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Equation of state of isospin asymmetric nuclear matter including relativistic random-phase approximation-type correlations

Norbert Fröhlich and Hans Baier
Institut für Radiumforschung und Kernphysik, University of Vienna, Boltzmanngasse 3, 1090 Vienna, Austria

Wolfgang Bentz
Department of Physics, Faculty of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan
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The equation of state of asymmetric matter including the relativistic exchange and correlation parts is calculated in the σ-ω model. The results are discussed in comparison with empirical informations and phenomenological approaches. Neutron star matter in β equilibrium and the resulting neutron star masses are also discussed. [S0556-2813(98)06506-6]

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I. INTRODUCTION

There has been much interest recently in the equation of state (EOS) of isospin-asymmetric ($N\neq Z$) nuclear matter, mainly in connection to the physics of supernova explosions [1] and neutron stars [2], and to the structure of neutron-rich nuclei [3] and collisions between them [4]. To achieve a consistent description of these phenomena, which are related to both the high and low density behavior of the EOS, relativistic meson-nucleon models are often used [5]. As long as one remains within the mean field approximation, one has to introduce nonlinear mesonic self-interactions [6–9] in order to get a reasonable incompressibility, which is a key parameter in supernova simulations [10,11] and analyses of nuclear breathing modes [12–16]. On the other hand, it has been shown [17] that a softening of the EOS can also be achieved by including the relativistic exchange and random-phase-approximation- (RPA-) type correlations terms in the framework of the σ-ω model in its simplest form [18]. The results obtained for symmetric matter and neutron matter indicate a basic agreement with the empirical information. In this work we will extend this EOS to asymmetric matter and neutron star matter in β equilibrium, and discuss the results in terms of phenomenological parametrizations, empirical data on giant monopole resonances, and neutron star masses.

II. MODEL FOR THE EOS

The derivation of the EOS of symmetric matter in Ref. [19] was based on the σ-ω model Lagrangian [18]

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/-M+g_\sigma\sigma-g_\omega\omega\!\!\!/)\psi+\tfrac{1}{2}[(\partial_\mu\sigma)^2-m_\sigma^2\sigma^2]-\tfrac{1}{4}(\partial_\mu\omega_\nu-\partial_\nu\omega_\mu)^2+\tfrac{1}{2}m_\omega^2\omega_\mu^2 \tag{1}$$

and the $1/N$ expansion, where N refers to the isospin SU(N), and in the present context one identifies $N=2$. In this scheme, the relativistic Hartree approximation (RHA) gives the leading contribution, and the next-to-leading term consists of the exchange and RPA-type correlations including vacuum polarization effects. Introducing a chemical potential term $\mu\bar{\psi}\gamma^0\psi$ into the Lagrangian, the thermodynamic potential for $T=0$ can be derived most conveniently by utilizing the path integral formalism [19]. The result up to the next-to-leading term is

$$\frac{\Omega}{V}=\frac{m_\sigma^2}{2}\langle\sigma\rangle^2-\frac{m_\omega^2}{2}\langle\omega^0\rangle^2-i\int\frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\ln S(k)+\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\ \mathrm{tr}\ln\Delta(k)+\mathrm{c.t.}, \tag{2}$$

where $\langle\cdots\rangle$ denotes the matter expectation value, and $S(k)=\{\tilde{k\!\!\!/}-M^*+i\epsilon[1-2\Theta(\tilde{k}_0)\Theta(\mu^*-\tilde{k}_0)]\}^{-1}$ is the nucleon Hartree propagator with $M^*=M-g_\sigma\langle\sigma\rangle$, $\tilde{k}^\mu=(k^0+\mu^*,\mathbf{k})$, and $\mu^*=\mu-g_\omega\langle\omega^0\rangle$. $\Delta(k)$ is the combined σ-ω meson RPA-type propagator [19], and Tr and tr denote the traces with respect to the generalized Dirac and Lorentz indices, respectively. The counterterms (c.t.) include the subtraction of the vacuum contributions, and are determined as in Ref. [19]. The Landau ghost singularities arising from the RPA-type meson propagators are eliminated following Redmond's method [20] based on the Källén-Lehmann representation

