Nuclear equation of state

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We present a discussion of the equation of state of cold nuclear matter predicted by our recently completed Thomas-Fermi model. The equation is in the form of a three-term polynomial in the cube root of the density, with coefficients that are functions of the relative neutron excess δ. The coefficients are tabulated in the range from $\delta=0$ (standard nuclear matter) to $\delta=1$ (neutron matter), making it very easy to calculate, for a given δ, the pressure, compressibility, saturation binding, and any other property of the Thomas-Fermi equation of state. We discuss the empirical information concerning abnormal densities and large neutron excess that is contained in the measured values of the surface energy, surface diffuseness, and the neutron skin.
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I. INTRODUCTION

There is currently considerable interest in the energy per particle of nuclear matter $e(\rho,\delta)$ considered as a function of the nuclear density ρ and the relative neutron excess δ, where $\rho=\rho_{\text{neutrons}}+\rho_{\text{protons}}$ and $\delta=(\rho_n-\rho_p)/\rho$. This fundamental quantity, the equation of state at zero temperature, plays a key role in theories of neutron stars and supernova explosions, as well as in the interpretation of nucleus-nucleus collisions at energies where nuclear compressibility comes into play. (For a review and references see, for example, Ref. [1].)

Direct information on $e(\rho,\delta)$ is difficult to come by for values of ρ away from those characterizing normal nuclei and for δ beyond the relatively small values characteristic of the most neutron-rich stable nuclei. One way to extrapolate beyond this limited regime is by using a nuclear model fitted to binding energies of finite nuclei and then applying the model to nuclear matter. In order to be reliable, such an extrapolation should be based on a well-founded, robust theory with as few adjustable parameters as possible, the theory having been fitted very precisely to measured binding energies and other relevant properties, and subsequently tested for its predictive powers under conditions not included in the determination of the parameters. We have recently completed a Thomas-Fermi model of nuclei that attempts to satisfy these requirements [2] , and in the present paper we shall describe the model's predictions concerning the equation of state $e(\rho,\delta)$.

II. THE THOMAS-FERMI MODEL

The model is based on the semiclassical Thomas-Fermi approximation of two fermions per h^3 of phase space, together with the introduction of a short-range (Yukawa) effective interaction between the particles. The strength of the interaction (different between like and unlike nucleons) is a function of the relative momentum of the interacting particles and of the densities at the particles' locations. There are altogether six relevant adjustable parameters which were fitted to 1654 measured binding energies and to the measured diffuseness of the nuclear surface. (The binding energies were first corrected for shell and even-odd effects and for the congruence/Wigner energy [3], because these effects are beyond the reach of the statistical Thomas-Fermi theory.) Without readjusting the fitted parameters, the resulting model was found (a) to reproduce measured nuclear sizes, (b) to extrapolate correctly to light nuclei with $N,Z<8$, not included in the parameter fit, (c) to extrapolate correctly to masses of strongly deformed fission saddle-point shapes, and (d) to predict the density dependence of the energy of neutron matter in substantial agreement with independent theoretical estimates.

III. THE EQUATION OF STATE

The model's prediction for the equation of state reads $e(\rho,\delta)=T_0\,\eta(\Omega,\delta)$, where

$$\eta(\Omega,\delta)=a\Omega^2-b\Omega^3+c\Omega^5. \tag{1}$$

Here $\Omega\equiv(\rho/\rho_0)^{1/3}$, and $\rho_0=0.16114$ fm^{-3}, $T_0=37.0206$ MeV are the saturation density and Fermi energy of standard nuclear matter as predicted by the model. The coefficients a, b, c are the following functions of δ:

$$a=\frac{3}{20}\Big[2(1-\gamma_{/\!/})(p^5+q^5) - \gamma_u\begin{cases}(5p^2q^3-q^5) & \text{for}\quad \rho_n\geqslant\rho_p\\ (5p^3q^2-p^5) & \text{for}\quad \rho_n\leqslant\rho_p\end{cases}\Big], \tag{2}$$

$$b=\frac{1}{4}[\alpha_{/\!/}(p^6+q^6)+2\alpha_u p^3q^3], \tag{3}$$

$$c=\frac{3}{10}[B_{/\!/}(p^8+q^8)+B_u p^3q^3(p^2+q^2)], \tag{4}$$

where $p=(1+\delta)^{1/3}$, $q=(1-\delta)^{1/3}$. [See Eqs. (21)–(24) in [2]. In Eq. (21) the bar over α is missing.] The relative interaction strengths $\gamma_{/\!/}$, γ_u, $\alpha_{/\!/}$, α_u, $B_{/\!/}$, B_u are given in terms of five of the six adjusted parameters and have the following values:

