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Rescattering in knockout reactions as manifested in ^{40}Ca$(p,p'p'')$ at an incident energy of 392 MeV

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The reaction ^{40}Ca$(p,p'p'')$ was investigated at an incident energy of 392 MeV. The energy and angular distributions are found to be consistent in shape as well as in absolute magnitude with the prediction of a reaction mechanism based on rescattering from a seminal knockout process. In this mechanism the struck nucleon behaves like an intranuclear projectile and initiates further rescatterings with the remainder of the target nucleus that is reminiscent of the interaction of a free projectile of similar kinetic energy. These results provide strong evidence for the validity of conclusions derived from earlier investigations at lower incident energies. The considerable uncertainty exhibited in previous work in the quantitative relationship between the yields of the initial knockout events and the secondary rescattering processes has been resolved in this study. Also, the participation of knockout from deeply bound shell-model orbitals has been clarified. [S0556-2813(98)05506-X]

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I. INTRODUCTION

It is well known that rescattering is a dominant process in nucleon knockout reactions with energetic protons. Although a proportion of the multiple scattering is manifested as events which are redistributed along a specific kinematic locus, a large fraction should proceed with energy transfer to the recoiling heavy system. Therefore, some of the knockout reactions that originate from collisions with valence nucleons in the target nucleus, and which subsequently undergo further rescattering, would in principle obscure the signature of pure knockout from deep-lying shell-model orbitals. In experiments designed specifically to study knockout to discrete final states, the region of phase space which is of interest is normally limited to a region where only a small part of the rescattering is observed. The coincident emission from the multistep processes simply provides a background upon which the discrete knockout is superimposed. Consequently, the presence of a multiple-scattering background is normally not a severe problem in actual experimental measurements, which are mostly performed at kinematic geometries where the knockout yield is maximized.

Although this situation might be favorable for cross section measurements in knockout reactions, the extent to which, for example, polarization observables are influenced may be more severe. Clearly this can only be evaluated if the reaction mechanism for the rescattering process is understood well.

At incident energies between 100 and 200 MeV, our understanding of the reaction mechanism whereby an energetic nucleon transfers energy to nuclear matter has improved during the last few years, largely as a result of experiments [1–7] in which coincident proton emission from the continuum was investigated. The results of these experiments suggest that the reaction mechanism can be interpreted as an initial nucleon-nucleon collision between the projectile and a nucleon bound in the target nucleus, with subsequent rescattering of the struck nucleon from the remainder of the target. Thus the struck nucleon seems to behave like an intranuclear projectile, and the angular and energy distributions of the protons observed in coincidence are in qualitative agreement with results from studies of inclusive (p,p') reactions. One of our recent coincidence experiments [7] demonstrated the remarkable extent to which the features of an initial intranuclear nucleon-nucleon collision are retained after the inelastic rescattering, even for a massive target nucleus.

A result of the existing studies is that, to a large extent, only the valence nucleons appear to take part in this process.

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However, this observation may merely be an artefact of the large distortion of the outgoing waves, especially for knockout from the more deeply bound shell-model orbitals at the relatively low incident energies (100–200 MeV) which were studied up to now. Consequently one might hope to see the participation of deeper states as the incident energy is increased. This provides the main motivation for the present investigation of the reaction ^{40}Ca$(p,p'p'')$ at an incident proton energy close to 400 MeV.

An additional advantage of the present work, in which knockout to the discrete levels is clearly resolved, is that it allows for a more reliable theoretical analysis of the experimental data. This, in turn allows a better test of the validity of the very simple theoretical model to predict absolute magnitudes of the cross section values.

The experimental details are presented in Sec. II and the theory is summarized in Sec. III. Details of the calculations are given in Sec. IV. Finally the results are discussed in Sec. V, with a summary and conclusions following in Sec. VI.

II. EXPERIMENTAL DETAILS

The experiment was performed at the accelerator facility of the Research Center for Nuclear Physics, Osaka, Japan. A dual magnetic spectrometer system was used.

The high-resolution magnetic spectrometer ''Grand Raiden'' was set up at a fixed forward angle of 25.5° and a central momentum which corresponds to 220 MeV in primary proton energy. The momentum acceptance of this detector for protons was equivalent to a kinetic energy range of 20 MeV.

