Predictive ability of QCD sum rules for decuplet baryons

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QCD sum rules for decuplet baryon two-point functions are investigated using a comprehensive Monte Carlo based procedure. In this procedure, all uncertainties in the QCD input parameters are incorporated simultaneously, resulting in realistic estimates of the uncertainties in the extracted phenomenological parameters. Correlations between the QCD input parameters and the phenomenological parameters are studied by way of scatter plots. The predicted couplings are useful in evaluating matrix elements of decuplet baryons in the QCD sum rule approach. They are also used to check a cubic scaling law between baryon couplings and masses, as recently found by Dey and co-workers. The results show a significant reduction in the scaling constant and some possible deviations from the cubic law. [S0556-2813(98)02201-8]

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I. INTRODUCTION

The QCD sum rule method [1] is a powerful tool in revealing the deep connection between hadron phenomenology and QCD vacuum structure via a few condensate parameters—vacuum expectation values of QCD local operators. This nonperturbative method has been successfully applied to a variety of problems to gain a field-theoretical understanding into the structure of hadrons (for a review of the early work, see Ref. [2]), and continues to be an active field [3].

The decuplet baryons have been studied in the past using the QCD sum rule method. In Ref. [4], Δ was first studied. It was later extended to Σ^* in Ref. [5]. In Ref. [6], sum rules for the decuplet family were given (they were also reported in Ref. [2]). Ratio method was used there to extract the masses. No attempt was made to extract the current couplings. No anomalous dimension corrections were considered. In Ref. [7], Δ was studied including leading-order α_s corrections. In Ref. [8], Δ was studied. A perusal of these works reveals discrepancies among the QCD sum rules derived where comparisons are possible. The analysis methods employed were relatively crude. Often a 10% or better accuracy was claimed in all the extracted quantities without the support of rigorous error analysis.

In this work, we decide to rederive the sum rules for all members of the decuplet family, consistently including operators up to dimension 8, first order strange quark mass corrections, flavor symmetry breaking of the strange quark condensates, anomalous dimension corrections, and factorization violation of the four-quark condensate. Furthermore, we try to assess quantitatively the errors in the phenomenological parameters, using a Monte Carlo based procedure [9]. This procedure incorporates all uncertainties in the QCD input parameters simultaneously, and translates them into uncertainties in the phenomenological parameters, with careful regard to operator-product expansion (OPE) convergence and ground state dominance. The goal is to get a realistic understanding of the predictive ability of the standard implementation of the QCD sum rule approach for the decuplet baryon two-point functions. Of particular interest is the baryon coupling to its current. This quantity is crucial to studying matrix elements of these baryons, such as magnetic moments, transition moments, axial charges, tensor charges, etc., because they utilize the coupling as normalization.

II. METHOD

The starting point is the two-point correlation function in the QCD vacuum

$$\Pi_{\mu\nu}(p)=i\int d^4x e^{ip\cdot x}\langle 0\,|T\{\eta_\mu(x)\,\overline{\eta_\nu}(0)\}|\,0\rangle, \qquad (1)$$

where η_μ is the interpolating field (or current) with the quantum numbers of the baryon in question. The interpolating field excites the ground state as well as the excited states of the baryon from the QCD vacuum. The ability of the interpolating field to annihilate the *ground state* baryon into the QCD vacuum is described by a phenomenological parameter λ_B (called current coupling or pole residue)

$$\langle 0\,|\,\eta_\mu|Bps\rangle=\lambda_B u_\mu(p,s), \qquad (2)$$

where u_μ is the Rarita-Schwinger spin-vector. This parameter plays an important role in evaluating matrix elements of the baryons.

The lowest dimensional interpolating fields for the decuplet are uniquely defined. Assuming SU(2) symmetry in the u and d quarks, they can be written as

$$\eta^{\Delta}_{\mu}(x)=\epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]u^c(x), \qquad (3)$$

$$\eta^{\Sigma^*}_{\mu}(x)=\sqrt{1/3}\epsilon^{abc}\{2[u^{aT}(x)C\gamma_\mu s^b(x)]u^c(x)+[u^{aT}(x)C\gamma_\mu u^b(x)]s^c(x)\}, \qquad (4)$$

$$\eta^{\Xi^*}_{\mu}(x)=\sqrt{1/3}\epsilon^{abc}\{2[s^{aT}(x)C\gamma_\mu u^b(x)]s^c(x)+[s^{aT}(x)C\gamma_\mu s^b(x)]u^c(x)\}, \qquad (5)$$

$$\eta^{\Omega}_{\mu}(x)=\epsilon^{abc}[s^{aT}(x)C\gamma_\mu s^b(x)]s^c(x). \qquad (6)$$

Here C is the charge conjugation operator and the superscript T means transpose.

