

Universal description of the 0_2^+ state in collective even- A nuclei

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A generalized description of the 0_2^+ state in collective even- A nuclei is obtained in the Q -excitation scheme. It is shown that for the whole symmetry triangle the two- Q and three- Q configurations are exhausted in a sum more than 90% of the norm of the 0_2^+ state. For the parameter range important for the description of nuclear data it is about 95% of the norm. The results obtained are applied to the description of the $E2$ -decay branching ratio of the 0_2^+ state in nondeformed nuclei. [S0556-2813(98)01206-0]

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I. INTRODUCTION

In the series of papers [1–4] an attempt has been undertaken to describe low-lying collective states of even-even nuclei in terms of the multiple- Q excitations of the ground state

$$|L^+, n\rangle = \mathcal{N}^{(L,n)} \underbrace{(Q \dots Q)}_n |0_1^+\rangle \quad (1)$$

where the operator Q

$$Q = s^+ \tilde{d} + d^+ s + \chi (d^+ \tilde{d})_2 \quad (2)$$

is a general quadrupole operator of the IBM-1. In the consistent- Q formalism (CQF) [5] the quadrupole operator Q is proportional to the $E2$ transition operator. The consideration has been performed in the framework of the extended consistent- Q formalism (ECQF) [6] of the sd -interacting boson model (IBM).

In the $O(6)$ dynamical symmetry limit the low-lying states with the $O(6)$ seniority quantum number $\sigma=N$ can be created acting by Q on the ground state. It was shown in [3] that these states can be described exactly as multiple- Q excitations with a fixed number of Q . In the $SU(3)$ dynamical symmetry limit only states belonging to the ground-state band can be produced acting by Q on the ground-state vector. These states are also described exactly by multiple- Q excitations with the number of Q equal to the half of the angular momentum. In a general case Q -configurations (1) form a basis which can be used to expand the eigenstates of the IBM-1 Hamiltonian. The question arises, how many basic states of the type (1) are required to describe eigenstates with a good accuracy. In [1,2] it was shown that the wave vectors of the yrast states can be described to a good accuracy (better than 90% of the norm) over the whole parameter space of the extended consistent Q Hamiltonian of IBM-1 by the simple universal expressions containing only one multiple- Q configuration. Then, it was shown in [4] that the second 2^+ state, which is a two-phonon state in the case of harmonic vibrator and a $K=2$ one-phonon vibrational state in a rotor limit, can be described with an accuracy better than 90% almost over the whole parameter space as a two- Q configuration

$$|2_{QQ}^+\rangle = \mathcal{N}^{(2,2)} \left[(QQ)^{(2)} - \frac{\langle 0_1^+ | (QQQ)^{(0)} | 0_1^+ \rangle}{\langle 0_1^+ | (QQ)^{(0)} | 0_1^+ \rangle} Q \right] |0_1^+\rangle. \quad (3)$$

The state vector (3) is orthogonal to the one- Q configuration. To describe a weak decay of the 2_2^+ state to the ground state it is necessary to take into account a small admixture of the one- Q configuration

$$|2_2^+\rangle = \mathcal{N}^{(2,1)} Q |0_1^+\rangle \quad (4)$$

to the state vector (3). In this case 2_2^+ state vector is described with an accuracy about 98% [4].

Although the consideration above was based on IBM, it was shown that the calculations are correct also for some other models [7,8]. So, we hope that the results mentioned above have a more general applicability than in the IBM only. It is also useful to stress that in expressions (1), (2), (3), and (4) the properties of the ground state as well as the quadrupole operator change with the Hamiltonian parameters. Therefore the properties of the states represented by the Q configurations are also changed. However, the relations among them remain approximately the same.

The aim of the present paper is to obtain a universal expression in terms of Q configurations for the 0_2^+ state vector. Together with the results obtained in [4] it will give us the expressions for the heads of the two main excited quasibands of even-even nuclei.

