Effective Q-Q interactions in constituent quark models

L. Ya. Glozman,[1] Z. Papp,[2] W. Plessas,[1] K. Varga,[2,3] and R. F. Wagenbrunn[1]

[1]*Institute for Theoretical Physics, University of Graz, A-8010 Graz, Austria*
[2]*Institute of Nuclear Research, Hungarian Academy of Sciences, H-4001 Debrecen, Hungary*
[3]*Department of Physics, Niigata University, Niigata 950-21, Japan*
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We study the performance of some recent potential models suggested as effective interactions between constituent quarks. In particular, we address constituent quark models for baryons with hybrid Q-Q interactions stemming from one-gluon plus meson exchanges. Upon recalculating two of such models we find them to fail in describing the N and Δ spectra. Our calculations are based on accurate solutions of the three-quark systems in both a variational Schrödinger and a rigorous Faddeev approach. It is argued that hybrid Q-Q interactions with a sizable one-gluon-exchange component encounter difficulties in describing baryon spectra due to the specific contributions from one-gluon and meson exchanges together. In contrast, a chiral constituent quark model with a Q-Q interaction relying solely on Goldstone-boson exchange is capable of providing a unified description not only of the N and Δ spectra but also of all strange baryons in good agreement with phenomenology. [S0556-2813(98)01006-1]

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I. INTRODUCTION

Traditional constituent quark models (CQM) adopted one-gluon exchange (OGE) [1] as the hyperfine interaction between constituent quarks (Q). Over the years it has become evident that CQM relying solely on OGE Q-Q interactions face some intriguing problems in light-baryon spectroscopy [2,3]. Most severe are

(i) the wrong level orderings of positive- and negative-parity excitations in the N, Δ, Λ, and Σ spectra;

(ii) the missing flavor dependence of the Q-Q interaction necessary, e.g., for a simultaneous description of the correct level orderings in the N and Λ spectra; and

(iii) the strong spin-orbit splittings that are produced by the OGE interaction but not found in the empirical spectra.

All of these effects can be traced back to inadequate symmetry properties inherent in the OGE interaction.

Beyond CQM with OGE dynamics only, hybrid models advocating in addition meson-exchange Q-Q interactions have been suggested, see, e.g., Ref. [4]. With respect to the N and Δ spectra, especially π and σ exchanges have been introduced to supplement the interaction between constituent quarks. Sometimes hybrid CQM have also been employed in constructing baryon-baryon interaction models, see, e.g., Refs. [4,5].

Recently two groups, viz. Valcarce, González, Fernández, and Vento (VGFV) [6] and Dziembowski, Fabre, and Miller (DFM) [7] came up with versions of hybrid constituent quark models. They presented N and Δ excitation spectra and claimed to have achieved a reasonable description thereof. In both works a sizable contribution from the OGE interaction is maintained. In either case the hyperspherical-harmonics method (HHM) [8,9] with several simplifying assumptions was used to solve the nonrelativistic three-Q problem. We have studied the models of VGFV and DFM in a detailed manner. In particular, we have checked the pertinent results for the N and Δ spectra in two very reliable ways, namely,

by calculating the three-Q systems through

(i) a precise solution of the Faddeev equations (via high-rank separable expansions) [10], and

(ii) a stochastic variational solution of the Schrödinger equation [11].

Both of these methods have been extensively tested before not only for three-Q systems like, e.g., for the CQM models of Refs. [12–14] but also for nuclear and atomic three-body problems. Existing benchmark results in the literature have been met accurately, in particular for spin-isospin/flavor dependent forces, and even a wider class of two-body potentials [10,11]; for instance, the stochastic variational method has recently passed successful tests in three-N (and also four-N) bound-state calculations with realistic N-N potentials containing noncentral components of various spin-spin and tensor types [15]. While in all cases our results for the light-baryon spectra coincide with each other for both of the above methods (i) and (ii), they are found in striking disagreement with the results given in the papers of VGFV [6] and DFM [7]. For example, the N-Δ splitting is found to be more than 600 MeV with the DFM interaction and even bigger than 2000 MeV for the VGFV model.

In the following section we give a detailed analysis of the VGFV and DFM models and outline their shortcomings. We present the corrected spectra and explain in which respects the physical implications drawn from the original results in the works of VGFV and DFM cannot be relied on. In Sec. III we discuss a constituent quark model whose hyperfine Q-Q interaction is derived from Goldstone-boson exchange (GBE) alone. In this case the effective Q-Q interaction in light baryons is thus mediated by π, η, and η' exchanges [14]. Along a modified parametrization of the corresponding pseudoscalar meson-exchange potentials in a semirelativistic framework it is demonstrated that no OGE (or at most a very weak one) is needed to reproduce the N and Δ spectra in close agreement with phenomenology. Furthermore all the strange baryons can also be described in a unified manner by

the same model without any further (free) parameter [16]. In the final section we give a detailed discussion of the effects (of the specific components) of various Q-Q hyperfine interactions on light-baryon spectra. In particular we argue why in any CQM a strong color-magnetic (OGE) interaction is unfavorable to reproducing simultaneously the right ordering of positive- and negative-parity states and a correct N-Δ splitting.

II. CONSTITUENT QUARK MODELS WITH HYBRID Q-Q INTERACTIONS

Hybrid CQM advocate sizable contributions from OGE (typically with a strong-coupling constant $\alpha_s \gtrsim 0.3$) and employ in addition meson exchanges for the effective interaction between constituent quarks.