The large acceptance spectrometer (LAS) was operated in coincidence with Grand Raiden and four overlapping field settings allowed the measurement of the energy distribution of emitted secondary protons from a threshold of approximately 50 MeV, up to the kinematic limit. Secondary scattering angles of 25.5°, 40°, 60°, 80°, 100°, and 120° on the opposite side of the beam were explored in the measurements. Horizontal acceptance angles, without use of defining slits, were ± 20 and ± 60 mr for Grand Raiden and LAS, respectively.

Standard focal plane detectors, electronics, and data taking system were used. Corrections for accidental coincidences were applied by the standard technique of subtraction of delayed coincidences in the time spectrum from events in the prompt coincidence peak. Due to the time structure of the beam these events appear as peaks in the time spectra that are clearly separated from each other.

The target was a self-supporting natural calcium (96.9% ^{40}Ca) foil of normal grade chemical purity. The thickness was measured as 12.75 ± 0.20 mg/cm^{-2}. In order to evaluate the oxygen contamination on the Ca target, a comparison of the coincidence yield with a completely oxidized target was performed. This suggested that the oxygen content of the target used for the $(p,p'p'')$ measurements contributed less than a few percent to our experimental cross sections and could therefore be neglected.

Momentum (kinetic energy) information was obtained from the data recorded with the focal plane detector system, thus enabling us to reconstruct binding energy spectra that displayed knockout to discrete final states with a missing-mass resolution of ~ 700 keV. In order to also have a visualization of the rescattering process, spectra were generated by projecting the cross section to the energy axis of the LAS. As a consistency check on the extraction of data, cross sections generated from the overlap regions in the LAS field settings were compared. These cross sections agreed mostly to well within 10%, as may be seen in Fig. 1. The absolute experimental cross section scale is estimated to be accurate to within 10%.

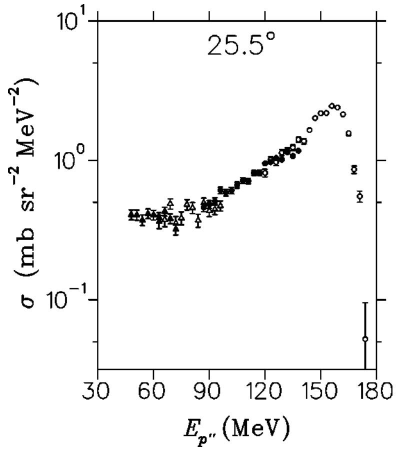

FIG. 1. Differential cross sections for coincident protons emitted in the reaction of 392 MeV protons with ^{40}Ca at an angle of 25.5° of the large acceptance spectrometer. Data obtained at four overlapping field settings are shown with different symbols. Statistical error bars are shown where these exceed the symbol size. The cross section units and scale are appropriate for the central momentum corresponding to 220 MeV in primary proton energy and momentum acceptance equivalent to an energy range of 20 MeV.

III. THEORY

Only a brief summary of the theory is given, as complete details are available in Refs. [7,8]. We symbolize our $(p,p'p'')$ reaction as $A(p,ac)B$, where it is assumed that after an initial collision between the projectile p and a bound nucleon b, further rescattering of the latter with the remainder of the target yields an observed particle c. The particle a is, classically of course, the scattered projectile. The cross section for the reaction can then be written as

$$\sigma = \sum_N \left[\int d\Omega_b \sum_\lambda \frac{d^3\sigma(\Omega_b, E_b)}{d\Omega_a \; d\Omega_b \; dE_a} \times \frac{1}{\sigma_N(E_b)} \frac{d^2\sigma(E_b, E_c)}{d(\Omega_c - \Omega_b) \; dE_c} \right].$$

In this expression N represents the type of nucleon (either proton or neutron) that is originally bound in a shell-model orbital λ in the target, and which is involved in the initial collision. The kinetic energies and angular information, respectively, of each participant in the interaction as given by the symbolic expression of the reaction, are denoted by E and Ω, with subscripts identifying specific particles. The quantity σ_N is the total cross section for the (b,c) reaction induced by an intranuclear projectile with a specific energy transferred to it by the projectile in the initial collision. The triple differential cross section is related to a knockout mechanism, but without inclusion of distortion in the wave function of the particle labeled b. The reason for this neglect is that the distortion is included implicitly in the subsequent rescattering interactions, as expressed by the double differential cross section.