II. Q CONFIGURATIONS FOR 0_2^+ STATE

Let us investigate a possibility of expressing the 0_2^+ state in terms of the multiple- Q configurations. The simplest are two- Q

$$(QQ)_0 |0_1^+\rangle \quad (5)$$

and three- Q

$$(QQQ)_0 |0_1^+\rangle \quad (6)$$

configurations. The expressions given above are not orthogonalized to the ground state. It should be done before to use them as a basic vectors for the expansion of the $|0_2^+\rangle$ state vector. Orthogonalizing vectors (5), (6) to the ground state and normalizing them we get the following state vectors:

$$|0_{\mathcal{Q}\mathcal{Q}}^+\rangle = \frac{[(\mathcal{Q}\mathcal{Q})_0 - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle]}{\sqrt{\langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle^2}} |0_1^+\rangle, \quad (7)$$

$$|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle = \frac{[(\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle]}{\sqrt{\langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle^2}} |0_1^+\rangle. \quad (8)$$

At the moment we do not know how many multi- \mathcal{Q} configurations it will be necessary to take into account to express 0_2^+ state vector. However, before going to the higher- \mathcal{Q} configurations let us first analyze the expansions of the $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ vectors in terms of the eigenstates of the ECQF Hamiltonian

$$|0_{\mathcal{Q}\mathcal{Q}}^+\rangle = \sum_{i=2,3,\dots} a_i |0_i^+\rangle, \quad \sum_{i=2,3,\dots} a_i^2 = 1, \quad (9)$$

$$|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle = \sum_{i=2,3,\dots} b_i |0_i^+\rangle, \quad \sum_{i=2,3,\dots} b_i^2 = 1, \quad (10)$$

where $|0_i^+\rangle$ are eigenstates of the ECQF Hamiltonian. For the coefficients a_i and b_i we have

$$a_i = \langle 0_i^+ | 0_{\mathcal{Q}\mathcal{Q}}^+ \rangle, \quad (11)$$

$$b_i = \langle 0_i^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle. \quad (12)$$

We have calculated the values of the a_2, a_3 and b_2, b_3 coefficients over the whole parameter space of the ECQF Hamiltonian. Some results of the calculations performed for the

number of bosons $N=12$ are presented in Table I, where there are given the values of $(\langle 0_2^+ | 0_{\mathcal{Q}\mathcal{Q}}^+ \rangle^2 + \langle 0_3^+ | 0_{\mathcal{Q}\mathcal{Q}}^+ \rangle^2)$ and $(\langle 0_2^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle^2 + \langle 0_3^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle^2)$ calculated for the set of values of ϵ/κ and those values of χ , which minimize $(\langle 0_2^+ | 0_{\mathcal{Q}\mathcal{Q}}^+ \rangle^2 + \langle 0_3^+ | 0_{\mathcal{Q}\mathcal{Q}}^+ \rangle^2)$ and $(\langle 0_2^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle^2 + \langle 0_3^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle^2)$ for every given ϵ/κ . The results show that already the first two excited 0^+ states (0_2^+ and 0_3^+) exhaust mainly more than 90% of a norm of the $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ vectors. Therefore the two-dimensional subspace based on $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ vectors approximately coincides with the two-dimensional subspace based on $|0_2^+\rangle$ and $|0_3^+\rangle$ vectors. It means that $|0_2^+\rangle$ and $|0_3^+\rangle$ state vectors can be presented with a good accuracy as a linear combination of $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ vectors. Since the last vectors are not orthogonal (although they are linearly independent in general case) to each other it is convenient to orthogonalize them. To do it we will take the two- \mathcal{Q} configuration $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ as it is and then subtract from $|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ its projection on $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$. The new normalized vector $|\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$, which is orthogonal to $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ can be presented as

$$|\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle = \frac{|0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle - \langle 0_{\mathcal{Q}\mathcal{Q}}^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle \cdot |0_{\mathcal{Q}\mathcal{Q}}^+\rangle}{\sqrt{1 - \langle 0_{\mathcal{Q}\mathcal{Q}}^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle^2}}, \quad (13)$$

where

$$\langle 0_{\mathcal{Q}\mathcal{Q}}^+ | 0_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+ \rangle = \frac{\langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle \langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle}{\sqrt{[\langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle^2][\langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle - \langle 0_1^+ | (\mathcal{Q}\mathcal{Q})_0 | 0_1^+ \rangle^2]}}. \quad (14)$$