A. The model by Valcarce, González, Fernández, and Vento

In the model of VGFV [6] the following Q-Q potential is used:

$$V^{\text{VGFV}} = V_{\text{OGE}} + V_{\text{OPE}} + V_{\text{OSE}} + V_{\text{conf}}. \tag{1}$$

The OGE potential is employed without tensor and spin-orbit parts in the actual calculations [17], i.e., in the form

$$V_{\text{OGE}}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s \vec{\lambda}_i^C \cdot \vec{\lambda}_j^C \left[\frac{1}{r_{ij}} - \frac{1}{4m_Q^2}\left(1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\frac{e^{-r_{ij}/r_0}}{r_{ij}r_0^2}\right], \tag{2}$$

where $\vec{\sigma}_i$ and $\vec{\lambda}_i^C$ are the quark spin and color matrices, r_{ij} is the interquark distance, and m_Q is the constituent quark mass; VGFV use $m_Q = 313$ MeV. α_s is the effective quark-gluon coupling constant and r_0 is a parameter involved in the smearing of the δ function. The one-pion-exchange (OPE) potential V_{OPE} is used in the form

$$V_{\text{OPE}}(\vec{r}_{ij}) = \frac{g_{\pi Q}^2}{4\pi}\frac{1}{12m_Q^2}\vec{\tau}_i \cdot \vec{\tau}_j \frac{\Lambda^2}{\Lambda^2 - \mu_\pi^2}$$

$$\times \left\{\left(\mu_\pi^2 \frac{e^{-\mu_\pi r_{ij}}}{r_{ij}} - \Lambda^2 \frac{e^{-\Lambda r_{ij}}}{r_{ij}}\right)\vec{\sigma}_i \cdot \vec{\sigma}_j\right.$$

$$+ \left[\left(1 + \frac{3}{\mu_\pi r_{ij}} + \frac{3}{\mu_\pi^2 r_{ij}^2}\right)\mu_\pi^2 \frac{e^{-\mu_\pi r_{ij}}}{r_{ij}}\right.$$

$$\left.\left.- \left(1 + \frac{3}{\Lambda r_{ij}} + \frac{3}{\Lambda^2 r_{ij}^2}\right)\Lambda^2 \frac{e^{-\Lambda r_{ij}}}{r_{ij}}\right]\hat{S}_{ij}\right\}, \tag{3}$$

where $g_{\pi Q}^2/4\pi$ is the effective pion-quark coupling constant, $\mu_\pi = 0.7$ fm^{-1} is the pion mass, and Λ is a parameter related to the extended πQ vertex. $\hat{S}_{ij}$ is the tensor operator $\hat{S}_{ij} = 3(\vec{\sigma}_i \cdot \hat{r}_{ij})(\vec{\sigma}_j \cdot \hat{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$. The one-sigma-exchange (OSE) potential V_{OSE} takes simply the form

$$V_{\text{OSE}}(\vec{r}_{ij}) = -\frac{g_{\pi Q}^2}{4\pi}\frac{\Lambda^2}{\Lambda^2 - \mu_\sigma^2}\left(\frac{e^{-\mu_\sigma r_{ij}}}{r_{ij}} - \frac{e^{-\Lambda r_{ij}}}{r_{ij}}\right) \tag{4}$$

TABLE I. Numerical values of fitting parameters of the VGFV Q-Q interaction. Three of the parameters, namely, α_s, $g_{\pi Q}^2/4\pi$, and Λ were taken from the hybrid N-N interaction model in Ref. [18].

α_s	$g_{\pi Q}^2/4\pi$	C (fm^{-2})	V_0 (MeV)	r_0 (fm)	Λ (fm^{-1})
0.485	0.545	0.980 [a]	+488 [b]	0.0367	4.2

[a]There is a misprint in the value of the confinement strength in Ref. [6]. The correct value given here was communicated to us by the Salamanca-Valencia group [19].
[b]The constant V_0 in the confinement potential, whose value is not quoted in Ref. [6], must be taken of this magnitude in order to reproduce the nucleon ground state at the correct energy in a converged calculation.

with the σ mass $\mu_\sigma = 3.42$ fm^{-1}. Finally the confinement potential is assumed in linear form

$$V_{\text{conf}}(\vec{r}_{ij}) = V_0 + Cr_{ij} \tag{5}$$

with the strength $C = 0.980$ fm^{-2} and a constant V_0 necessary to adjust the nucleon ground-state level to its phenomenological value. The whole Q-Q potential (1) involves six fitting parameters (α_s, $g_{\pi Q}$, r_0, Λ, C, V_0) if one considers the constituent-quark and meson masses as predetermined; the actual values of the fitting parameters are summarized in Table I.

VGFV solved the three-Q system along the Schrödinger equation with the HHM [8,9]. However, only a very restricted basis set was used for the expansion of the three-Q wave functions (two terms $K = 0,2$ for positive-parity states and one term $K = 1$ for negative-parity states). VGFV published the results shown in Fig. 1(a).

We have recalculated the N and Δ spectra with the VGFV potential in Eq. (1) along both of our methods, i.e., (i) the precise solution of the Faddeev equations (via high-rank separable expansions) and (ii) the stochastic variational solution of the Schrödinger equation (as explained in the previous section; for details see also Refs. [10,11]). The results from both calculations agree perfectly with each other and they are shown in Fig. 1(b).