IV. CALCULATIONS

A. Knockout cross section

The initial knockout interaction was calculated in the distorted-wave impulse approximation (DWIA) [9] with the computer code THREEDEE [10]. Two optical potential parameter sets for the calculation of distorted waves were investigated [11,12]. The one set taken from the work of Hama *et al.* [11] is based on a Schrödinger-equivalent reduction of a Dirac-based global fit to elastic-scattering data. The other set by Madland [12] is derived from that of Schwandt *et al.* [13] in order to generate global parameters that are also applicable to energies above 200 MeV. The wave functions for the bound states were calculated for the various shell-model orbitals with fixed geometries for the proton [14] and neutron [15] optical potentials. For each orbital the well depth of the potential was adjusted to reproduce the required characteristics of the bound-state wave function at empirical [16,17] separation energies as determined by the quantum numbers.

As is usual, the off-shell nucleon-nucleon interaction in the DWIA was approximated on-shell by interpolation of available nucleon-nucleon phase shifts, that are incorporated into the code of THREEDEE. Relative spectroscopic factors were based on the single-particle shell-model spectroscopic sum-rule limit, except for the $1f_{7/2}$ state, which had to be scaled to the results of Doll *et al.* [16]. The overall spectroscopic strength was subsequently adjusted to give a fair representation of the cross sections observed for knockout to discrete states in our present work. This procedure then fixes the absolute magnitude of the initial interaction that serves as a source reaction to the rescattering process of the struck nucleon.

B. Rescattering process

The rescattering of the bound nucleons was interpreted in the spirit of the multistep direct process [18,19]. However, instead of using, for example, the theory of Feshbach, Kerman, and Koonin [19] to evaluate the cross sections and angular distributions, we interpolated the available (p,p') experimental data [20] on ^{40}Ca over the required range of effective incident energies. In view of the large extent of these data, such a procedure is acceptable for the present purposes. Also, it is known [20] that these data, which were measured at the National Accelerator Center, can be successfully described in terms of a statistical multistep direct model [19].

The contribution from the (n,p) reaction, which would originate from collisions of the projectile with bound neutrons, was assumed to be similar in emission-energy dependence as that of the (p,p') yield. The work of Subramanian *et al.* [21] combined with that of Bertrand and Peelle [22] suggests that this assumption is not unreasonable.

Finally, the total preequilibrium cross sections, which effectively scale the contributions from (n,p) rescattering relative to that of the (p,p') reaction, were based on the geometry-dependant hybrid model and calculations were performed with the code ALICE [23]. A ratio of 0.7 is suggested by the values of 319 and 227 mb found for the proton- and neutron-induced reactions, respectively. This ratio is also consistent with the conclusions of Chadwick *et al.* [24] and Richter *et al.* [25], who infer that the difference between (p,p') and (n,p) inclusive yields on a medium-mass target nucleus is expected to be of the order of 10−30%. It should be noted, however, that the final calculated coincidence cross sections are not very sensitive to the proportions contributed by the two possible reaction types. The total reaction cross section for protons was taken as 540 mb [26], of which half is assumed to be associated with $\sigma_N(N \equiv p)$. Again, calculations with the ALICE code suggest that σ_p should be within 20% of our adopted value.