Now we have two orthogonalized and normalized multi- \mathcal{Q} vectors $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$, which can be used to present $|0_2^+\rangle$ state in terms of \mathcal{Q} configurations

$$|0_2^+\rangle = \alpha_2 |0_{\mathcal{Q}\mathcal{Q}}^+\rangle + \alpha_3 |\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle + \dots \quad (15)$$

Let us calculate the coefficients α_2 and α_3 over the whole symmetry triangle. The results of the calculations are presented in Figs. 1, 2 for different sets of parameters ϵ/κ and χ

over the whole symmetry triangle. It is seen from these results that two multi- \mathcal{Q} configurations $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ exhaust in a sum more than 90% of the norm of the $|0_2^+\rangle$ state. For the parameter range important for a description of nuclear data it is about 95% of the norm. Thus, the 0_2^+ state is described with a good accuracy as a linear combination of $|0_{\mathcal{Q}\mathcal{Q}}^+\rangle$ and $|\tilde{0}_{\mathcal{Q}\mathcal{Q}\mathcal{Q}}^+\rangle$ vectors over the whole symmetry triangle. However, the relative contributions of the separate components to the 0_2^+ state vary strongly over the triangle. Near the

TABLE I. The sums of the squares of the scalar products ($\langle 0_2^+ | 0_{QQ}^+ \rangle^2 + \langle 0_3^+ | 0_{QQ}^+ \rangle^2$) and ($\langle 0_2^+ | 0_{QQQ}^+ \rangle^2 + \langle 0_3^+ | 0_{QQQ}^+ \rangle^2$) calculated for the set of values of ϵ/κ and those χ , which minimize these sums for every given ϵ/κ . $N=12$.

ϵ/κ	$\langle 0_2^+ 0_{QQ}^+ \rangle^2 + \langle 0_3^+ 0_{QQ}^+ \rangle^2$	$\langle 0_2^+ 0_{QQQ}^+ \rangle^2 + \langle 0_3^+ 0_{QQQ}^+ \rangle^2$
0.1	0.985	0.946
0.6	0.989	0.945
1.1	0.991	0.944
1.6	0.993	0.943
2.1	0.994	0.942
3.1	0.995	0.940
4.1	0.996	0.939
5.1	0.996	0.937
10.1	0.990	0.923
15.1	0.978	0.904
20.1	0.964	0.881
25.1	0.956	0.924
30.1	0.966	0.970

$O(6)$ angle of the triangle, i.e., for small $|\chi|$ and small ϵ/κ , the 0_2^+ state mainly coincides with the $\tilde{0}_{QQQ}^+$ configuration. With an increase of the ϵ/κ a quasicrossing of the two lowest 0^+ states takes place and at the vicinity of a quasicrossing the states are exchanged by the main components. With an increase of the $|\chi|$ two configurations 0_{QQ}^+ and $\tilde{0}_{QQQ}^+$ interact more strongly and an interval of the values of ϵ/κ , where the exchange by the components takes place, becomes wider. Simultaneously, the state with the 0_{QQ}^+ configuration as the main component goes down in energy. Thus, at small $|\chi|$ and ϵ/κ , the 0_2^+ state is described with a good accuracy as a $\tilde{0}_{QQQ}^+$ configuration. At large $|\chi|$ or ϵ/κ (or both, simultaneously) the 0_2^+ state coincides mainly with the 0_{QQ}^+ configuration. There is a transitional region where two configurations are strongly mixed. This region is very narrow in ϵ/κ at $\chi \approx 0$. With an increase of $|\chi|$ a width of this region increases. It is illustrated in Fig. 3 where the energies of the 0_2^+ and 0_3^+ states are shown as functions of ϵ/κ for several values of χ . The calculations are done for $N=12$. It is seen that for $\chi = 0$ there is a level crossing. With an increase of $|\chi|$ the level repulsion is increased.