From the comparison of the spectra in Figs. 1(a) and 1(b), which have both been normalized to reproduce the nucleon ground state at 939 MeV (in case of our calculations the constant V_0 in the confinement potential (5) gets thereby modified to $V_0 = +488$ MeV), several differences are evident. Most striking is the unrealistic N-Δ splitting, which turns out to be larger than 2000 MeV; notice that the Δ ground state has a mass of ≈ 3200 MeV. Also some of the N^* levels, especially in the negative-parity $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$ bands, show excitations above the N ground state by far too large. Furthermore, additional levels (not given in Ref. [6]) appear. For instance, in the $\frac{1}{2}^-$ band we reproduce the lowest state, which is predominantly $S = \frac{1}{2}$, $L = 1$, at a mass of ≈ 1550 MeV. The next excitations, whose spin contents are still $S = \frac{1}{2}$, $L = 1$ (with probabilities $\approx 99\%$), are found between 2000 and 2500 MeV. Only much further above, at ≈ 3500 MeV, the $S = \frac{3}{2}$, $L = 1$ excitation [marked by a dagger in Fig. 1(b)] appears. This is the state to be compared with the second $\frac{1}{2}^-$ excitation (also marked by a dagger) in the spectrum given by VGFV in Fig. 1(a). Exactly the same
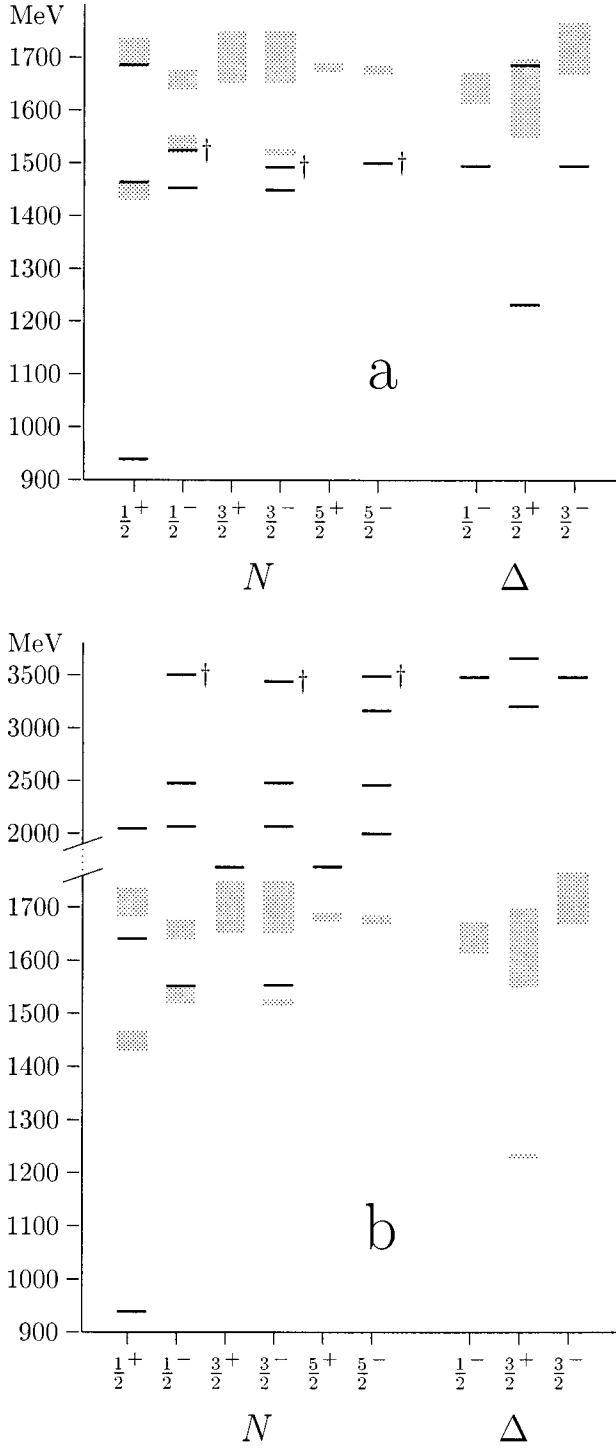
FIG. 1. N and Δ spectra in case of the model VGFV: (a) as given in Ref. [6], (b) recalculated. In both cases the nucleon mass is "normalized" to its phenomenological value of 939 MeV. In the recalculated spectra the N-Δ mass difference is 2270 MeV. The negative-parity states marked by a dagger correspond to the spin content of predominantly $S = \frac{3}{2}$.

structure is recovered in the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ bands, with even an additional $S = \frac{1}{2}$, $L = 3$ state appearing in the latter. In Ref. [6] also the prediction for the Roper resonance $N(1440)$ is given with a wrong energy. In fact, it lies above 1600 MeV and thus also above the first negative-parity excitation [see Fig. 1(b)]. Consequently the problem with the wrong level order-

ings of positive- and negative-parity states persists, contrary to the claim made by VGFV.

We conclude that the HHM as used by VGFV is inadequate to reproduce the correct spectra in the case of the Q-Q potential (1)–(5) with the parameters chosen in Ref. [6]. The most important reason is that for certain spin-isospin channels, specifically for $S_{ij} = T_{ij} = 0$, the potentials (2) and (3) become very deep at rather short distances r_{ij}. This is especially due to the OGE potential whose δ-function term in the central and spin-spin parts involves an extremely small parameter r_0 (see Table I). If also the OGE tensor part as given in formula (2) of Ref. [6] was employed, the three-Q spectrum would even not be bounded from below. In dealing with the OGE potential as in Eq. (2) above (with such a small r_0 as employed by VGFV) great care must be exerted in the expansion of test functions. While the actual calculation is not so dangerous in the Δ case (here, $S_{ij} = T_{ij} = 1$, thus the OGE is repulsive and the OPE is attractive at short ranges), the N case is rather tricky to compute (here, $S_{ij} = T_{ij} = 0$, and both the OGE and OPE are extremely attractive). In this situation the HHM result with only a few (two) basis functions is simply not converged. If a sufficiently rich basis is employed, the N ground state (as well as some of its excitations) falls down by more than 2000 MeV and additional levels of spin content $S = \frac{1}{2}$ are found below the $S = \frac{3}{2}$ states in the $J^P = \frac{1}{2}^-$, $J^P = \frac{3}{2}^-$, and $J^P = \frac{5}{2}^-$ bands. The convergence of the energies to the values given in Fig. 1(b) can be followed in a transparent manner especially in the case of our variational solution of the Schrödinger equation, when gradually increasing the test-function space. From the fully converged results of the stochastic variational method, which then perfectly agree with the Faddeev solutions, one must finally conclude that the VGFV model, with the parameter set in Table I, fails in describing the N and Δ spectra.