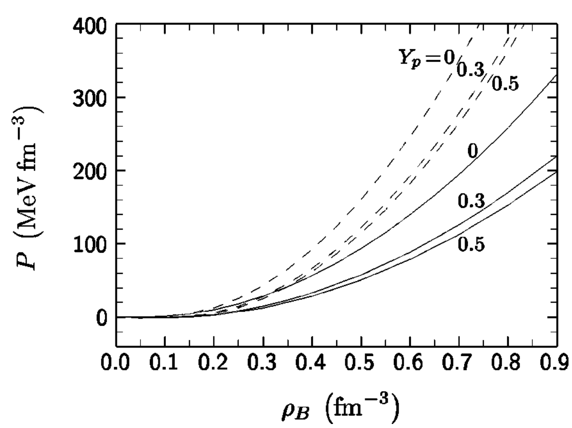

FIG. 1. Pressure as a function of baryon density for various proton fractions. The solid lines correspond to the $1/N$ and the dashed lines to the RHA EOS.

TABLE I. Parameters used in the full calculation $1/N$ and the RHA. Also shown are the parameters of the Lagrangian including nonlinear terms (NL), which approximately reproduce the $1/N$ EOS in a mean field approximation. M^* is the nucleon effective mass at the saturation density. $m_\omega = 783$ MeV and $m_\varrho = 770$ MeV are fixed in each parameter set.

	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	$g_\varrho^2/4\pi$	m_σ (MeV)	c_3 (fm^{-1})	c_4	M^*/M
$1/N$	2.24	3.66	1.48	550.0	0	0	0.89
RHA	6.23	8.18	1.46	550.0	0	0	0.73
NL	3.61	3.45	1.62	550.0	−34.61	501.33	0.86

(for details see Ref. [21]). It has been shown in Ref. [22] that the extension of Eq. (2) for $T>0$ leads to a thermodynamic consistent theory.

The extension to asymmetric matter is done by breaking the isospin symmetry according to $\mu^* \rightarrow \tau^+ \mu_p^* + \tau^- \mu_n^*$, where $\mu^*_{p(n)} = \mu_{p(n)} - g_\omega \langle \omega^0 \rangle \pm g_\varrho \langle \varrho_3^0 \rangle$ (+ for p and − for n) and $\tau^\pm \equiv \frac{1}{2}(1 \pm \tau_z)$, and adding a term $-\frac{1}{2} m_\varrho^2 \langle \varrho_3^0 \rangle^2$ to Eq. (2). That is, we include the neutral ϱ meson and its coupling to the nucleon on the Hartree level, which is the minimum ingredient required to obtain a reasonable symmetry energy [17]. (The inclusion of the ϱ meson in the higher order terms is a formidable task and is left for future work.) The original $1/N$ expansion is recovered in the limit of isospin symmetric matter. With regard to this correspondence, the resulting EOS will be denoted as ‘‘$1/N$ EOS’’ in this paper. The RHA EOS is obtained by discarding the term involving the meson propagator Δ in Eq. (2).

III. RESULTS AND DISCUSSIONS

A. Pressure and comparison to empirical information on the EOS

The pressure of the system, which is given by $-\Omega/V$ with the chemical potentials eliminated in favor of the densities, is shown in Fig. 1 as a function of the baryon density for various proton fractions $Y_p \equiv Z/N$. The parameters used in the calculation are shown in Table I. As in previous works [17,19,22], m_σ is fixed to 550 MeV, and g_σ, g_ω, and g_ϱ are fitted to the saturation properties of symmetric matter. (We take $\rho_0 = 0.148$ fm^{-3} for the saturation density, $E_B = 15.75$ MeV for the binding energy per nucleon, and $a_4 = 32.5$ MeV for the symmetry energy.) From Fig. 1 we see that the EOS is softened due to the inclusion of the higher order terms for all values of the proton fraction. As discussed in detail in Ref. [19], the inclusion of the higher order terms leads to a larger M^* and a smaller g_ω, which results in a smaller incompressibility and a reduction of the pressure at higher densities. (For $1/N$ we obtain $K_v = 302$ MeV, compared to the RHA value of 452 MeV.[1]) As a result of the relation $P(\rho_B,\beta) = P(\rho_B,0) + \beta^2 \rho_B^2 da_4/d\rho_B + O(\beta^4)$ the increase of P with increasing asymmetry $\beta \equiv (N-Z)/A = 1 - 2Y_p$ for all densities reflects the fact that in our calculation the symmetry energy increases monotonously with the baryon density (see Fig. 24 of Ref. [17]).