The QCD sum rules are derived by calculating the correlator in Eq. (1) using operator-product expansion, on the one hand, and matching it to a phenomenological representation, on the other. The obtained tensor structure has the form

$$\Pi_{\mu\nu}(p)=\Pi_1(p^2)g_{\mu\nu}+\Pi_2(p^2)g_{\mu\nu}\hat{p}+\cdots, \tag{7}$$

where the hat notation denotes $\hat{p}=p^\alpha\gamma_\alpha$. Two QCD sum rules can be derived from the two invariant functions $\Pi_1(p^2)$ and $\Pi_1(p^2)$ for each member using standard procedure. The sum rule from Π_1 is called chiral-odd since it involves dimension-odd condensates only. Similarly, the sum rule from Π_2 is called chiral-even since it involves dimension-even condensates only. These two structures are considered because they receive contributions from spin-3/2 states only. The chiral-odd sum rules at the structure $g_{\mu\nu}$ are given for Δ,

$$\frac{4}{3}aE_1L^{16/27}M^4-\frac{2}{3}m_0^2aE_0L^{2/27}M^2-\frac{1}{18}abL^{16/27}$$
$$=\tilde{\lambda}_\Delta^2M_\Delta e^{-M_\Delta^2/M^2}, \tag{8}$$

for Σ^*,

$$\frac{4}{9}(f_s+2)aE_1L^{16/27}M^4-\frac{2}{9}(f_s+2)m_0^2aE_0L^{2/27}M^2$$
$$-\frac{1}{54}(f_s+2)abL^{16/27}+\frac{1}{2}m_sE_2L^{-8/27}M^6-\frac{1}{24}m_sbE_0$$
$$\times L^{-8/27}M^2+\frac{2}{3}m_s\kappa_va^2L^{16/27}=\tilde{\lambda}_{\Sigma^*}^2M_{\Sigma^*}e^{-M_{\Sigma^*}^2/M^2}, \tag{9}$$

for Ξ^*,

$$\frac{4}{9}(2f_s+1)aE_1L^{16/27}M^4-\frac{2}{9}(2f_s+1)m_0^2aE_0L^{2/27}M^2$$
$$-\frac{1}{54}(2f_s+1)abL^{16/27}+m_sE_2L^{-8/27}M^6$$
$$-\frac{1}{12}m_sbE_0L^{-8/27}M^2+\frac{4}{3}m_sf_s\kappa_va^2L^{16/27}$$
$$=\tilde{\lambda}_{\Xi^*}^2M_{\Xi^*}e^{-M_{\Xi^*}^2/M^2}, \tag{10}$$

and for Ω,

$$\frac{4}{3}f_saE_1L^{16/27}M^4-\frac{2}{3}f_sm_0^2aE_0L^{2/27}M^2-\frac{1}{18}f_sabL^{16/27}$$
$$+\frac{3}{2}m_sE_2L^{-8/27}M^6-\frac{1}{8}m_sbE_0L^{-8/27}M^2$$
$$+2\,m_sf_s^2\kappa_va^2L^{16/27}=\tilde{\lambda}_\Omega^2M_\Omega e^{-M_\Omega^2/M^2}. \tag{11}$$

The chiral-even sum rules at the structure $g_{\mu\nu}\hat{p}$ are given for Δ,

$$\frac{1}{5}E_2L^{4/27}M^6-\frac{5}{72}bE_0L^{4/27}M^2+\frac{4}{3}\kappa_va^2L^{28/27}$$
$$-\frac{7}{9}m_0^2a^2L^{14/27}\frac{1}{M^2}=\tilde{\lambda}_\Delta^2e^{-M_\Delta^2/M^2}, \tag{12}$$

for Σ^*,

$$\frac{1}{5}E_2L^{4/27}M^6-\frac{5}{72}bE_0L^{4/27}M^2+\frac{4}{9}(2f_s+1)\kappa_va^2L^{28/27}$$
$$-\frac{7}{27}(2f_s+1)m_0^2a^2L^{14/27}\frac{1}{M^2}+\frac{1}{3}m_s(4-f_s)$$
$$\times aE_0L^{4/27}M^2-\frac{1}{18}m_s(14-5f_s)m_0^2aL^{-10/27}$$
$$=\tilde{\lambda}_{\Sigma^*}^2e^{-M_{\Sigma^*}^2/M^2}, \tag{13}$$