B. The model by Dziembowski, Fabre, and Miller

In the DFM model the effective Q-Q interaction (called "complete Hamiltonian" in Ref. [7]) consists of

$$V^{\text{DFM}} = V_{\text{OGE}} + V_\chi + V_{\text{conf}}. \tag{6}$$

The OGE potential [1] was taken in the usual form but again without spin-orbit and tensor parts—just as in the case of the VGFV model in Eq. (2). Also a linear confinement as in Eq. (5) was used. The potential V_χ relating to broken chiral symmetry was assumed after the suggestion of GBE in Ref. [2]. In the actual calculation, DFM took the central part of the pseudoscalar-octet meson-exchange potential (i.e., for π and η) whose radial dependence was parametrized as

$$V_\chi(\vec{r}_{ij}) = \frac{g_{\pi Q}^2}{4\pi} \frac{1}{12 m_Q^2} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F$$

$$\times \left(\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda^2 \frac{e^{-\Lambda r_{ij}}}{r_{ij}} \right), \quad \gamma = \pi, \eta. \tag{7}$$

Here, $\vec{\lambda}_i^F$ are the Gell-Mann flavor matrices, $m_Q = 336$ MeV is the constituent quark mass, μ_γ are the meson masses, and $\Lambda = 1/r_0$ is a parameter, again from the smearing of the δ function. Contrary to VGFV, in the DFM model no σ ex-

TABLE II. Numerical values of fitting parameters of the DFM Q-Q interaction. The cutoff parameter Λ in the meson-exchange potentials (7) was chosen to be the same as in the OGE part, i.e., $\Lambda = 1/r_0$.

α_s	$g^2_{\pi Q}/4\pi$	C (fm^{-2})	V_0 (MeV)	r_0 (fm)
0.35	1.15	1.014	-86 [a]	0.238

[a]The constant V_0 in the confinement potential, whose value is not quoted in Ref. [7], must be taken of this magnitude in order to reproduce the nucleon ground state at the correct energy in a converged calculation.

change was employed. Indeed, the effect of this spin- and flavor-independent scalar interaction can always be incorporated effectively in the confinement potential (5). The fitting parameters of the DFM Q-Q interaction are given in Table II. The DFM group solved the three-Q system also along the Schrödinger equation with the HHM [8,9]. However, they used an even more restricted basis than VGFV, employing only a single state for both positive- as well as negative-parity states and leaving out mixed-symmetry spin-flavor configurations. In case of their ''complete Hamiltonian'' (i.e., the hybrid model with OGE and one-meson exchange and the parameter set of Table II) they obtained the results shown in Fig. 2(a).

We have recalculated the DFM model again by both the stochastic variational and Faddeev methods (as described in the previous sections) and found the N and Δ spectra as shown in Fig. 2(b). For adjusting the N ground state to its empirical value of 939 MeV in a converged calculation one has to take a constant $V_0 = -86$ MeV in the confinement potential (5). Considerable differences are evident when comparing Figs. 2(a) and 2(b). Again, most striking is the huge N-Δ splitting, which turns out to be more than two times larger than in reality. In Fig. 2(b) the whole Δ spectrum is shifted up by almost 400 MeV as compared to Fig. 2(a); in fact, the nucleon has fallen down by this amount of energy in the converged calculation. In the nucleon spectrum the discrepancy for the $\frac{5}{2}^-$ excitation is about 300 MeV, and the differences in the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ resonances are about 100 MeV. In the first excited states of the $J^P = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-$ nucleon bands the discrepancies are still between 20 and 50 MeV. The higher excitations in both the positive- and negative-parity bands show unrealistic splittings from the N ground state ($\gtrsim 900$ MeV). Similarly to the VGFV model, additional states not given in the work by DFM appear in the $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ bands: the $S = \frac{3}{2}$, $L = 1$ states are found only above a second $S = \frac{1}{2}$, $L = 1$ excitation. From the recalculated spectra it is also evident that in the DFM model the wrong level orderings of positive- and negative-parity excitations persist. For example, the Roper resonance $N(1440)$ lies above the first negative-parity excitations $N(1535)$-$N(1520)$ in the corrected spectrum of Fig. 2(b).

