Statistical calculations of the damping width of giant resonances

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The damping widths of giant resonances is calculated using a scheme developed for multistep compound processes. The results were found to be reasonable and compare well to those indirectly extracted from data. [S0556-2813(98)01205-9]

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I. INTRODUCTION

The calculation of the widths of giant multipole resonances (GR) in nuclei has received much attention since the pioneering work of Brown and Bolsterli [1]. Microscopically, one relies on the random phase approximation (RPA) or a more refined version of it. To allow us to obtain the escape width $\Gamma^{\uparrow}$ the particles in the RPA are allowed to be in the continuum with the mean field here replaced by the appropriate real part of the optical model potential. This method is referred to as the continuum RPA (CRPA). In light nuclei, such as ^{16}O, the escape width is a large function of the total width of the GR. However, in heavier nuclei the escape width is but a small fraction of the total width owing to the large number of noncollective states in which the GR is embedded and coupled to. The difference $\Gamma_{GR}-\Gamma_{GR}^{\uparrow}$ is the damping width $\Gamma_{GR}^{\downarrow}$. Clearly in such cases one needs to have ways to calculate $\Gamma^{\downarrow}$.

There are two ways in which such a calculation can be executed. One involves the enlargement of the space of the RPA to include $2p-2h$ degrees of freedom. The resulting $1p-1h+2p-2h$ RPA is called the second RPA (SRPA). The calculation becomes formidable since the dimension of the $2p-2h$ subspace is usually large and one relies on approximations [2]. The other way, that is not used much, is a natural extension of the CRPA. Here one uses for the continuum particles the full complex optical model and allows the holes to have complex energies. The complexity of the interactions simulates the coupling both to the continuum and to the more complicated configurations such as $2p-2h$, $3p-3h$, etc. The latter is the origin of the damping of the collective state. Results obtained using this method are comparable in quality to those of the SRPA [3].

In all of the above, the major emphasis is the response function which supplies a measure of the excitation cross section through its imaginary part; the strength function. It is necessary, though, to emphasize that the GR eventually decays into the different available channels, and whatever input one uses in the formation process should somehow manifest itself in the decay process. Our group here in São Paulo has for several years studied this question and devised models that could allow one to obtain useful structure information from the analysis of the decay modes of the GR [4–6]. In particular, quite recently, we have presented evidence for preequilibrium contribution in the proton decay of the giant quadrupole resonance in ^{40}Ca [7], excited in a ^{40}Ca+^{40}Ca reaction at 50 MeV/A. By preequilibrium, we mean emission of protons from the $2p-2h$ configurations whose coupling to the collective $1p-1h$ giant quadrupole resonance supplies a reasonable measure of the latter's damping width. If observed in other systems, the preequilibrium decay of GR's should supply a means to extract information about the damping width. Thus it is important to develop a practical method for its calculation.

The purpose of this paper is to present a detailed discussion of the calculation of the damping width of the GR and supply a reaction theory approach to its extraction from the data. In Sec. II we present a brief account of the excitation and decay theory of the GR and discuss in some detail the preequilibrium component of the decay process. In Sec. III we supply a mean to calculate the damping width within the statistical multistep compound reaction theory [8,9]. Finally in Sec. IV we perform the calculation for ^{40}Ca, ^{90}Zr , and ^{208}Pb both for the dipole and quadrupole mode, and present some concluding remarks.

II. EXCITATION AND DECAY THEORY OF GIANT RESONANCES

A model for the decay of GR's which contains the semidirect, preequilibrium, and statistical parts, consistently connected by unitarity constraint, was developed about a decade ago by Dias, Hussein, and Adhikari [4] (see Fig. 1). This model, based on the theory of [8], was later refined and further applied. A similar, but slightly more general version of the model was developed by Piza and Foglia [5]. In this section we use Ref. [5] to isolate and analyze the preequilibrium part.