for Ξ^*,

$$\frac{1}{5}E_2L^{4/27}M^6-\frac{5}{72}bE_0L^{4/27}M^2+\frac{4}{9}f_s(f_s+2)\kappa_va^2L^{28/27}$$
$$-\frac{7}{27}f_s(f_s+2)m_0^2a^2L^{14/27}\frac{1}{M^2}+\frac{2}{3}m_s(f_s+2)$$
$$\times aE_0L^{4/27}M^2-\frac{1}{9}m_s(2f_s+7)m_0^2aL^{-10/27}$$
$$=\tilde{\lambda}_{\Xi^*}^2e^{-M_{\Xi^*}^2/M^2}, \tag{14}$$

and for Ω,

$$\frac{1}{5}E_2L^{4/27}M^6-\frac{5}{72}bE_0L^{4/27}M^2+\frac{4}{3}f_s^2\kappa_va^2L^{28/27}$$
$$-\frac{7}{9}f_s^2m_0^2a^2L^{14/27}\frac{1}{M^2}+3\,m_sf_saE_0L^{4/27}M^2$$
$$-\frac{3}{2}m_sf_sm_0^2aL^{-10/27}=\tilde{\lambda}_\Omega^2e^{-M_\Omega^2/M^2}. \tag{15}$$

In the above equations, $a=-(2\pi)^2\langle\bar{u}u\rangle$, $b=\langle g_c^2G^2\rangle$, $\langle\bar{u}g_c\sigma\cdot Gu\rangle=-m_0^2\langle\bar{u}u\rangle$, $\tilde{\lambda}_B=(2\pi)^2\lambda_B$. The ratio $f_s=\langle\bar{s}s\rangle/\langle\bar{u}u\rangle=\langle\bar{s}g_c\sigma\cdot Gs\rangle/\langle\bar{u}g_c\sigma\cdot Gu\rangle$ accounts for the flavor symmetry breaking of the strange quark. The four-quark condensate is not well-known and we use the factorization approximation $\langle\bar{u}u\bar{u}u\rangle=\kappa_v\langle\bar{u}u\rangle^2$ and investigate its possible violation via the parameter κ_v. The anomalous dimension corrections of the various operators are taken into account via the factor $L=[\alpha_s(\mu^2)/\alpha_s(M^2)]=[\ln(M^2/\Lambda_{\rm QCD}^2)/\ln(\mu^2/\Lambda_{\rm QCD}^2)]$, where $\mu=500$ MeV is the renormalization scale and $\Lambda_{\rm QCD}$ is the QCD scale parameter. As usual, the excited state contributions are modeled using terms on the OPE side surviving $M^2\to\infty$ under the assumption of duality, and are represented by the factors $E_n(x)=1-e^{-x}\Sigma_nx^n/n!$ with $x=w_B^2/M_B^2$ and w_B an effective continuum threshold. Note that w_B is in principle different for different member of the decuplet and we will treat it as a free parameter in the analysis.

These sum rules constitute a complete, up-to-date set of QCD sum rules for the decuplet baryon two-point functions under standard implementation of the approach. The reader can find differences in a number of Wilson coefficients and in the anomalous dimension corrections, when comparing with the existing calculations.

III. ANALYSIS

The basic steps of the Monte Carlo based analysis are as follows [9]. Given the uncertainties in the QCD input parameters, randomly selected, Gaussianly distributed sets are generated, from which an uncertainty distribution in the OPE can be constructed. Then a χ^2 minimization is applied to the sum rule by adjusting the phenomenological fit parameters. This is done for each QCD parameter set, resulting in distributions for phenomenological fit parameters, from which errors are derived. The Borel window over which the two sides of a sum rule are matched is determined by the following two criteria: (a) *OPE convergence*—the highest-dimension-operators contribute no more than 10% to the QCD side; (b) *ground-state dominance*—excited state contributions should not exceed more than 50% of the phenomenological side. The former effectively establishes a lower limit, the latter an upper limit. Those sum rules which do not have a valid Borel window under these criteria are considered unreliable and therefore discarded.