Though the DFM potential (6), being less deep than the VGFV potential at short distances (due to the larger value of r_0), is much easier to compute than the Q-Q interaction of VGFV, the HHM still was not applied on a large enough basis in Ref. [7]. As a result the N level (and some of its excitations) were not converged. If in our stochastic variational calculation we restrict the test-function space in a
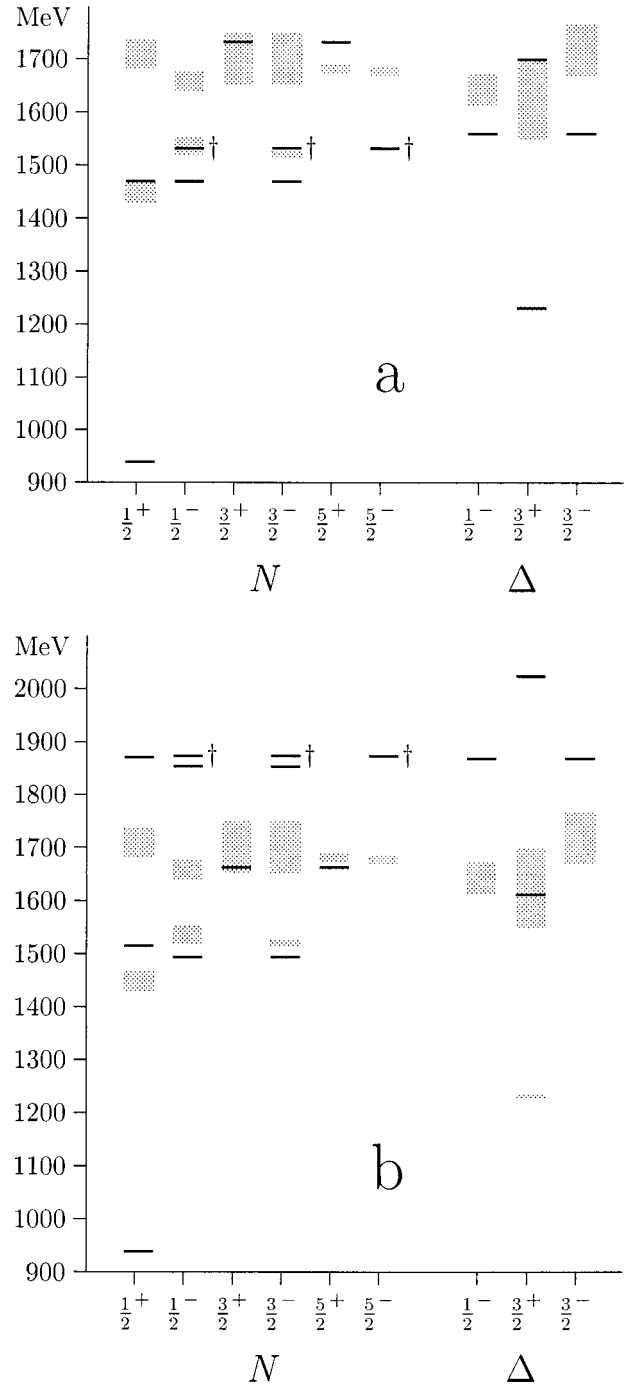

FIG. 2. N and Δ spectra in case of the model DFM: (a) as given in Ref. [7], (b) recalculated. In both cases the nucleon mass is ''normalized'' to its phenomenological value of 939 MeV. In the recalculated spectra the N-Δ mass difference is 670 MeV. The negative-parity states marked by a dagger correspond to the spin content $S = \frac{3}{2}$.

similar manner as DFM, we can roughly reproduce their results given in Ref. [7]. However, as it becomes evident from the fully converged results in Fig. 2(b), the DFM model, with the parameter set in Table II, is not capable of describing the N and Δ spectra.

III. THE GBE CONSTITUENT QUARK MODEL

In this section we discuss a constituent quark model whose hyperfine Q-Q interaction relies solely on GBE. We

like to demonstrate that in this case a correct description of the N and Δ spectra, and in addition of all strange spectra [16], can be achieved without the need of advocating OGE at all. We construct our CQM in a semirelativistic framework [20], i.e., taking the kinetic-energy operator in relativistic form

$$H_0 = \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2}, \qquad (8)$$

with m_i the masses and $\vec{p}_i$ the three-momenta of the constituent quarks. It is obvious that this intermediate step towards a fully covariant treatment of the three-Q system (which is still beyond scope at present) already constitutes an essential improvement over nonrelativistic approaches. Thereby one can at least avoid several disturbing shortcomings of nonrelativistic CQM. For example, if one calculates the expectation value of the kinetic energy in nonrelativistic CQM, such as the hybrid models of the previous section or the model of Ref. [14], one finds that the ratio of the mean velocity v of a constituent quark to the velocity c of light is $v/c > 1$. This clearly indicates that three-Q systems must not be treated in a nonrelativistic manner. By the use of the relativistic kinetic-energy operator in Eq. (8) one avoids such problems *a priori* and $v/c < 1$ is always ensured.

In the GBE constituent quark model, the dynamical part of the Hamiltonian consists of the Q-Q interaction

$$V = V_\chi + V_{\text{conf}}, \qquad (9)$$

where V_{conf} is taken in the usual linear form of Eq. (5) and V_χ is derived from GBE. The latter leads to a spin- and flavor-dependent interaction between constituent quarks i and j, whose spin-spin component is manifested by the sum of the pseudoscalar meson-exchange potentials [2,14]

$$V_\chi^{\text{octet}}(\vec{r}_{ij}) = \left[\sum_{a=1}^{3} V_\pi(\vec{r}_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^{7} V_K(\vec{r}_{ij}) \lambda_i^a \lambda_j^a \right.$$
$$\left. + V_\eta(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad (10)$$

and

$$V_\chi^{\text{singlet}}(\vec{r}_{ij}) = \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j V_{\eta'}(\vec{r}_{ij}), \qquad (11)$$

with $\vec{\sigma}_i$ and $\vec{\lambda}_i$ the quark spin and flavor matrices, respectively. In the simplest derivation, when pseudoscalar or pseudovector couplings are employed in the meson-quark vertices and the boson fields satisfy the linear Klein-Gordon equation, one obtains, in static approximation, the well-known meson-exchange potentials

$$V_\gamma(\vec{r}_{ij}) = \frac{g_8^2}{4\pi} \frac{1}{12 m_i m_j} \left\{ \mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\},$$
$$\gamma = \pi, K, \eta, \qquad (12a)$$