In the excitation of the GR with heavy ions, we use the recently developed exit-doorway model for the formation process [10]. Since we are discussing here specifically the ^{40}Ca+^{40}Ca at intermediate energies (50 MeV/A), we may comfortably use the theory developed in Ref. [4], namely, the cross section is given by

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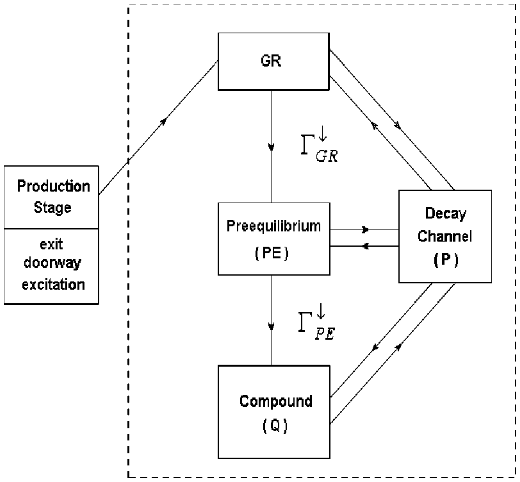

FIG. 1. A schematic representation of the splitting of the host nucleus phase space and the coupling between the parts.

$$\left\langle \frac{d\sigma_{c'c}}{dE^*} \right\rangle = \sigma_c(E^*) \left[\frac{d\mu}{dE^*} \frac{\Gamma^c_{\rm GR}}{\Gamma^{\downarrow}_{\rm GR}} + (1-\mu_2) \times \frac{d\mu}{dE^*} \frac{\tau^{c'}_{\rm PE} + \Gamma^{c'}_{\rm GR}(d\mu/dE^*)}{\sum_{f'} \tau^{f'}_{\rm PE} + (d\mu/dE^*)\,\Gamma^{f'}_{\rm PE}} + \frac{d\mu}{dE^*} \frac{\tau^{c'}_{\rm CN} + \mu_2 \tau^{c'}_{\rm PE} + \mu_2 \Gamma^{c'}_{\rm GR}(d\mu/dE^*)}{\sum_{f'} \tau^{f'}_{\rm PE} + (d\mu/dE^*)\,\Gamma^{f'}_{\rm PE}} \right]. \tag{1}$$

The formation cross section $\sigma_c(E^*)$ for a given value of the impact parameter, is given by

$$\sigma_c(E^*) = \frac{1}{2\pi} \frac{\Gamma_{\rm GR}}{(E^* - E_{\rm GR})^2 + (1/4)\,\Gamma^2_{\rm GR}} \times \sum_{\mu} \left| \int_{-\infty}^{\infty} dt' \, \exp\left(\frac{iE^* t'}{\hbar} \right) [V^{02}_{\mu}(t')]^* a_0(t') \right|^2, \tag{2}$$

where

$$\dot{a}_0(t) = -\sum_{\mu} V^{02}_{\mu}(t) \int_{-\infty}^{t} dt' [V^{02}_{\mu}(t')]^* \times \exp\left[-\frac{i}{\hbar}\left(E^* - \frac{i\Gamma_1}{2} \right)(t-t') \right] a_0(t'), \tag{3}$$

where $V^{02}_{\mu}(t) \equiv V^{02}_{\mu}[r(t)]$ is the Coulomb quadrupole coupling with magnetic quantum number μ evaluated at a given value of the classically determinated separation distance $r(t)$. For a quadrupole coupling, we have $V^{02}_{\mu}(t) \equiv \alpha_{\mu}/r^3(t)$, and $a_0(t)$ semiclassical elastic probability amplitude at $r(t)$. $E_{\rm GR}$ and $\Gamma_{\rm GR}$ are the position and width of the giant resonance in the host nucleus, respectively. The strength α_{μ} depends on the $B(E2)$ (see discussion to follow and Ref. [11]).

In Ref. [10], it was shown that by writing

$$a_0(t) = 1 + \sum_{\mu} A_{\mu}(t) \tag{4}$$

one obtains a second order differential equation for the auxiliary amplitudes $A_{\mu}(t)$

$$\ddot{A}_{\mu}(t) - \left[\frac{\dot{V}^{02}_{\mu}(t)}{V^{02}_{\mu}(t)} - \frac{i}{\hbar}\left(E^* - \frac{i\Gamma}{2} \right) \right] \times \dot{A}_{\mu}(t) - \frac{|V^{02}_{\mu}(t)|^2}{\hbar^2} \left[1 + \sum_{\mu'=-2,-1,0,1,2} A_{\mu'}(t) \right] = 0, \tag{5}$$

which can be easily solved. Application of the formation theory above has already been reported [11]. Preliminary results for the total (angle or impact parameter) integrated cross section, Eq. (2), for ^{40}Ca+^{40}Ca at 50 MeV/A gave for the quadrupole giant resonance [$E_{\rm GR}$=17 MeV $B(E2)$ =962 fm^4 e^2] the value σ_c=21 mb in reasonable agreement with experiment [12]. This small value of σ_c implies that the simpler perturbation calculation with $a_0(t')$ in Eq. (2) taken to be unity is adequate. More detailed account for the formation calculation will be reported later.