The QCD input parameters and their uncertainty assignments are given as follows. The quark condensate in standard notation is taken as $a=0.52\pm0.05$ GeV^3, corresponding to a central value of $\langle\bar{u}u\rangle=-(236)^3$ MeV^3. For the gluon condensate, early estimates from charmonium [1] place it at $b=0.47\pm0.2$ GeV^4, a value commonly used in QCD sum rule analysis. But more recent investigations support much larger values [10–13]. Here we adopt $b=1.2\pm0.6$ GeV^4 with 50% uncertainty. The mixed condensate parameter is placed at $m_0^2=0.72\pm0.08$ GeV^2. For the four-quark condensate, there are claims of significant violation of the factorization hypothesis [10–12]. Here we use $\kappa_v=2\pm1$ and $1\leq\kappa_v\leq4$. The QCD scale parameter is restricted to $\Lambda_{\rm QCD}=0.15\pm0.04$ GeV. We find variations of $\Lambda_{\rm QCD}$ have little effects on the results. The strange quark mass is taken as $m_s=0.15\pm0.02$ GeV. The value of f_s has been determined in [2,5] and is given by $f_s=0.83\pm0.05$ after converting to our notation by $\gamma=f_s-1$. These uncertainties are assigned conservatively and in accord with the state-of-the-art in the literature. While some may argue that some values are better known, others may find that the errors are underestimated. In any event, one will learn how the uncertainties in the QCD parameters are mapped into uncertainties in the phenomenological fit parameters.

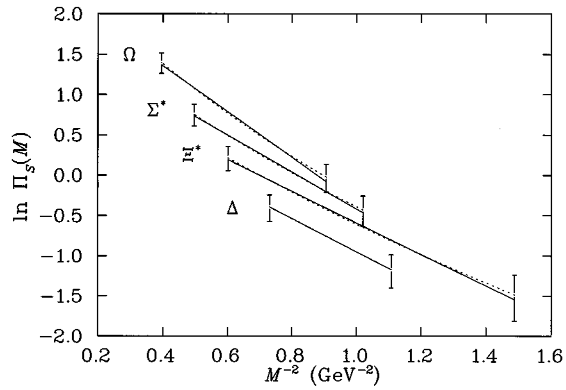

FIG. 1. Monte Carlo fits of the chiral-odd sum rules at the structure $g_{\mu\nu}$. Each sum rule is searched independently. The solid line corresponds to the ground state contribution, the dotted line the rest of the contributions (OPE minus continuum). The error bars are only shown at the two ends for clarity.

In illustrating how well a sum rule works, we choose to plot the logarithm of the two sides of the sum rule against the inverse of M^2. In this way, the right-hand side will appear as a straight line whose slope is $-M_B^2$ and whose intercept with the y axis gives a measure of the coupling strength. The linearity (or deviation from it) of the left-hand side gives an indication of OPE convergence and the quality of the continuum model. The two curves should match in the defined Borel region for a good sum rule. Figure 1 shows such a plot for the chiral-odd sum rules at the structure $g_{\mu\nu}$. The resulting fit parameters are given in Table I. The experimental masses are taken from Particle Data Group [14]. Figure 2 shows a similar plot for the chiral-even sum rules. The resulting fit parameters are given in Table II.

First, let us note that the chiral-odd Δ sum rule (8) is the only one that allows a three-parameter search. In the other sum rules, there is not enough information in the OPE for a three-parameter search. What happens numerically is that the search algorithm return a continuum threshold that is either zero or smaller than the mass, which is clearly unphysical. In order to proceed, we decide to fix the continuum thresholds at certain values and perform a two-parameter search on the masses and couplings. By adjusting the continuum thresh-

TABLE I. Monte Carlo analysis of the chiral-odd sum rules for the decuplet. The third column is the percentage contribution of the excited states to the phenomenological side at the lower end of the Borel region (it increases to 50% at the upper end). The results are obtained from consideration of 1000 QCD parameter sets.