$$V_{\eta'}(\vec{r}_{ij}) = \frac{g_0^2}{4\pi} \frac{1}{12 m_i m_j} \left\{ \mu_{\eta'}^2 \frac{e^{-\mu_{\eta'} r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\},$$
$$(12b)$$

with μ_γ being the meson masses ($\gamma = \pi, K, \eta, \eta'$). While the octet of pseudoscalar mesons may be considered as the manifestation of the Goldstone bosons, the singlet η' is *a priori* no Goldstone boson due to the axial anomaly. However, in the large-N_C limit, the axial anomaly would vanish [21] and the η' become the ninth Goldstone boson of the spontaneously broken $U(3)_R \times U(3)_L$ chiral symmetry [22]. Under real conditions with $N_C = 3$, we may thus consider the contribution from η' exchange but with a coupling different from the octet mesons (see the discussion later on). The potentials (12) are strictly applicable only for pointlike particles. Since one deals with structured particles (constituent quarks and pseudoscalar mesons) of finite extension, one must smear out the δ function. The corresponding term turns out to be of crucial importance for the effective Q-Q interaction in baryons: it has a sign appropriate to reproduce the level splittings and it dominates over the Yukawa part towards shorter distances.

There are several choices of smearing the δ function in Eqs. (12). A reasonable constraint is that the volume integral of the potential should vanish—as it is required for pseudoscalar exchange interactions with finite-mass bosons (mesons); for pure Goldstone bosons of vanishing mass this constraint would not apply, however. In Ref. [14] we have employed the Gaussian-type smearing of the δ function

$$4\pi \delta(\vec{r}_{ij}) \rightarrow \frac{4}{\sqrt{\pi}} \alpha^3 e^{-\alpha^2 (r_{ij} - r_0)^2}, \qquad (13)$$

which involves two parameters corresponding to the position (r_0) and the breadth (α) of a bell-shaped curve. In this case the volume integral of the chiral potential V_χ does not vanish.

We have found [23] that a Yukawa-type smearing like

$$4\pi \delta(\vec{r}_{ij}) \rightarrow \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}}, \quad \gamma = \pi, K, \eta, \eta' \qquad (14)$$

involving the ''cutoff'' parameters Λ_γ, works equally well for the effective Q-Q interaction in baryon spectra. With the δ-function representation (14) the volume integral of V_χ vanishes, thus meeting the requirement mentioned above.

The δ-function representation (14) relates to the finite extension of the meson-quark vertices. If we employ the phenomenological values for the different meson masses μ_γ, we should also allow for a different cutoff parameter Λ_γ corresponding to each meson exchange: with a larger meson mass also Λ should increase. Otherwise, the meson-exchange interactions (12) could receive unwarranted contributions from certain meson exchanges (e.g., it could become effectively attractive instead of repulsive or vice versa at short distances). In our attempt to keep the number of free parameters as small as possible, we have avoided fitting each individual cutoff parameter Λ_γ, however. Instead, we have succeeded in describing their dependence on the meson mass via the linear scaling prescription

$$\Lambda_\gamma = \Lambda_0 + \kappa \mu_\gamma, \tag{15}$$

which involves only the two parameters Λ_0 and κ. Their numerical values have been determined from a fit to the baryon spectra.

In Eq. (12a) we have foreseen a single octet meson-quark coupling constant $g_8^2/4\pi$. Indeed, in the chiral limit there is only one coupling constant $g_8^2/4\pi$ for all Goldstone bosons. Due to explicit chiral symmetry breaking the coupling constants for π, K, and η may become different. Nevertheless, due to the present lack of firm insight and in order to prevent a proliferation of free parameters, we rely at this stage on a universal coupling constant $g_8^2/4\pi$, which we assume to be equal to the pion-quark coupling constant $g_{\pi Q}^2/4\pi$. The latter may be deduced from the pion-nucleon coupling using the Goldberger-Treiman relations for both the pion-quark and pion-nucleon vertices:

$$\frac{g_8^2}{4\pi} = \frac{g_{\pi Q}^2}{4\pi} = \left(\frac{g_Q^A}{g_N^A}\right)^2 \left(\frac{m_Q}{m_N}\right)^2 \frac{g_{\pi N}^2}{4\pi}. \tag{16}$$

Here, g_Q^A and g_N^A are the quark and nucleon axial coupling constants, m_Q and m_N the light-quark, respectively, nucleon masses, and $g_{\pi N}^2/4\pi = 14.2$, the phenomenological πN coupling constant [24]. The nucleon and quark axial coupling constants can be related by the ratio

$$\frac{g_Q^A}{g_N^A} = \frac{3}{5} = 0.6. \tag{17}$$

If one takes a constituent-quark mass of $m_Q = 340$ MeV (as it is suggested from nucleon magnetic-moment studies) one can deduce the πQ coupling constant of the size $g_{\pi Q}^2/4\pi = 0.67$. Thus we fix the octet meson-quark coupling to this value, as it was done already in Ref. [14].

We remark, however, that the πQ coupling constant could also assume a value slightly different from 0.67. For instance, if one takes for the Q axial coupling constant $g_Q^A = 1$ (as it should be in the large-N_C limit [25]) and employs for the N axial coupling constant its phenomenological value [24] $g_N^A = 1.25$, then one obtains the ratio

$$\frac{g_Q^A}{g_N^A} = 0.8. \tag{18}$$

Thus one would end up with a pion-quark coupling constant $g_{\pi Q}^2/4\pi = 1.19$. If one took into account that $1/N_C$ corrections would reduce the Q axial coupling constant to $g_Q^A = 0.87$ [26], then

$$\frac{g_Q^A}{g_N^A} = 0.7, \tag{19}$$

and consequently $g_{\pi Q}/4\pi = 0.91$. Therefore we may ultimately expect the πQ coupling constant within the interval $0.67 \lesssim g_{\pi Q}^2/4\pi \lesssim 1.19$ and this is even dependent on the value of the constituent-quark mass adopted. Still, at this stage, we

TABLE III. Numerical values of fitting parameters of the semirelativistic GBE Q-Q interaction. The octet meson-quark coupling constant was fixed to $g_8^2/4\pi = 0.67$.