To facilitate the comparison to other works, we note that our $1/N$ EOS can be well reproduced in the mean field approximation by adding a nonlinear interaction term $\delta\mathcal{L} = -\frac{1}{3} c_3 \sigma^3 - \frac{1}{4} c_4 \sigma^4$ to the σ-ω Lagrangian. The parameters of this nonlinear (NL) Lagrangian, which we fixed such as to reproduce our $1/N$ pressure as close as possible within $\rho_0 \leqslant \rho_B \leqslant 3\rho_0$, are shown in Table I. This NL set leads to a different M^* as compared to $1/N$. As compared with the parameter set NL1, which was determined in Ref. [8] by a fit to properties of proton-magic nuclei and used in Ref. [7] to discuss supernova matter, we note that both our values of c_3 and c_4 carry the opposite sign and c_4 is much larger in magnitude. Because of this distinction, we obtain for $\langle\sigma\rangle$ roughly half of the value of Ref. [8], which subsequently leads to some change in the incompressibility, whereas the direct contribution of the nonlinear term to the incompressibility is prevailed by the separately large contributions of the quadratic terms in σ and ω. Thus, like in the linear model, the incompressibility is still mainly determined by the values of $\langle\sigma\rangle$ and g_ω.

The fact that the softening of the EOS, which in our treatment is due to the relativistic correlation terms, can be achieved in a mean field model by including nonlinear interaction terms has been known for a long time [9]. These nonlinear terms also play an important role in the chiral effective Lagrangian approach [23] to describe properties of nuclear matter and nuclei, and studies based on the quark-meson coupling model [24,25] have shown that they can be thought to reflect the compositeness of the nucleon. In this connection, it is interesting to observe that the effects of the relativistic correlation terms in our approach can be incorporated into interaction terms of similar nature.

For the purpose of comparing our results with empirical information and phenomenological approaches, it is useful to adopt the parametrization from Baron, Cooperstein, and Kahana [10] (BCK) for the supernova problem,

$$P = \frac{K(\beta)\rho_s(\beta)}{9\gamma}\left[\left(\frac{\rho_B}{\rho_s}\right)^\gamma - 1\right]. \tag{3a}$$

Here γ is the high density adiabatic index which is assumed to be independent of β. The incompressibility $K(\beta)$ and the saturation density $\rho_s(\beta)$ are described by the first terms in an expansion around $\beta=0$,

$$K(\beta) = K_v(1 - a_s\beta^2), \quad \rho_s(\beta) = \rho_0(1 - b_s\beta^2). \tag{3b}$$

[1]The range of K_v obtained in the ‘‘macroscopic approach’’ is about 200–350 MeV; see Refs. [12,13] and references therein. Recently, however, arguments in favor of the ‘‘microscopic approach’’ have been presented [14], which leads to $K_v = 200-230$ MeV in nonrelativistic theories [15] and 250–270 MeV in relativistic theories [16].