Sum rule	Region (GeV)	Cont. (%)	w (GeV)	$\tilde{\lambda}^2$ (GeV^6)	Mass (GeV)	Exp. (GeV)
Δ	0.95 to 1.17	31	1.65 ± 0.22	2.26 ± 0.89	1.43 ± 0.12	1.232
Σ^*	0.82 to 1.29	8	1.80	2.83 ± 0.32	1.394 ± 0.052	1.384
Ξ^*	0.99 to 1.42	13	2.00	4.32 ± 0.47	1.505 ± 0.037	1.533
Ω	1.05 to 1.59	8	2.30	7.19 ± 0.75	1.676 ± 0.031	1.672

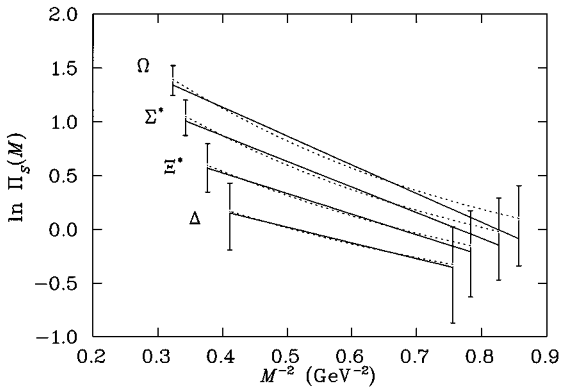

FIG. 2. Similar to Fig. 1, but for the chiral-even sum rules (without α_s corrections) at the structure $g_{\mu\nu}\hat{p}$. For better viewing, the curves for each baryon have been shifted downward by 0.2 units relative to the previous one.

olds to values that reproduce the experimentally known masses, the current couplings $\tilde{\lambda}_B^2$ are left as predictions from the self-consistency requirement of the corresponding sum rules. We have tried a different approach: fixing the masses at known values and search for the continuum thresholds and couplings. This was not successful due to the same reasons as above. It is satisfying to observe that the obtained continuum thresholds in Table I roughly coincide with the first excited state in each channel from Particle Data Group [14]: $\Delta(1600)$, $\Sigma^*(1840)$, $\Xi^*(1950)$, $\Omega(2250)$.

The predicted Δ mass lies above the experimental value by less than two standard deviations. This small overestimation of M_Δ is a typical result in the QCD sum rule approach. For example, in Ref. [4] it was found $M_\Delta \simeq 1.37$ GeV, $\tilde{\lambda}_B^2 \simeq 2.3$ GeV^6, and $w=2.2$ GeV; and in Ref. [8] it was found $M_\Delta \simeq 1.36$ GeV, $\tilde{\lambda}_B^2 \simeq 1.53$ GeV^6, and $w=1.58$ GeV. In these works, the traditional value of $b=0.47$ GeV^4 was used. Using this value in the present study yields $M_\Delta \simeq 1.385$ GeV, $\tilde{\lambda}_B^2 \simeq 2.00$ GeV^6, and $w=1.61$ GeV. The couplings from these studies are consistent with each other. It is worth mentioning that both M_Δ and $\tilde{\lambda}_B^2$ as determined from the QCD sum rule method agree with those from other calculations. For example, a lattice QCD calculation [15] gives $M_\Delta \simeq 1.43$ GeV, $\tilde{\lambda}_B^2 \simeq 2.13$ GeV^6; and an instanton liquid model calculation [16] gives $M_\Delta \simeq 1.43$ GeV, $\tilde{\lambda}_B^2 \simeq 1.70$ GeV^6. The reason for the overestimation of the Δ mass in QCD sum rule method remains an open question. It is likely that the power corrections are large in this sum rule and the OPE is not sufficiently convergent. On the phenomenological side, one notes that the continuum contribution in this sum rule is the largest among the decuplet family, greater than 30% in the entire Borel region used. This also indicates that the continuum model may not be adequate.

Further examination of Figs. 1 and 2 reveals that the errors in the chiral-odd sum rules are generally smaller than those in the chiral-even sum rules. More importantly, the convergence of the OPE in the chiral-odd sum rules are better than that in the chiral-even sum rules (as evidenced by the deviation of the dotted line from linearity). Consequently, the results from the chiral-odd sum rules are more reliable than those from the chiral-even sum rules. This fact is a general feature of baryon two-point functions as recently emphasized in Ref. [17]. It can be traced to the fact that even and odd parity excited states contribute with different signs. The contributions of positive- and negative-parity excited states partially cancel each other in the chiral-odd sum rules, whereas they add up in the chiral-even sum rules. Further evidence for the unreliability of chiral-even sum rules will be discussed below.