$(g_0/g_8)^2$	C (fm^{-2})	V_0 (MeV)	Λ_0 (fm^{-1})	κ
1.34	2.33	-416	2.87	0.81

need not vary the size of the octet meson-quark coupling constant and simply fix it to the ''canonical'' value of ≈ 0.67.

Due to the special character of the singlet η' meson (see the discussion above), its coupling was allowed to deviate from the universal octet coupling. In the actual fit, the ratio $(g_0/g_8)^2$ turns out larger than 1. It is comfortable to see that this is in line with results deduced in Refs. [27,28] from the Gottfried sum-rule violation, the $\bar{u}/\bar{d}$ ratio, as well as the spin content of the nucleon.

In the semirelativistic GBE model we have a confinement strength [of the linear confining potential, Eq. (5)] of the order of $C \approx 2.3$ fm^{-2}. We note that this value appears to be quite realistic, as it is consistent both with Regge slopes and also the string tension extracted in lattice QCD [29]. The size of the confinement strength required in our semirelativistic GBE model represents another satisfying improvement over nonrelativistic CQM where a much smaller value of C must be chosen (see the examples of the previous section or Refs. [14,23]).

For the whole Q-Q potential in Eq. (9) our model now involves a total of five free parameters whose numerical values are given in Table III. We remark, however, that this is only one fit out of a possible set of others that lead to a similar quality in the description of the baryon spectra. For instance, we have found that one could increase the octet coupling constant $g_8^2/4\pi$ and compensate the change by readjusting the other parameters, particularly the cutoffs Λ_γ. Similarly, the ratio $(g_0/g_8)^2$ could be lowered, and even the constituent-quark masses may be chosen differently. Retuning the other parameters, including the confinement strength C, one can within certain limits always obtain a fit of comparable quality.

From the results shown below and the subsequent discussion it will become clear that the decisive advantage of the GBE model lies in the symmetry structure that comes with the spin-flavor operators in the chiral potentials (10) and (11). When these operators are evaluated between spin-flavor wave functions of the baryonic states, they produce factors with signs and magnitudes [2,3] that appear just adequate to produce the fine-structure splittings as observed in experiment. The gross features of the spectra are already provided even if one does not adhere to a particular parametrization of the spatial dependence of the chiral potential, as it was done in Ref. [2].

The GBE model with the semirelativistic three-Q Hamiltonian relying on Eqs. (8) and (9) was solved along the stochastical variational method [11] in momentum space. The convergence was carefully tested, especially with respect to the N ground state and the ''dangerous'' levels in the excitation spectra.

In Fig. 3 we present the ground-state and excitation levels of the N and Δ spectra as produced by the semirelativistic
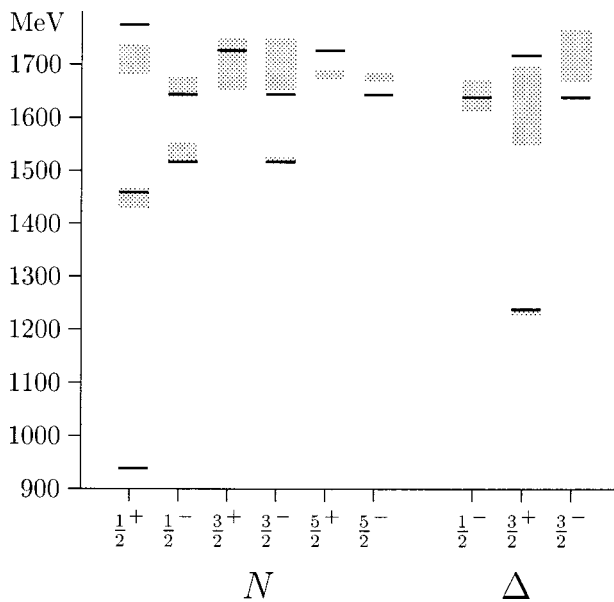
FIG. 3. Energy levels of the 14 lowest light-baryon states with total angular momentum and parity J^P in case of the semirelativistic GBE constituent quark model with parameters as given in Table III. The shadowed boxes represent the experimental values with their uncertainties; the N and Δ ground-state levels coincide with the experimental values.
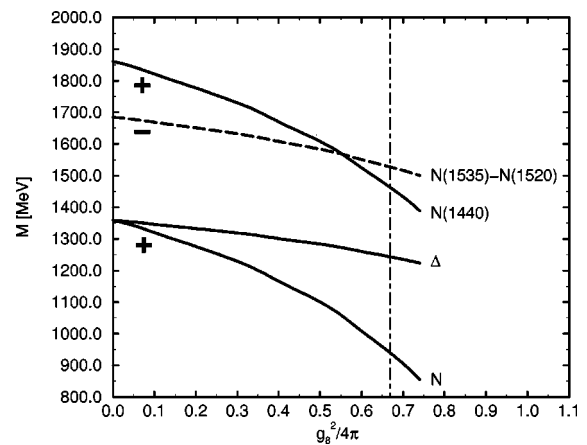

FIG. 4. Level shifts as a function of the strength of the Goldstone-boson exchange interaction. Solid and dashed lines correspond to positive- and negative-parity states, respectively.