In the factor multiplying $\sigma_c(E^*)$ in Eq. (1), we have the details of the decay modes of the GR:

$$\frac{d\mu}{dE^*} = \frac{1}{2\pi} \frac{\Gamma^{\downarrow}_{\rm GR}}{(E^* - E_{\rm GR})^2 + (1/4)\,\Gamma^2_{\rm GR}} ,$$

where $\tau^{c'}_{\rm PE} = \{1 - \exp[-2\pi \Gamma^f_{\rm PE}(E^*) \rho_{\rm PE}(E^*)]\}$ is the transmission coefficient of the preequilibrium stage into the final channel, and $\tau^{f'}_{\rm CN}$ is the transmission coefficient of the last stage of the reaction in the compound nucleus, into the final channel.

The mixing parameter μ_2 measures the fragmentation of the preequilibrium stage owing to its coupling to the compound nucleus and is given by

$$\mu_2 \equiv \frac{\Gamma^{\downarrow}_{\rm PE}}{\Gamma_{\rm PE}} .$$

The last factor inside the square bracket is the contribution to the decay channel from the equilibrated compound stage. In the next section we concentrate our discussion on the second term inside the square bracket of Eq. (1) arising from the preequilibrium stage. In a recent publication [7] we gave evidence that this component may be appreciable. If so, one can extract from the data the mixing parameter μ_2 and then the damping width.

III. STATISTICAL THEORY OF THE DAMPING WIDTH

In the calculation of the contribution of preequilibrium emission, we use the formalism originally developed by

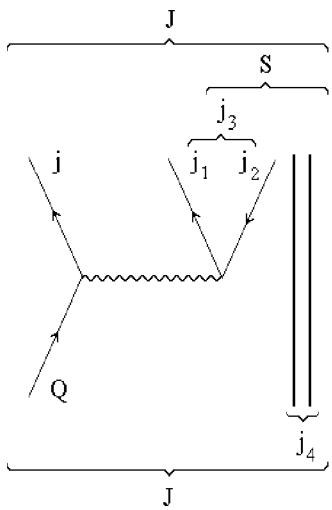

FIG. 2. Diagrammatic representation of the $\Delta n = +2$ process.

Feshbach, Kerman, and Koonin [8] and later improved and generalized by Oblozinsky [13]. In these approaches one assumes a factorization of the excitation energy and angular momentum dependences in the density of states. Thus one writes for the damping width associated with the nth stage the following form [8]:

$$\langle \Gamma_{nJ}^{\downarrow n+2}(E)\rangle = X_{nJ}^{\downarrow n+2} Y_n^{\downarrow n+2}(E). \tag{6}$$

The X and Y functions are described below.

A. The *X* function

The X function, which contains the details of angular momentum coupling, the distribution of the spin of the single particle levels and the radial integral of the stage $n \rightarrow$ stage $n+2$ transition matrix element is given by

$$X_{nJ}^{\downarrow n+2} = 2\pi \sum_{jQj_3j_4} (2j+1)(2j_3+1) \times \frac{R_{n-1}(j_4)R_1(Q)R_1(j)}{R_n(J)} \begin{pmatrix} j & Q & j_3 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \times F(j_3) I^2(j,j_1,j_2,Q)\Delta(j_4,Q,J)\ , \tag{7}$$

where the different factors are defined below. The angular momentum coupling scheme is shown in Fig. 2.