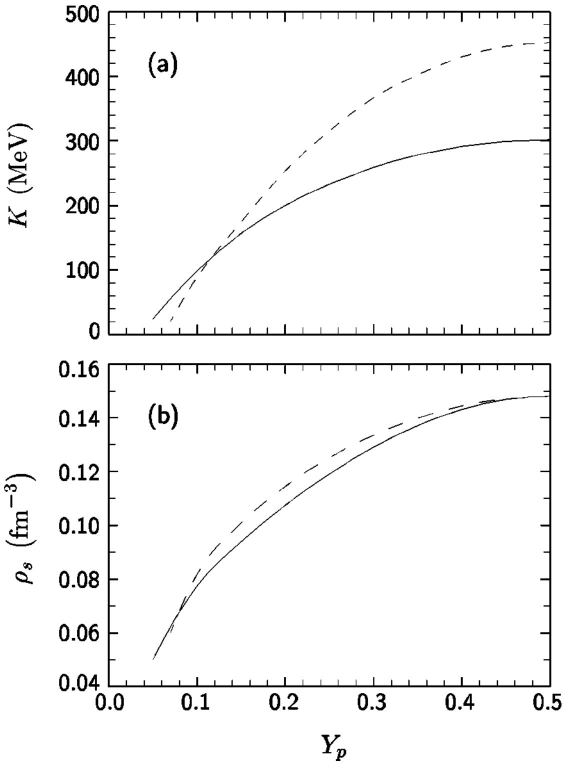

FIG. 2. Incompressibility (a) and saturation density (b) as a function of proton fraction. The solid and dashed lines show the result of the $1/N$ and RHA EOS, respectively.

Our results for $K(\beta)$ and $\rho_s(\beta)$ are shown in Fig. 2, and within the range $0.3 \leqslant Y_p \leqslant 0.5$ they can be approximated by setting $a_s = 0.91$ and $b_s = 0.79$. With the choice $\gamma = 2.7$ Eq. (3) reproduces adequately our $1/N$ results of Fig. 1 in the range $\rho_s \leqslant \rho_B \leqslant 3\rho_s$ and $0.2 \leqslant Y_p \leqslant 0.5$. The values for K_v, γ, a_s, and b_s are listed in Table II in comparison to the parameters employed by Baron *et al.* [10], which lead to a prompt supernova explosion of a $M = 15 M_\odot$ star, and to the ones derived by Bombacci *et al.* [26] from Brueckner-Hartree-Fock calculations based on the Paris potential.

We note that our value of a_s differs very much from the

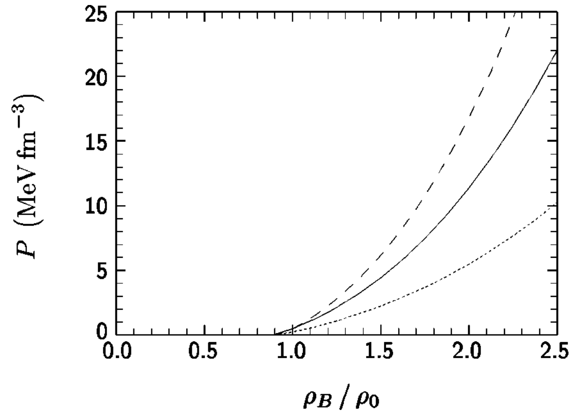

FIG. 3. The pressure according to the parametrization Eq. (3) using the $1/N$ (solid line), RHA (dashed line), and BCK (dotted line) parameters of Table II.

values of Refs. [10,26]. Roughly speaking, a large a_s gives a soft EOS if the other parameters are held fixed. In Fig. 3 we plot the pressure (3) according to the $1/N$, RHA, and BCK parameters listed in Table II for $Y_p = 0.33$, which is considered to be significant for supernova matter. We see that, due to the differences in K_v and a_s, our $1/N$ EOS is distinctly stiffer than the one of Baron *et al.* [10].

If the relations in Eq. (3b) are considered as the leading terms in an expansion around $\beta^2 = 0$, it follows from the definition of the symmetry energy a_4 that the coefficients a_s and b_s can be expressed in terms of derivatives of $a_4(\rho_B)$ and the third derivative of the binding energy per nucleon of symmetric matter, $E_B(\rho_B) \equiv \int d\rho_B P(\rho_B)/\rho_B^2$ [27,28],

$$a_s = -\frac{1}{K_v}\left[K_{\rm sym} + L\left(\frac{\tilde{K}}{K_v} - 6\right)\right] \equiv -\frac{K_{vs}}{K_v},$$

$$b_s = \frac{3L}{K_v}, \tag{4}$$

TABLE II. Parameters characterizing the four EOS's discussed in the main text. The values for a_s and b_s without parentheses are obtained from the expansion of E/A yielding Eqs. (4) and (5), and those in parentheses are obtained from a least squares fit of the numerical results to the expressions (3b) within the range $0.2 \leqslant Y_p \leqslant 0.5$. BCK refers to model 43 of Ref. [10] and BKL to Ref. [26]. The empirical values of $a_s = -K_{vs}/K_v$ and $a_c = -K_c/K_v$ are taken from Table 3 of Ref. [12].