III. DECAY TRANSITIONS

It was shown in [2] that for the yrast states, which can be described approximately as a pure multi- Q configurations, there exists a simple selection rule for the $E2$ -transition probabilities. [Namely, the $E2$ transitions between the states differing by more than one quadrupole operator Q in the expression of the state vector are weak in comparison with the $E2$ transitions between the states differing by one quadrupole operator Q .] For instance, it was shown that the branching ratios

$$\frac{B(E2; 3_{QQQ}^+ \rightarrow 2_Q^+)}{B(E2; 3_{QQQ}^+ \rightarrow 2_{QQ}^+)} \quad (16)$$

and

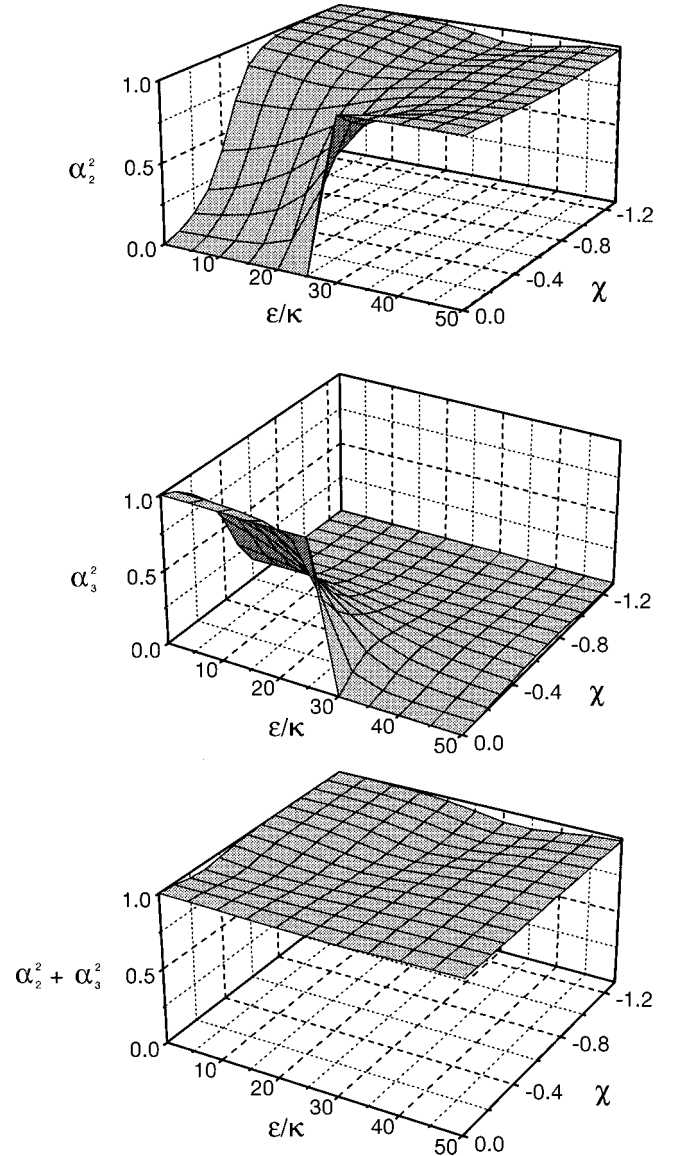


FIG. 1. The squares of the scalar products $\alpha_2^2 \equiv \langle 0_{QQ}^+ | 0_2^+ \rangle^2$, $\alpha_3^2 \equiv \langle \tilde{0}_{QQQ}^+ | 0_2^+ \rangle^2$ and their sum $\alpha_2^2 + \alpha_3^2$ for the whole ECQF IBM-1 space calculated gridwise throughout the symmetry triangle for $N=12$ bosons.

$$\frac{B(E2; 5_{QQQQ}^+ \rightarrow 4_{QQ}^+)}{B(E2; 5_{QQQQ}^+ \rightarrow 4_{QQQ}^+)} \quad (17)$$

are very small in the full symmetry triangle.