The spin distribution function, $R_n(j)$, is given by

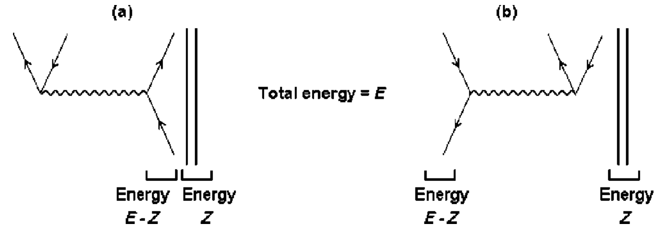

FIG. 3. The two damping processes, corresponding to a particle or a hole interacting with a bound nucleon, exciting an additional particle-hole pair. The energy of the interacting particle (or hole) is $E-Z$.

$$R_n(j) = \frac{(2j+1)}{\pi^{1/2} n^{3/2} \sigma^3} \exp\left[-\frac{[j+1/2]^2}{n\sigma^2} \right], \tag{8}$$

the angular momentum function $F(j_3)$ of the pair of states is given by

$$F(j_3) = \sum_{j_1} \sum_{j_2} (2j_1+1)(2j_2+1) R_1(j_1) R_1(j_2) \times \begin{pmatrix} j_1 & j_2 & j_3 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2, \tag{9}$$

while the radial integral $I(j_1,j_2,j,Q)$ is given by

$$I(j_1,j_2,j,Q) = \left(\frac{4}{3}\pi r_0^3 \right) V_0 \frac{1}{4\pi} \times \int_{r=0}^{\infty} R_1(r) R_2(r) R(r) R_Q(r) \frac{dr}{r^2}, \tag{10}$$

where $R_1(r)$, $R_2(r)$, $R_Q(r)$, and $R(r)$ represent the radial oscillator wave functions of particles (hole) with angular momentum j_1, j_2, Q, and j, respectively. The function $\Delta(j_4,Q,J)$ in Eq. (7) guarantees the angular momentum coupling selection rules, i.e., $\Delta(j_4,Q,J)=1$ if $|j_4-Q| \leqslant J \leqslant |j_4+Q|$ or zero otherwise.

B. The *Y* function

The Y functions in Eq. (6) describes the available phase space for the transition and it is calculated by considering the density of particle-hole states in the initial and final configurations. Schematically, the processes that contribute to Y are depicted in Fig. 3. These represent the scattering of a particle (a) and a hole (b) from a bound nucleus and resulting in the excitation of a particle-hole (two excitons) pair.

The contribution of the hole scattering Fig. 3(a) is given by [13]

$$ {}_bY_n^{\downarrow n+2} = \frac{1}{2} \frac{g^4}{\omega(p,h,E)} \left[\frac{\omega(p,h-1,E^{N+1}) - \omega[p,h-1,(E-B)^{N+1}]}{(N-1)N(N+1)} + \frac{B\omega[p,h-1,(E-F)^N]}{N} + \frac{\omega[p,h-1,E^2(E-F)^{N-1}] - \omega[p,h-1,(E-B)^2(E-F)^{N-1}]}{2(N-1)} \right] \tag{11}$$

and that of the particle scattering by

$$ {}_aY_n^{\downarrow n+2}=\frac{1}{2}\,\frac{g^4}{\omega(p,h,E)}\left[\frac{\omega(p-1,h,E^{N+1})-\omega[p-1,h,(E-B)^{N+1}]}{(N-1)N(N+1)}-\frac{B\omega[p-1,h,(E-B)^N]}{(N-1)N}-\frac{B^2\omega[p-1,h,(E-B)^{N-1}]}{2(N-1)}\right]. \tag{12} $$

The available density of states for the damping is the sum of Eqs. (11) and (12):

$$ Y_n^{\downarrow n+2}={}_aY_n^{\downarrow n+2}+{}_bY_n^{\downarrow n+2}. \tag{13} $$

