variational calculations also reproduce the empirical symmetry energy coefficient [19,20]. The coefficient of the α^4 term is very small in the density region considered here. This means that the approximation of neglecting this term as adopted in Ref. [18,19] is quite reasonable.

In Fig. 3 we compare our results for the density dependence of the symmetry energy with those of Ref. [20] based on the BHF calculation and of Ref. [19] based on the variational calculation. There are significant differences between the results of these three calculations. In relativistic approaches [27,28], the symmetry energy increases almost linearly with density and is considerably larger than those in nonrelativistic and variational calculations [19,20]. The difference between the DBHF and BHF calculations is mainly due to the relativistic effects. In the simple mean-field approximation to the Walecka-type model, the symmetry energy has a contribution from the ''kinetic energy'' difference, which is inversely proportional to $E_F^*=\sqrt{k_F^2+m^{*2}}$. This contribution is thus larger in relativistic approaches because of the dropping nucleon mass. This also accounts for part of the difference between our results and that of Ref. [19], which is nonrelativistic in nature. The remaining difference can be explained by the differences in the nucleon-nucleon potentials used in the two calculations. In the variational calculations [19], the major contribution to the ''potential'' part of the symmetry energy comes chiefly from the second-order tensor interaction, which is progressively blocked with increasing density. With the strong ρ coupling of the Bonn potential, the second-order tensor force is relatively weak, compared with that of Ref. [19], so that this is not a large effect in our calculation, where the main contribution to the symmetry energy comes from ρ-meson exchange. The differences in the symmetry energy in these three calculations will have a profound impact on the properties of neutron stars. We hope that future experiments with radioactive ion beams will help to shed light on this problem.

Phenomenologically, Prakash, Ainsworth, and Lattimer proposed the following parametrization for the density dependence of the symmetry energy,

$$S(u)=(2^{2/3}-1)\,\frac{3}{5}\,E_F^0[u^{2/3}-F(u)]+S_0F(u), \qquad (12)$$

with

$$F_1(u)=2u^2/(1+u), \qquad (13)$$

$$F_2(u)=u, \qquad (14)$$

$$F_3(u)=\sqrt{u}, \qquad (15)$$

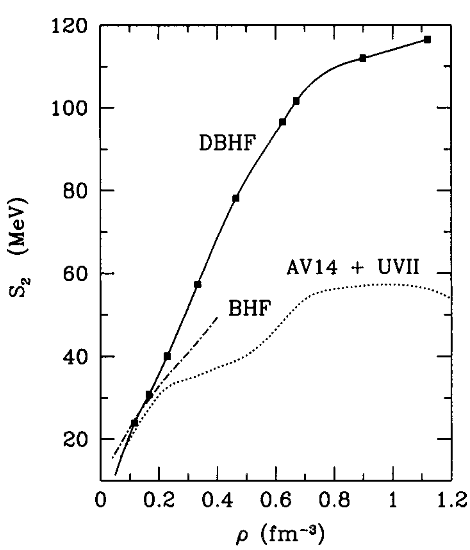

FIG. 3. Comparison of our results with those of Refs. [19] and [20].

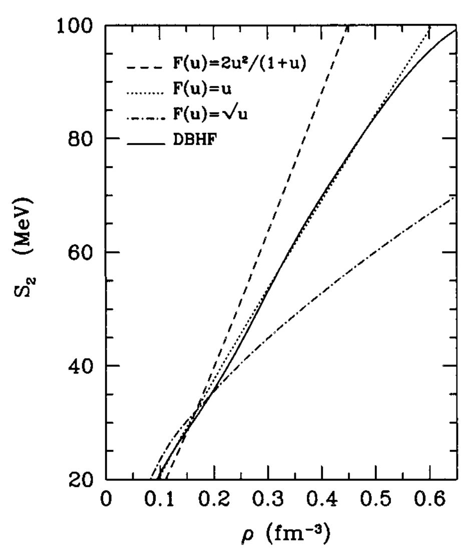

FIG. 4. Comparisons of our results with phenomenological parametrizations of Prakash, Ainsworth, and Lattimer [4].

where $u=\rho/\rho_0$ and E_F^0 is Fermi energy at saturation density ρ_0. In Fig. 4, the density dependences of three forms of $F(u)$ are compared with our results, and it is seen that the $F(u)=u$ case is very close to our results.

In summary, we studied nuclear symmetry energy in the formalism of the Dirac-Brueckner approach with the Bonn one-boson-exchange nucleon-nucleon interaction. The symmetry energy coefficient at the saturation density obtained in this work is about 30 MeV. This is in good agreement with the empirical value of about 34 ± 4 MeV and in agreement with other approaches such as the BHF [20] and variational [19] calculations. The higher-order dependence of the symmetry energy or nuclear equation of state on the asymmetry parameter is found to be small. The symmetry energy in our study is found to increase almost linearly with the density and agrees with the linear parametrization of Prakash, Ainsworth, and Lattimer [4]. At higher densities, the symmetry energy in our calculation is considerably larger than those in the BHF and variational calculations. The difference can be understood as coming from the both the relativistic effects in the ''kinetic energy'' contribution and a strong ρ-meson coupling in the Bonn potential that increases the ''potential energy'' contribution to the symmetry energy. We expect that future experiments with radioactive beams will be able to discriminate these predictions.

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