The effects of perturbative corrections to first order in α_s were also studied in this work. These corrections have been calculated in [7] and are given by

$$\left(\frac{11}{9}-\frac{4}{3}\gamma_E\right)\frac{\alpha_s}{\pi},$$

and

$$\left(\frac{17}{27}-\frac{5}{6}\gamma_E\right)\frac{\alpha_s}{\pi}, \tag{16}$$

to the dimension-three (a) and dimension-five ($m_0^2 a$) terms, respectively, in the chiral-odd sum rules (8) to (11); and

$$\left(\frac{539}{90}-\frac{1}{3}\gamma_E\right)\frac{\alpha_s}{\pi},$$

and

TABLE II. Similar to Table I, ut for the chiral-even sum rules. The second row in each sum rule shows the results with the leading-order α_s corrections included.

Sum rule	Region (GeV)	Cont. (%)	w (GeV)	$\tilde{\lambda}^2$ (GeV^6)	Mass (GeV)	Exp. (GeV)
Δ	1.15 to 1.56	10	2.20	4.13 ± 0.65	1.19 ± 0.13	1.232
	1.15 to 1.39	23	2.00	3.79 ± 0.51	1.23 ± 0.11	
Σ^*	1.13 to 1.63	6	2.40	5.63 ± 0.48	1.36 ± 0.13	1.384
	1.13 to 1.47	15	2.20	5.49 ± 0.41	1.38 ± 0.11	
Ξ^*	1.10 to 1.71	3	2.60	7.71 ± 0.37	1.52 ± 0.13	1.533
	1.10 to 1.56	8	2.40	7.92 ± 0.43	1.53 ± 0.10	
Ω	1.08 to 1.76	2	2.70	9.16 ± 0.43	1.61 ± 0.12	1.672
	1.08 to 1.61	5	2.50	9.54 ± 0.53	1.61 ± 0.10	

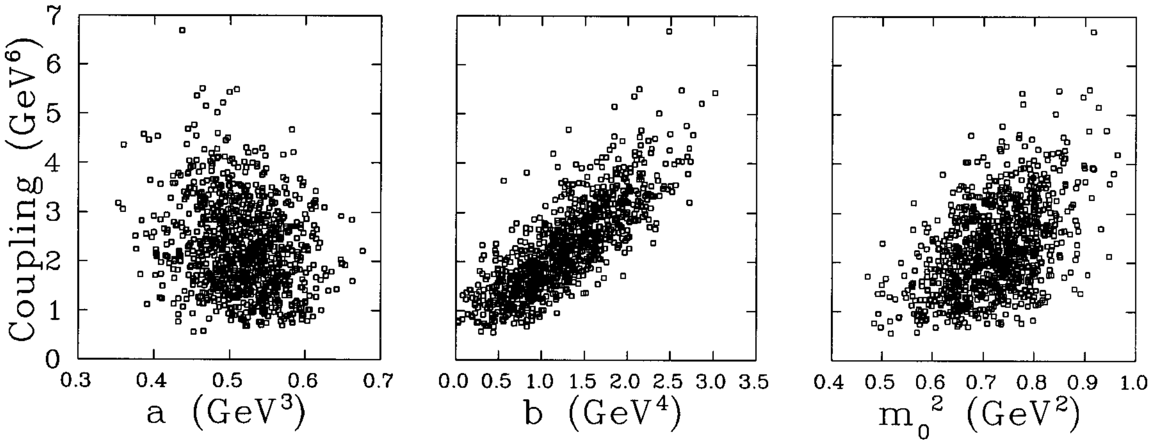

FIG. 3. Scatter plots showing correlations between $\widetilde{\lambda}_\Delta^2$ and the QCD input parameters for the chiral-odd sum rule (8). The results are drawn from 1000 QCD parameter sets. The distributions for M_Δ and w_Δ have qualitatively the same shapes as for $\widetilde{\lambda}_\Delta^2$, and are not shown here.

$$-\left(\frac{113}{108}+\frac{22}{3}\gamma_E\right)\frac{\alpha_s}{\pi}, \tag{17}$$

to the dimension-zero (the identity operator) and dimension-six (a^2) terms, respectively, in the chiral-even sum rules (12) to (15). Here $\gamma_E \simeq 0.58$ is the Euler constant. At the scale of about 1 GeV^2, $\alpha_s/\pi \simeq 0.12$. So the leading-order α_s corrections amount to about 5% in the chiral-odd sum rules and about 70% in the chiral-even sum rules. Their effects on the spectral parameters are given in Table II as a second entry for each of the chiral-even sum rules. We find the effects of α_s corrections in the chiral-odd sum rules are small and can be safely neglected. They were not listed in Table I. On the other hand, inclusion of α_s corrections in the chiral-even sum rules leads to a shrinkage of the valid Borel regions and an increase of the continuum contributions, signs of deterioration of the sum rules. Since the identity operator term is closely tied to the continuum model, it turns out that the effects of the α_s corrections can be compensated by simply shifting down the continuum threshold by about 200 MeV.