The particle-hole density $\omega(p,h,E)$ which appears in Eqs. (11) and (12) is given by

$$ \omega(p,h,E)=\frac{g^N}{p!h!(N-1)!}\sum_{i=0}^{p}\sum_{k=0}^{h}(-1)^{i+k}\binom{p}{i}\binom{h}{k} \times\Theta(E-\alpha_{ph}-iB-kF) \times(E-A_{ph}-iB-kF)^{N-1}, \tag{14} $$

where $N=p+h, B$ is the binding energy, F is the Fermi energy, and $g\sim(3A/4\pi^2)$ is the density of single particle states. A_{ph} and α_{ph} are given by

$$ \alpha_{ph}=\frac{1}{2}\left(\frac{p^2+p}{g}+\frac{h^2-h}{g}\right) \tag{15} $$

and

$$ A_{ph}=\frac{1}{4}\left(\frac{p^2+p}{g}+\frac{h^2-3h}{g}\right). \tag{16} $$

Note that in Eqs. (11) and (12), a compact notation was used for the densities:

$$ \omega(p,h,U^{N+\nu})=\begin{cases}\dfrac{g^{p+h}}{p!h!(p+h-1)!}\displaystyle\sum_{i=0}^{p}\sum_{k=0}^{h}(-1)^{i+k}\binom{p}{i}\binom{h}{k},\Theta(U-iB-kF)(U-iB-kF)^{N+\nu} & \text{for}\quad U>0,\\ 0 & \text{for}\quad U\leqslant 0.\end{cases} \tag{17} $$

The above formalism for $Y_n^{\downarrow n+2}(E)$ developed by Oblozinsky [13] should be contrasted with the simpler one of Feshbach, Kerman, and Koonin (FKK) [8] where it is given by

$$ Y_{n,\mathrm{FKK}}^{\downarrow n+2}(E^*)=g\,\frac{(gE^*)^2}{2(n+1)}. \tag{18} $$

It would be interesting to check the accuracy of Eq. (18). In the next section we give a detailed comparison between the damping widths calculated with Eqs. (11) and (12) and the damping widths obtained with Eq. (18) for several nuclei.

IV. NUMERICAL RESULTS AND CONCLUSIONS

We present in Figs. 4, 5, and 6 the result of our calculation for the nuclei ^{40}Ca, ^{90}Zr, and ^{208}Pb. The calculations have been done by considering a basis including all bound single particle and hole states and using for g and σ the expressions given by FKK [8]

$$ g\sim\frac{3A}{4\pi^2},\qquad \sigma=\left[\frac{\sqrt{12}}{45\pi}\,\frac{A^{5/3}}{g}\right]^{1/2}. $$

The variation in the intervals of the excitation energy used in the calculus of the width is done around the centroids of the giant resonances in each nucleus. As one can see from the figures the FKK result is a very good approximation for all cases studied.

Of particular interest is the damping width of the giant quadrupole resonance in ^{40}Ca. We find the value of 1 MeV at $E^*\sim 17$ MeV. Our recent finding that preequilibrium contribution to the proton decay of this nucleus is significant, implying that our estimate $\Gamma_{\mathrm{GR}}^{\downarrow}\simeq\Gamma_{1p1h}^{2p2h\downarrow}$ is reasonable. Therefore to a certain extent the detection of a preequilib-

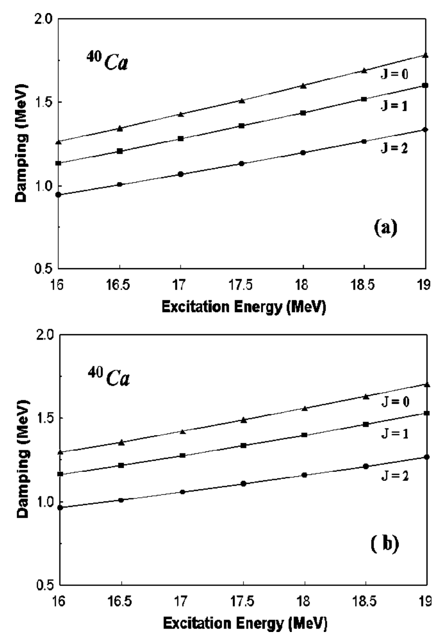

FIG. 4. Damping width $(1p-1h\rightarrow 2p-2h)$ for the GR's $E0,E1$, $E2$ in ^{40}Ca calculated in the FKK approximation (a) and (b) the approximation of [9,13].