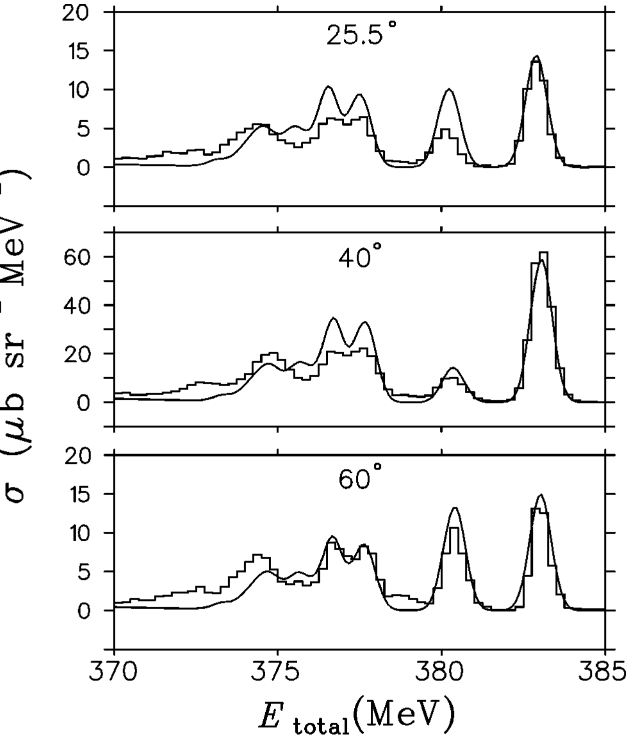

FIG. 2. Binding-energy spectra for the reaction ^{40}Ca$(p,2p)^{39}$K shown as a function of the total observed energy. Experimental data are shown as histograms for various secondary angles. The theoretical distributions, calculated as described in the text, are shown as curves.

V. RESULTS AND DISCUSSION

Binding energy spectra are shown in Fig. 2 for knockout to discrete low-lying states in ^{39}K at various coplanar angle pairs. The experimental data are compared with calculated binding-energy spectra based on the known levels [16,17] and assuming that the observed width of the states is dominated by the experimental resolution. As was mentioned in Sec. IV B, the relative strengths are taken from the single-particle sum-rule limit and the overall normalization is arbitrary (40−50 % of the sum rule limit, as will be explained later). The agreement shown in Fig. 2 is satisfactory.

In Fig. 3 experimental data for the complete range of measured binding energy is shown, together with theoretical

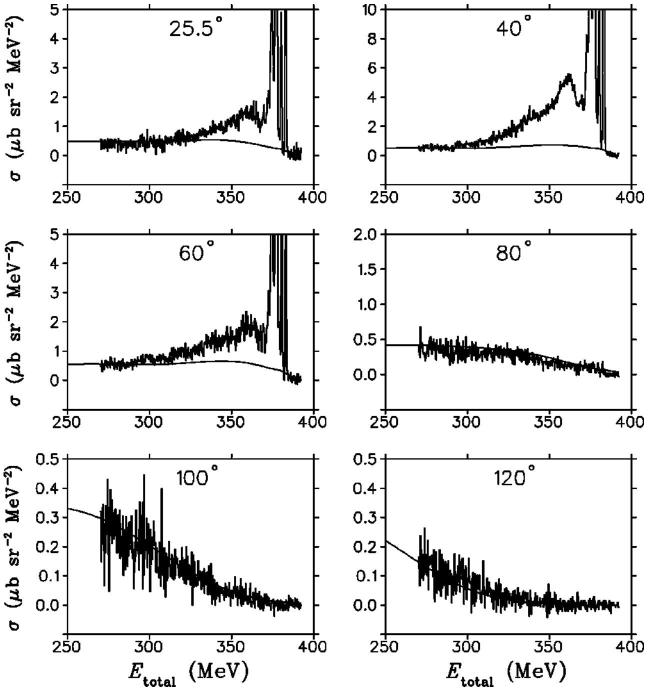

FIG. 3. The full binding-energy spectra. The curves represent the theoretical calculation for the ^{40}Ca(p,$p'p''$) reaction, as described in the text.

cross sections for the (p,$p'p''$) reaction, which forms a background upon which yields of the (p,2p) reaction to discrete states are superimposed. The fact that knockout to discrete states is not observed at angles of 80° and beyond, is consistent with the rapid falloff predicted by the DWIA theory for the source reaction. This is illustrated in Fig. 4 for the sum of contributions from the various shell-model orbitals for in-plane and out-of-plane secondary angles at a fixed primary angle of 25.5°. Results are shown for the distorting potential of Madland [12] (as are theoretical calculations presented in all figures), but the two sets that were investigated gave almost identical distributions, apart from a difference of about 25% in absolute cross section value.