When considering the uncomfortably large α_s corrections in the chiral-even sum rules, the possibility of a significant dimension-two power correction from a summation of the perturbative series [18,19], coupled with the large uncertainties associated with the four-quark condensate, one has to conclude that the QCD side of these sum rules are really poorly known. The spectral properties extracted from them are most likely unreliable. Therefore, we caution against the use of these chiral-even sum rules in favor of the chiral-odd sum rules.

Since all the parameters in the Monte Carlo analysis are correlated, one can study the correlations between any two parameters by looking at their scatter plots. Such plots are useful in revealing how a particular sum rule resolves the spectral properties. Figure 3 shows the correlations of vacuum condensates with the Δ coupling for the chiral-odd sum rule (8). Similar plots hold for the mass and the continuum threshold and are not shown. The uncertainties in the quark condensate reveal anticorrelations with the spectral parameters, while those in the gluon condensate and the mixed condensate display positive correlations. The fact that all of

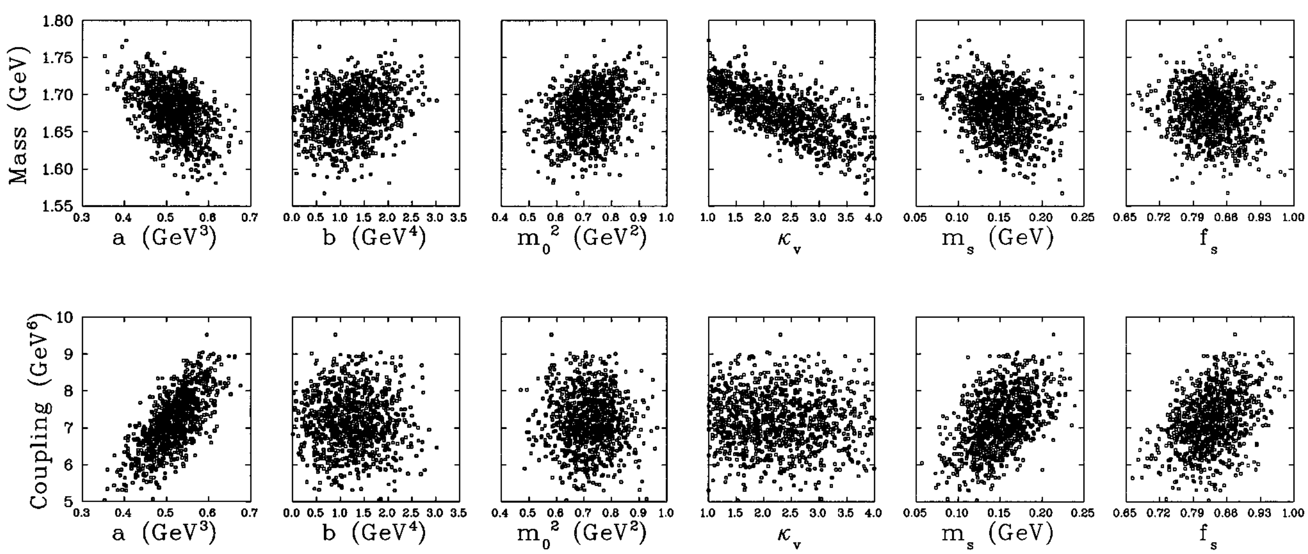

FIG. 4. Similar to Fig. 3, but for the chiral-odd Ω sum rule (11).

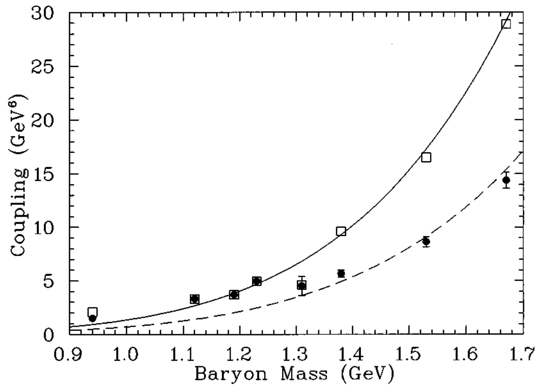

FIG. 5. Baryon current coupling ($\tilde{\lambda}_B^2$) versus the baryon mass (M_B). The empty squares are the points extracted by Ref. [20], and the filled circles are from this work. The solid line represents the fit to the square points: $\tilde{\lambda}_B^2=(1.16)^2M_B^6$, as obtained in Ref. [20]. The dashed line is the fit to our points: $\tilde{\lambda}_B^2=(0.84)^2M_B^6$.