Because of this selection rule we can expect that the ratio $B(E2; 0_{QQQ}^+ \rightarrow 2_{QQ}^+) / B(E2; 0_{QQQ}^+ \rightarrow 2_Q^+)$ should be much larger than the ratio $B(E2; 0_{QQ}^+ \rightarrow 2_{QQ}^+) / B(E2; 0_{QQ}^+ \rightarrow 2_Q^+)$ at least for small $|\chi|$. With an increase in $|\chi|$ the $E2$ transitions between the configurations with the same numbers of Q increase also.

Let us consider the even-even nuclei which do not belong to the region of the well deformed ones. In this case $|\chi|$ is relatively small and for small ϵ/κ the 0_2^+ state is dominated by the 0_{QQQ}^+ configuration. At large ϵ/κ the 0_2^+ state is mainly described by the 0_{QQ}^+ configuration. In all these cases 2_2^+ state is well approximated by the 2_{QQ}^+ configuration and

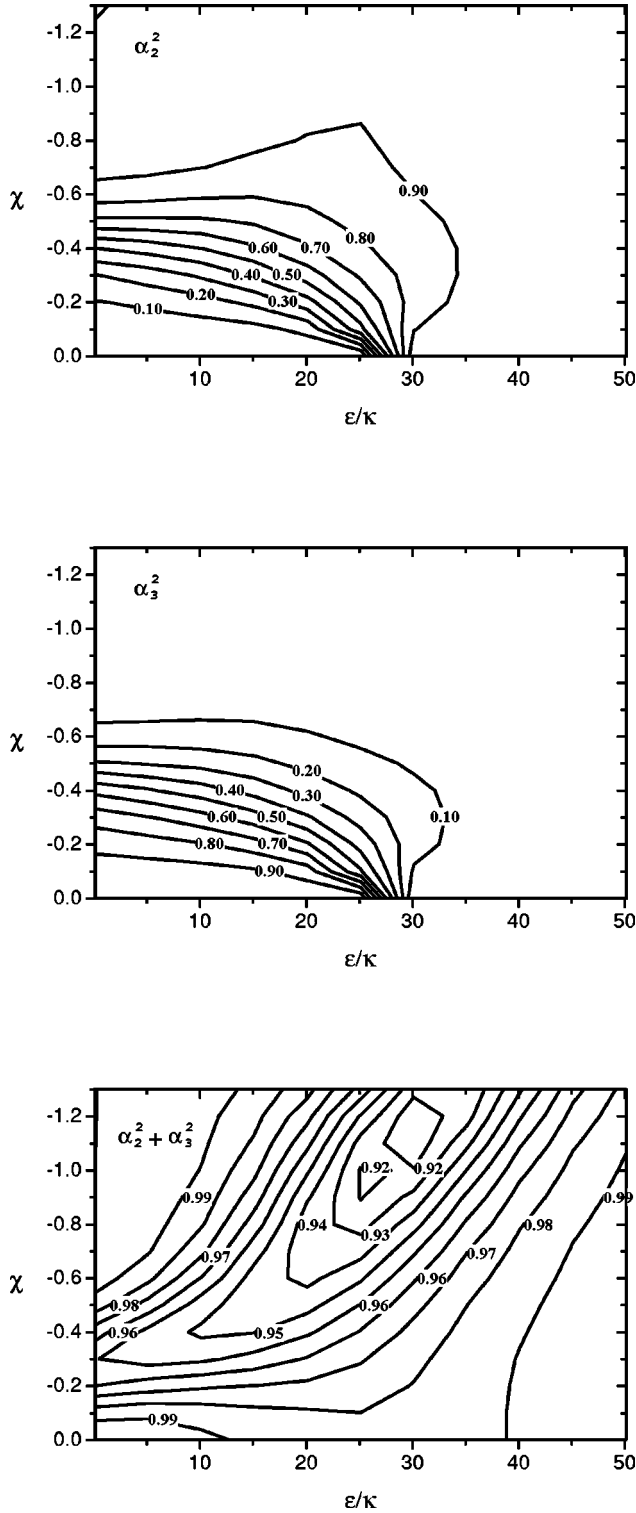


FIG. 2. Contour plots of the squares of the scalar products $\alpha_2^2 \equiv \langle 0_{QQ}^+ | 0_2^+ \rangle^2$, $\alpha_3^2 \equiv \langle \bar{0}_{QQ}^+ | 0_2^+ \rangle^2$ and their sum $\alpha_2^2 + \alpha_3^2$ for the whole ECQF IBM-1 space.

the 2_1^+ state is well approximated by the 2_Q^+ configuration. Thus, we get that for relatively small ϵ/κ the ratio

$$R' \equiv \frac{B(E2; 0_2^+ \rightarrow 2_2^+)}{B(E2; 0_2^+ \rightarrow 2_1^+)} \quad (18)$$