	K_v (MeV)	γ	a_s	b_s	a_c	b_c
$1/N$	302	2.7	0.820 (0.91)	0.847 (0.79)	0.0111	0.00733
RHA	452	2.9	1.220 (1.23)	0.605 (0.60)	0.0129	0.00489
BCK	180	2.5	2.0	0.75		
BKL	185	2.5	2.03	1.12		
Empirical						
Set 1	150		-0.446 ± 0.67		-0.039 ± 0.014	
Set 2	200		0.235 ± 0.51		-0.013 ± 0.010	
Set 3	250		0.643 ± 0.40		0.0028 ± 0.008	
Set 4	300		0.915 ± 0.34		0.013 ± 0.007	
Set 5	350		1.109 ± 0.29		0.021 ± 0.006	

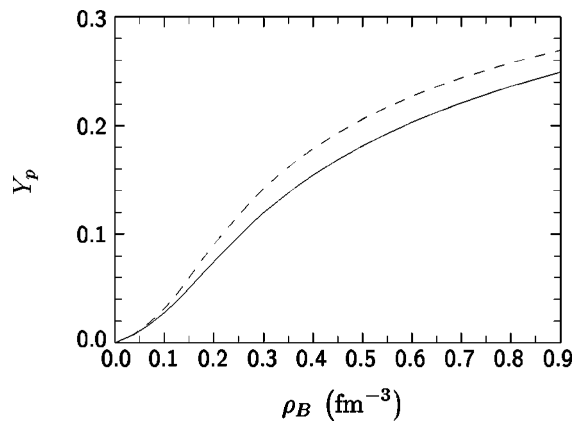

FIG. 4. Proton fraction in β equilibrium as a function of baryon density. The solid line is obtained from the $1/N$ and the dashed line from the RHA EOS.

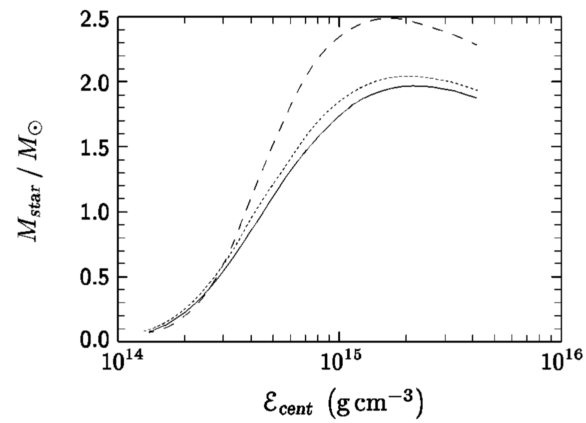

FIG. 5. Neutron star masses in units of solar mass against central mass density. The solid and dashed lines correspond to the $1/N$ and RHA EOS under condition of β equilibrium, respectively. The dotted line shows the $1/N$ result for pure neutron matter.

where $L=3\rho_0 a_4'$, $K_{\rm sym}=9\rho_0^2 a_4''$, $\tilde{K}=-27\rho_0^3 E_B'''$, and the primes indicate the derivatives with respect to ρ_B at the saturation density of symmetric matter ρ_0. Our values for a_s and b_s obtained from these expressions are also listed in Table II. They are very similar to the ones obtained above by a fit to the approximate expressions, Eq. (3b). Pearson [12] has extracted empirical values for K_{vs} for several input values of K_v by fits to the measured nuclear breathing mode energies.[2] His results are listed in the lower part of Table II. Our $1/N$ EOS has roughly the same K_v as that of set 4, and our value for a_s is in reasonable agreement with the corresponding empirical one.