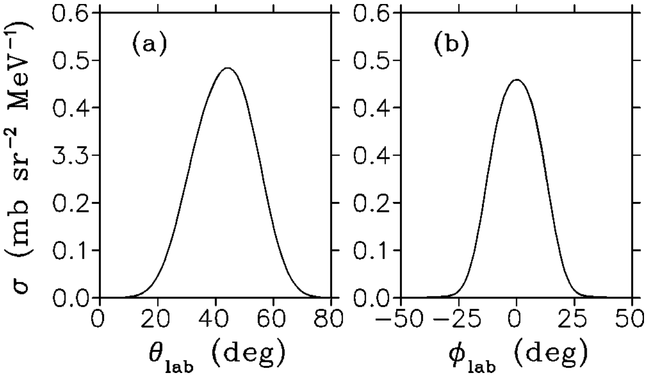

FIG. 4. Differential cross section for the ^{40}Ca(p,2p)^{39}K source reaction, as predicted by the DWIA theory, as functions of the in-plane (a) and out-of-plane (b) angles of one of the observed protons. The other proton, with a kinetic energy of 220 MeV, is scattered to a fixed angle of 25.5°.

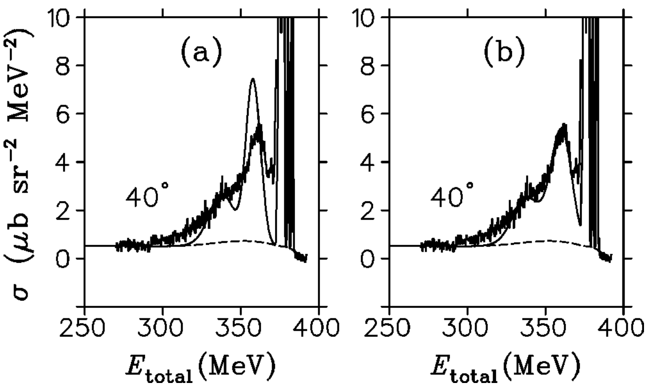

FIG. 5. As in Fig. 3. Theoretical spectra for the sum of discrete knockout and rescattering yields, compared with the experimental data. The continuous curve shown in (a) corresponds to a calculation with widths and positions of the 1s and 1p states taken from the experimental work of Volkov *et al.* [27]. In (b) the result is displayed if these quantities are adjusted for better agreement with the present experimental data. The calculated rescattering background is shown as dashed curves.

The ability of the DWIA theory (added to the rescattering background) to reproduce the experimental binding energy spectra for the (p,2p) reaction is, apart from the results already shown in Fig. 2, demonstrated for the energy region of deep-lying hole states in Fig. 5. Results are shown with widths and positions of the 1s and 1p states taken from the experimental work of Volkov *et al.* [27], as well as the results obtained if these quantities are adjusted for better agreement with our experimental data. The absolute magnitudes, however, are treated as before. It is seen that the two sets of results do not differ appreciably in their prediction of the rescattered component.

Final comparisons between the experimental coincident cross sections and an incoherent sum of the theoretical (p,$p'p''$) and (p,2p) values, given by the expression of Sec. III and the DWIA respectively, are shown in Fig. 6. Results are plotted as a function of the energy of the proton observed in the large acceptance spectrometer for various secondary angles. This specific choice of representation allows for a somewhat better comparison between the theoretical prediction and the experimental data. The reason for this is that these projections effectively remove the sensitivity to the assumption regarding the experimental widths of the discrete knockout to narrow states by averaging the yield over binding energy. It should be noted that the cross section units and scale are appropriate for the central momentum corresponding to 220 MeV in primary proton energy and momentum acceptance equivalent to an energy range of 20 MeV.

Excellent agreement between the theoretical and experimental energy distributions in Fig. 6 is obtained. For the calculations the absolute magnitude was fixed at a value corresponding to 50% of the single-particle spectroscopic sum-rule limit for the optical model potential of Hama *et al.* [11] and 40% for the Madland set [12]. These values are somewhat low compared to an expected value [28] of 60−70%, but not unacceptably so. Therefore we conclude that, to within the implicit uncertainty, the absolute magnitudes of the cross sections as presented are reasonable.