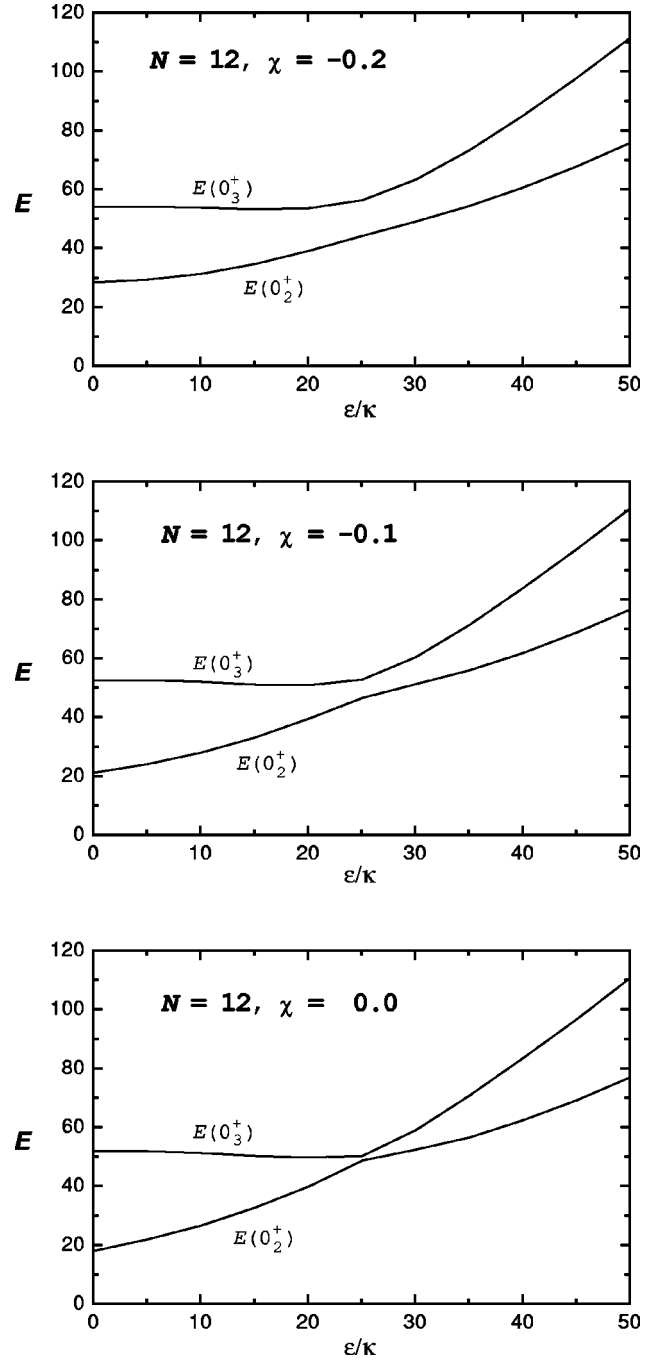


FIG. 3. Energies of the 0_2^+ and 0_3^+ states as the functions of ϵ/κ calculated for $N=12$ and $\chi=0.0, -0.1, -0.2$.

is large. With a ϵ/κ increase this ratio should decrease. Another observable which also varies monotonously with ϵ/κ is

$$R_{4/2} \equiv \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}. \quad (19)$$

This ratio is larger for larger values of ϵ/κ and decreases with a ϵ/κ decrease. In Fig. 4 are shown the correlations of two observables R' and $R_{4/2}$ for nondeformed nuclei. It is seen that the data, which are taken from [9–12], confirm the assertion formulated above.

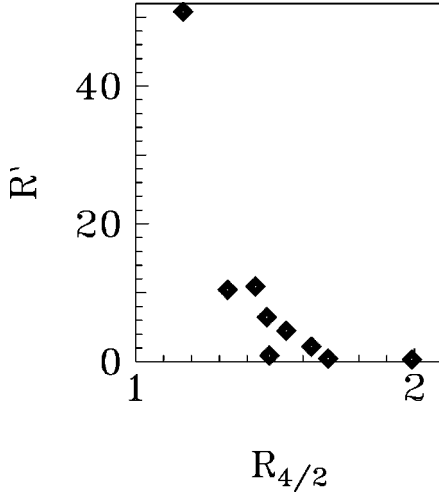


FIG. 4. Correlation of the observables R' and $R_{4/2}$ for nondeformed nuclei. The data are taken from [9–12].

IV. TWO EXCITED 0^+ STATES: FLUCTUATIONS

From the results obtained in Sec. II (see also Fig. 3) it follows that near the $O(6)$ dynamical symmetry limit, but not exactly in this limit, two lowest excited 0^+ states can be described as approximately pure three- Q and two- Q configurations. The lowest one is