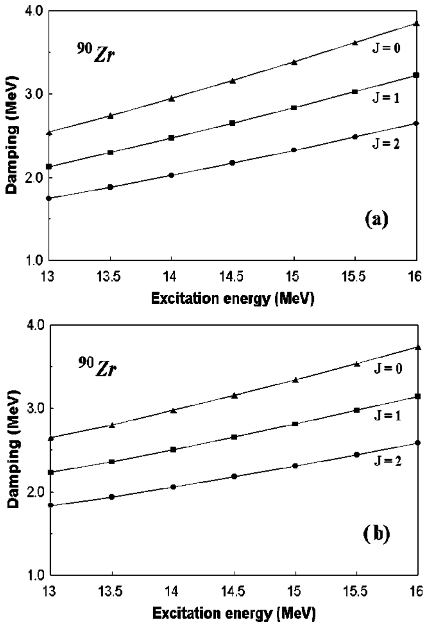

FIG. 5. The same as Fig. 4 for the ^{90}Zr nuclei.

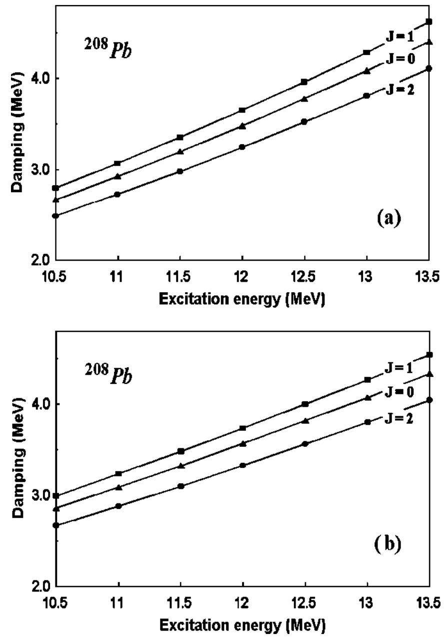

FIG. 6. The same as Fig. 4 for the ^{208}Pb nuclei.

rium emission of the GR help ''measure'' its damping width.

The results presented here should be of use to understand better the decay mechanism of the giant resonances in nuclei. In particular, the recent observation and decay analysis of double giant resonances [14] should particularly benefit from our discussion. Further application of our theory is underway.

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[1] G. Brown and M. Bolsterli, Phys. Rev. Lett. **3**, 472 (1959).

[2] See, e.g., G. F. Bertsch, P. F. Bortignon, and R. A. Broglia, Rev. Mod. Phys. **55**, 287 (1983); S. Drozdz, S. Nishizaki, J. Speth, and J. Wambach, Phys. Rep. **197**, 1 (1990).

[3] Y. C. Wang, F. E. Serr, E. J. Moniz, N. Teruya, H. Dias, and M. S. Hussein, Int. J. Mod. Phys. E **4**, 833 (1992).

[4] H. Dias, M. S. Hussein, and S. Adhikari, Phys. Rev. Lett. **57**, 1998 (1986).

[5] A. F. R. de Toledo Piza and G. A. Foglia, in *Proceedings of the International Nuclear Physics Conference*, São Paulo, Brazil, 1989, edited by Hussein *et al.* (World Scientific, Singapore, 1989), Vol. 2.

[6] H. Dias, M. S. Hussein, and B. V. Carlson, Phys. Lett. B **173**, 355 (1986).

[7] N. Teruya and H. Dias, Phys. Rev. C **50**, R2668 (1994); C. A. P. Ceneviva, N. Teruya, H. Dias, and M. S. Hussein, *ibid.* **55**, 1246 (1997).

[8] H. Feshbach, A. Kerman, and S. Koonin, Ann. Phys. (N.Y.) **125**, 429 (1980).

[9] R. Bonetti, M. B. Chadwich, P. E. Hodgson, B. V. Carlson, and M. S. Hussein, Phys. Rep. **202**, 173 (1991).

[10] L. F. Canto, A. Rommanelli, M. S. Hussein, and A. F. R. de Toledo Piza, Phys. Rev. Lett. **72**, 2147 (1994).

[11] C. A. Bertulani, L. F. Canto, M. S. Hussein, and A. F. R. de Toledo Piza, Phys. Rev. C **53**, 334 (1996).

[12] Ph. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995).

[13] P. Oblozinsky, Nucl. Phys. **A453**, 127 (1986).

[14] For a recent publication see, e.g., K. Boretzky *et al.*, Phys. Lett. B **384**, 30 (1996).