GBE constituent quark model with the Q-Q potential from Eqs. (9)–(12), the Yukawa-type representation (14) of the δ function, and the parameter set given in Table III. The light-Q masses were taken as $m_u = m_d = m_Q = 340$ MeV, and for the meson masses μ_γ the phenomenological values were employed.

Tensor force effects are not yet included in these results, as is evident from the absence of any fine-structure splittings in the theoretical LS multiplets. It is clear, however, that tensor forces can at most play a subordinate role in the N and Δ spectra. First of all, this follows from the smallness of the level splittings of corresponding LS multiplets in the experimental spectra. Second, from a first numerical estimation of the influence of the pseudoscalar meson-exchange tensor forces prevailing in the GBE model we have found only small effects on the baryon states.

From the results of Fig. 3 it becomes evident that within the GBE constituent quark model an improved description of both the N and Δ spectra is achieved in good agreement with phenomenology. Practically all levels are found in their "experimental boxes" and specifically the orderings of positive- and negative-parity excitations are correct.

IV. DISCUSSION

As already emphasized before and explained in detail in previous works [2,3,14], the successes of the GBE constituent quark model are due to the specific spin-flavor symmetry inherent in the chiral potential V_χ in Eqs. (10) and (11). It allows for a level structure called for by the experimental data. This is not only true in the N and Δ cases but holds also for strange baryons [23]. All octet/decuplet states together with their excitations can now be described in a unified manner [16]; the only exception is the empirically large

$\Lambda(1405)$-$\Lambda(1520)$ splitting, which remains unexplained at this stage.

From the results of the GBE constituent quark model exemplified in the previous section we also learn that *a priori* no (additional) OGE is needed for the reproduction of the baryon spectra, contrary to the claims made for the hybrid models, e.g., in Refs. [6] and [7]. Still, one could accommodate a certain contribution from OGE but only with a rather small effect from the color-magnetic interaction, i.e., with a strong-coupling constant $\alpha_s < 0.3$, say. As soon as the color-magnetic forces become strong, like it is the case in the hybrid models discussed in Sec. II (cf. their values for α_s in Tables I and II), one faces the traditional difficulties in the light-baryon spectra. This can be understood from a closer inspection of the effects of the different types of hyperfine interactions relating to OGE and GBE dynamics, especially with regard to the N-Δ splitting and the correct ordering of positive- and negative-parity states, the $\frac{1}{2}^+$ $N(1440)$ and the $\frac{1}{2}^-$-$\frac{3}{2}^-$ $N(1535)$-$N(1520)$ resonances, in particular. While the N-Δ splitting can be produced by either the color-magnetic or the π-exchange part of the GBE potential, the inversion of the positive- and negative-parity states definitely needs a certain strength of the quark-meson coupling. In Fig. 4 we have demonstrated how the level crossing comes about when the octet coupling constant is gradually increased.

If in a meson-exchange plus OGE interaction with a sizable contribution from the color-magnetic interaction ($\alpha_s > 0.3$, say) the N-Δ splitting, to which both types the OGE and π exchanges contribute in the same manner, is to be described correctly, the π exchange must remain weaker, as compared to the pure GBE model. As a result the proper ordering of positive- and negative-parity levels is not achieved. Only a bigger influence from the π exchange would allow the first positive-parity excitations in the N spectrum to fall below the first negative-parity excitations. If, on the other hand, the π exchange is made strong enough to achieve this level inversion, as demanded by phenomenology, the color-magnetic contribution must remain very small, practically negligible. Indeed, with a reasonable size of the octet coupling constant (see the discussion in the previous section), the π exchange alone produces an N-Δ splitting

that is already too large. It gets reduced to the correct magnitude by the addition of the η and η' exchanges. These force components produce effects in the N-Δ splitting opposite to the π exchange or the color-magnetic interaction but have only minor influences on the relative position of the first positive- and negative-parity nucleon excitations. Once the octet coupling constant is fixed and the cutoff parameters Λ are tuned to reasonable values (cf. our scaling law in the previous section and the discussion given there), the η exchange is also completely fixed and a large N-Δ splitting can only be reduced by increasing the influence from η' exchange. Consequently a major contribution of a color-magnetic (OGE) force, on top of the required strength of the octet pseudoscalar meson exchange, could only be compensated by increasing the singlet coupling constant to a larger value. While the magnitude we find for the η' coupling is already bigger than the octet coupling, but in line with the value reported in the literature from other sources [27,28], a further increase is certainly not advised. From all that it follows quite naturally that the color-magnetic interaction can at most play a subordinate role in CQM for light baryons. A number of additional arguments deduced from the level structure of the strange baryons further support this qualification of the flavor-independent OGE interaction [2,3,16].

Finally we note that a nonrelativistic treatment of three-Q systems cannot really work. Evidently, light (and strange) three-Q systems already bear large relativistic effects of kinematical origin. They call at least for the kinetic energy part of the full Hamiltonian to be employed in relativistic form, as was done for the GBE model in Sec. III. Otherwise the lack of taking into account (kinematical) relativistic effects explicitly will be compensated within the Q-Q potential parameters in the fit to the baryon spectra. Consequently, in this procedure the model parameters will assume rather unrealistic values, e.g., the confinement strength would turn out to be only half of the one resulting for the string tension from lattice QCD and the one needed to fit Regge slopes. As a result, any nonrelativistic CQM can at most be considered as a parametrization of the baryon energy levels, rather than as a dynamical model for light three-Q systems. Certainly it will not prove acceptable for future applications such as the description of electromagnetic form factors, hadronic decays, and other dynamical observables that are determined by the behavior of the baryon wave functions and are generally much influenced by Q-Q potential parameters.

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