the three- Q configuration. The energy interval between these states can be regulated by the additional forces, for instance by the IBM pairing forces with the sign which provides a lowering of the 0^+ state with a smaller value of σ . Such forces do not disturb the wave functions if $|\chi|$ is near zero. These two multi- Q configurations are characterized by the different $E2$ -decay properties: three- Q configuration decay mainly to the 2_2^+ state, however, two- Q configuration decay mainly to the 2_1^+ state. The example is given by ^{196}Pt [13]. In this case $B(E2; 0_2^+ \rightarrow 2_2^+)/B(E2; 0_2^+ \rightarrow 2_1^+) = 6.32$ and $B(E2; 0_3^+ \rightarrow 2_2^+)/B(E2; 0_3^+ \rightarrow 2_1^+) = 0.16$. A similar situation is realized in ^{194}Pt for 0_2^+ and 0_4^+ states obtained in the Coulomb excitation experiment [9]. In this case $B(E2; 0_2^+ \rightarrow 2_2^+)/B(E2; 0_2^+ \rightarrow 2_1^+) = 10.9$ and $B(E2; 0_4^+ \rightarrow 2_2^+)/B(E2; 0_4^+ \rightarrow 2_1^+) = 0.97$. However, we should take care of a possible mixing effect of the three- Q and two- Q configurations in ^{194}Pt .