the condensates have significant correlations with the phenomenological parameters suggests that all three terms in the OPE side of this sum rule play an important role in determining the outcome. This may be an indication that the OPE in this sum rule is not yet sufficiently convergent. The fact that all three phenomenological parameters are correlated with the vacuum condensates in the same manner suggests that attempts to fine tune the condensates will increase or decrease the phenomenological parameters simultaneously in this sum rule.

Figure 4 shows similar scatter plots for Ω at the chiral-odd sum rule (11). Here three more parameters come into play: κ_v, m_s, and f_s. It is interesting to observe that the mass and the coupling have different correlation patterns. They are opposite for the quark condensate. Similar is true for the strange quark mass, although less pronounced. The mass is negatively correlated with κ_v, while the coupling has little correlation with it. The correlations with b, m_0^2, and f_s appear fairly weak.

We have examined scatter plots for all of the sum rules. We find that qualitatively, the strange baryon (Σ^*, Ξ^*, and Ω) sum rules have very similar patterns within the same chirality, despite some subtle differences. The patterns in the chiral-even sum rules are very different from those in the chiral-odd sum rules. We will not elaborate further about them.

In a recent work [20], the authors observed that baryon current couplings and their masses obey a simple cubic scaling law: $\lambda_B\sim M_B^3$, across the octet as well as the decuplet. This result is based on the couplings extracted from existing QCDSR calculations. Some theoretical justification was given based on general scaling arguments for QCD and a simple light-cone constituent quark model. Using our results for the decuplet, we are able to check more carefully this claim.

In Fig. 5, following Ref. [20], we plot the couplings $\tilde{\lambda}_B^2\equiv 2(2\pi)^4\lambda_B^2$ as a function of the masses, using points extracted by Ref. [20] and our points. The nucleon point is taken from Ref. [9] which is the most recent analysis of the nucleon mass sum rule. It lies slightly below the point used in Ref. [20]. For the octet Λ, Σ, and Ξ, there exist no new analyses to our knowledge, so the same points are used. The important difference is that our values from Table I for the decuplet Σ^*, Ξ^*, and Ω are significantly smaller than the ones extracted by Ref. [20]. The Δ points roughly agree. This is true even for the values extracted from the less reliable chiral-even sum rules in Table II, which are slightly larger than those from the chiral-odd sum rules. Note that we have taken into account a factor of 2 difference in the definition of $\tilde{\lambda}_B^2$ in the comparisons. We attempted to fit the new points with a cubic law and obtained $\tilde{\lambda}_B^2=0.71M_B^6$ compared to their result of $\tilde{\lambda}_B^2=1.35M_B^6$. Notice, however, that there are considerable deviations from the points in the new fit, especially the octet strange baryon points. This calls for a reexamination of the QCD sum rules for the octet Λ, Σ, and Ξ. Given the great importance of a scaling law between baryon current couplings and their masses and its phenomenological consequences, more investigations are clearly needed to resolve the deviations, perhaps coupled with other methods such as lattice QCD.

IV. CONCLUSION

We have rederived and reanalyzed in detail the QCD sum rules for decuplet baryon two-point functions using a comprehensive Monte Carlo based procedure. Predictions were obtained for the current couplings with an accuracy on the order of 10% for Σ^*, Ξ^*, and Ω, and 40% for Δ (see Table I), using realistic error estimates for the QCD input parameters. The results are useful in evaluating matrix elements of decuplet baryons in the QCD sum rule approach. Our calculations confirmed the general feature that chiral-odd sum rules are more reliable than chiral-even sum rules as far as baryon two-point functions are concerned. Correlations between the QCD input parameters and the phenomenological parameters are studied by way of scatter plots. Some insights are gained on how a particular sum rule resolves the spectral properties. Finally, the couplings obtained in the present study are used to check a possible cubic scaling law between baryon couplings and their masses, as recently claimed in Ref. [20]. We find significant reduction in the scaling constant and possible deviations from the cubic law. More studies are needed to clarify this important issue.

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