It is also encouraging that the absolute magnitude of the (p,$p'p''$) reaction scales exactly to the (p,2p) source reaction. It should be mentioned, however, that this extracted

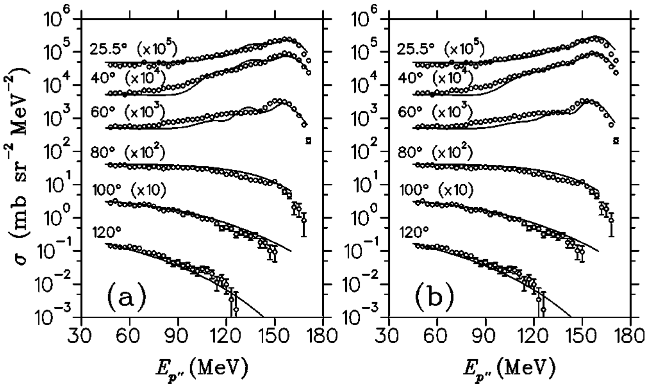

FIG. 6. Differential cross sections for coincident protons emitted in the reaction of 392 MeV protons with ^{40}Ca at a primary angle of 25.5° and at various secondary angles as indicated. The cross section units and scale are appropriate for an average primary proton energy of 220 MeV and over a range of 20 MeV, as discussed in the text. Experimental data are shown with statistical error bars where these exceed the symbol size. The theoretical curves represent calculations as described in the text. (a) shows the theoretical results without adjustment of the widths and positions of the 1*s* and 1*p* states of Volkov *et al.* [27], whereas (b) shows the slight improvement between the theoretical curves and the experimental results if these quantities are optimized.

scaling is uncertain to within approximately 20% due to the specific assumptions regarding the input to the calculations, as explained in Sec. IV B. Consequently, the fact that absolutely no further rescaling is needed for the results shown in Fig. 6 is to some extent fortuitous.

From the work of Förtsch *et al.* [7] on ^{197}Au$(p,p'p'')$ at an incident energy of 200 MeV, it appears that there is a need to renormalize the theoretical cross sections by a factor of $3-6$[1] relative to the knockout component. However, a large uncertainty in those values may be due to the fact that knockout to discrete levels was not explicitly resolved in Ref. [7], thus making an accurate theoretical analysis difficult. Our present results clearly indicate that such a drastic renormalization is not required.

VI. SUMMARY AND CONCLUSIONS

The reaction ^{40}Ca$(p,p'p'')$ was studied at an incident energy of 392 MeV in order to investigate the dominant reaction mechanism leading to events that are not observed as knockout to discrete states. Coincident proton spectra were measured at a primary angle of 25.5° and secondary coplanar angles ranging from 25.5° to 120° on the opposite side of the incident proton beam to the primary detector. The primary magnetic spectrometer at a fixed angle detected protons with energies in a narrow range of 20 MeV at an average of 220 MeV, whereas the other spectrometer was used to measure coincident protons from a threshold energy of 50 MeV up to the kinematic limit.

The $(p,p'p'')$ results were modeled as an initial nucleon-nucleon collision between the projectile and a nucleon bound in the various shell-model orbitals of the target nucleus, followed by rescattering of the struck nucleon as if it were acting as an intranuclear projectile. The initial collision was described with the DWIA theory, which was also compared with the $(p,2p)$ knockout data to discrete final states that were clearly identified in our measurements. The rescattering of the struck nucleons was treated in an empirical way by using existing spectra of the inclusive ^{40}Ca(p,p') reaction that is known to be describable in a statistical multistep-direct model.

The theoretical treatment reproduces the experimental coincidence spectra accurately, with the cross section of the secondary $(p,p'p'')$ reaction scaling with the value of the $(p,2p)$ source reaction remarkably well. In addition, the overall absolute theoretical cross section compared with the experimental data implies spectroscopic factors for nucleon removal that are in reasonable agreement with expectation.

The present work presents very strong evidence for the validity of conclusions from studies at lower incident energies regarding the dominant reaction mechanism that leads to rescattering effects in knockout reactions. In addition, earlier concerns as to the relationship between the yield of the initial knockout events and the secondary rescattering processes have been clarified. Finally, the insight provided by our work should provide invaluable guidance to future knockout studies of deep-lying hole states.

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[1] Due to an incorrect implementation of σ_N in Ref. [7], the values of the normalization quoted in that work should be adjusted to approximately half, as is presented here.

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