These experimental data give us a possibility to get information about the fluctuations of $(QQQ)_0$, i.e., of the γ degree of freedom. Indeed in the framework of the Q -excitations formalism we can derive the following relations for the branching ratios discussed above:

$$\frac{B(E2; 0_{QQQ}^+ \rightarrow 2_{QQ}^+)}{B(E2; 0_{QQ}^+ \rightarrow 2_Q^+)} = \frac{(\sqrt{35/2}K_3)^2}{\bar{K}_4} \frac{1}{(1 - (\sqrt{35/2}K_3)^2/\bar{K}_4)} \times \left(\frac{K_6 - K_5}{K_5 - 1} \right)^2 \quad (20)$$

and

$$\frac{B(E2; 0_{QQ}^+ \rightarrow 2_{QQ}^+)}{B(E2; 0_{QQ}^+ \rightarrow 2_Q^+)} = \frac{(\sqrt{35/2}K_3)^2}{\bar{K}_4} \frac{1}{(1 - (\sqrt{35/2}K_3)^2/\bar{K}_4)} \times \left(\frac{K_5 - \bar{K}_4}{\bar{K}_4 - 1} \right)^2, \quad (21)$$

where K_3 and \bar{K}_4 have been introduced in [14]

$$K_3 \equiv \frac{|\langle 0_1^+ | (QQQ)_0 | 0_1^+ \rangle|}{\sqrt{5}\sqrt{5}\langle 0_1^+ | (QQ)_0 | 0_1^+ \rangle^{3/2}}, \quad (22)$$

$$\bar{K}_4 \equiv \frac{\langle 0_1^+ | (QQ)_0 (QQ)_0 | 0_1^+ \rangle}{\langle 0_1^+ | (QQ)_0 | 0_1^+ \rangle^2}, \quad (23)$$

and

$$K_5 \equiv \frac{\langle 0_1^+ | (QQQ)_0 (QQ)_0 | 0_1^+ \rangle}{\langle 0_1^+ | (QQQ)_0 | 0_1^+ \rangle \langle 0_1^+ | (QQ)_0 | 0_1^+ \rangle}, \quad (24)$$

$$K_6 \equiv \frac{\langle 0_1^+ | (QQQ)_0 (QQQ)_0 | 0_1^+ \rangle}{\langle 0_1^+ | (QQQ)_0 | 0_1^+ \rangle^2}. \quad (25)$$

Expressions (20), (21) have been derived by neglecting the noncommutativity of the components of Q , which produces a small correction for these quantities.

Consider as an example ^{196}Pt , which is the closest to the $O(6)$ limit nucleus [13]. It is important because for $\chi \neq 0$ it can be necessary to take into account a mixing of the three- Q and two- Q configurations. The values of K_3 and \bar{K}_4 can be extracted from the data on $B(E2)$'s and are given for some nuclei in [14]. For ^{196}Pt $\sqrt{35/2}K_3 = 0.446$ and $\bar{K}_4 = 1.06$. Substituting these values into Eqs. (20), (21) and using the experimental data on the branching ratios [15] we get for ^{196}Pt

$$K_5 = 1.11,$$

$$K_6 = 1.69. \quad (26)$$

The deviations of \bar{K}_4 , K_5 , and K_6 from 1 characterize the amplitudes of the fluctuations of the corresponding dynamical quantities. From the results obtained it follows that in the case of ^{196}Pt the fluctuations of $(QQQ)_0$ are much larger than those of $(QQ)_0$, i.e., the fluctuations of γ are much larger than those of β . This result is natural for the nucleus closed to the $O(6)$ limit. However, by this consideration we have demonstrated the ability of the Q -excitation scheme to extract from the data an interesting physical information not only about the average values of the physical quantities but also about their fluctuations.

V. SUMMARY

Within the IBM-1 we have derived analytic expression for the 0_2^+ state in terms of multi- Q configurations. The expression is approximately valid outside of the dynamical symmetries where the exact wave functions can be obtained only by numerical diagonalization of the Hamiltonian. Together with the result obtained in [4] it gives us the expressions for the bandheads of the two main excited bands of even-even nuclei: the (“quasi”-) β band and the (“quasi”-) γ band in terms of Q configurations.

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