It has been pointed out [12] that further useful information on the EOS can be obtained by considering also the Coulomb contribution to the incompressibility [29]. If one assumes nuclear matter to be confined within a radius $R=r_0 A^{1/3}$ with $r_0^{-3}=(4\pi/3)\rho_0$, one can derive the additional Coulomb terms $-a_c Z^2 A^{-4/3}$ and $-b_c Z^2 A^{-4/3}$, which should be added to the terms inside the parentheses on the right-hand side of Eq. (3b),

$$a_c=-\frac{1}{K_v}\frac{3\alpha}{5r_0}\left(\frac{\tilde{K}}{K_v}-8\right)\equiv-\frac{K_c}{K_v},\quad b_c=\frac{9\alpha}{5K_v r_0},\qquad (5)$$

with α=1/137. Since, for fixed K_v, a_c depends only on $\tilde{K}$, the empirically observed correlation between K_c and K_v (see Table II) implies a relation between the second and third derivatives (or K_v and $\tilde{K}$) of the binding energy. Our value for a_c listed in Table II is consistent with the empirical value of set 4; i.e., our EOS follows the K_v-$\tilde{K}$ relation observed in Ref. [12].

We note that one might discuss our results for the coefficients b_s and b_c listed in Table II by relating them to the recently determined root mean square matter radii of isotopes [30]. However, for this purpose a calculation for finite nuclei is preferable, and therefore we do not pursue this issue further here.

[2]Recent works in favor of the ‘‘microscopic approach’’ indicate that the correlations observed in Ref. [12] might be probably too tight due to the truncation of the ‘‘leptodermous expansion’’ to the leading terms (see also footnote 1). Therefore the comparisons presented in this section have mainly qualitative character.

B. Matter in β equilibrium

We now turn to the discussion of the β-equilibrium state, which is characterized by charge neutrality $(\rho_p=\rho_e+\rho_\mu)$ and the condition $\mu_e=\mu_\mu=\mu_n-\mu_p$. The leptons are treated as a Fermi gas. In Fig. 4 we show the resulting proton fraction as a function of the baryon density. According to Ref. [31], a proton fraction of more than 11–13 % is required to allow the kinematics of the direct Urca process, which leads to a fast cooling of the neutron star. In our calculation this criterion is satisfied for $\rho_B>0.27$ fm^{-3} or $M_{\rm star}>1.0M_\odot$ (see below). Since neutron star masses are observed at $1.4M_\odot$, one can assume a considerable contribution of the direct Urca process to the cooling of neutron stars.

The neutron star mass can be calculated as a function of the central mass density by integrating the Tolman-Oppenheimer-Volkoff equation [32]. The result is shown in Fig. 5 for the $1/N$ EOS in β equilibrium, in comparison to the pure neutron matter case and the RHA EOS in β equilibrium. The stiffer the EOS, the higher is the mass of the most massive stable star and the smaller is its central density. Since the softening of the EOS due to the proton admixture overcomes the increase of the pressure due to the leptons, the maximum star mass is decreased somewhat as compared to the pure neutron matter case. Nevertheless, the properties of the star, like the radius, density profile, and surface redshift ratio, are not changed drastically as compared to the neutron matter results of Ref. [17], where it was shown that they are consistent with observations.

IV. CONCLUDING REMARKS

In conclusion, we have shown that the EOS of isospin-asymmetric matter is softened considerably due to the relativistic exchange and correlation terms. Our EOS is in basic agreement with bulk properties of neutron stars and with empirical information derived from the ‘‘macroscopic approach’’ to the nuclear breathing modes, but is somewhat stiff compared to recent analyses based on the ‘‘microscopic approach.’’ Concerning the supernova problem, it is very likely that our EOS is too stiff to lead to an explosion according to the prompt shock mechanism. In this respect, it would be very interesting to use the more likely delayed shock mechanism [33,1] to derive detailed information on the stiffness of the nuclear EOS.

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