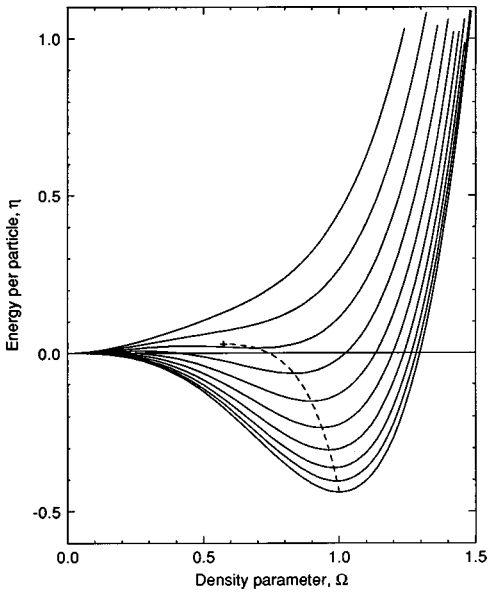

FIG. 1. The dimensionless energy per particle $\eta(\Omega,\delta)$ plotted as function of Ω, the cube root of the relative density ρ/ρ_0, for ten values of the relative neutron excess $\delta=0,0.2,0.3,0.4,\ldots,1.0$. The dashed curve follows the loci of the energy minima up to the critical point marked by a cross.

$$\gamma_\ell = 0.25198, \quad \gamma_u = 0.88474,$$
$$\alpha_\ell = 0.70110, \quad \alpha_u = 1.24574, \tag{5}$$
$$B_\ell = 0.22791, \quad B_u = 0.80020.$$

Figure 1 displays the dimensionless energy per particle $\eta(\Omega,\delta)$ as a function of Ω for $\delta=0,0.2,0.3,0.4,\ldots,1.0$. It will be seen that neutron matter ($\delta=1$) is unbound in our model. A minimum in η appears below the critical value $\delta_c=0.8213$, where $\Omega=\Omega_c=0.5735$ (i.e., $\rho=0.03039$ fm^{-3}) and $\eta_c=0.02979$ (i.e., $e=1.1029$ MeV). The saturation energy per particle becomes negative for $\delta<0.7783$ and attains the standard value $\eta=-0.43859$ (i.e., $e=-16.24$ MeV) at $\delta=0$. Figure 2 shows, as a function of δ, the density ρ, the energy per particle e, and the compressibility coefficient K_0 along the sequence of minima in Fig. 1.

In order to facilitate the application of Eq. (1) we have prepared Table I. It lists, as a function of δ, the coefficients a,b,c and the corresponding values of Ω and η, as well as ρ and e. The equilibrium value of Ω was obtained by solving the cubic resulting from equating to zero the derivative $\partial\eta/\partial\Omega$, which leads to $\Omega=0$ or

$$2a-3b\Omega+5c\Omega^3=0. \tag{6}$$

The relevant solutions are

$$\Omega=\sqrt{b/5c}\,[\cos(\theta/3)\pm\sqrt{3}\sin(\theta/3)], \tag{7}$$

where

$$\theta=\cos^{-1}(ab\sqrt{5c/b}). \tag{8}$$

The positive sign in Eq. (7) is associated with the energy minimum, the negative with a maximum. The vanishing of θ (and the coalescence of the solutions) corresponds to a point of inflection in η at the critical value δ_c, which is thus determined by the condition

$$5ca^2/b^3=1. \tag{9}$$

The critical value of Ω is then

$$\Omega_c=\sqrt{b/5c} \tag{10}$$

and the critical energy follows from Eq. (1).

The compressibility coefficient at the minimum, K_0, also listed in Table I, is given by

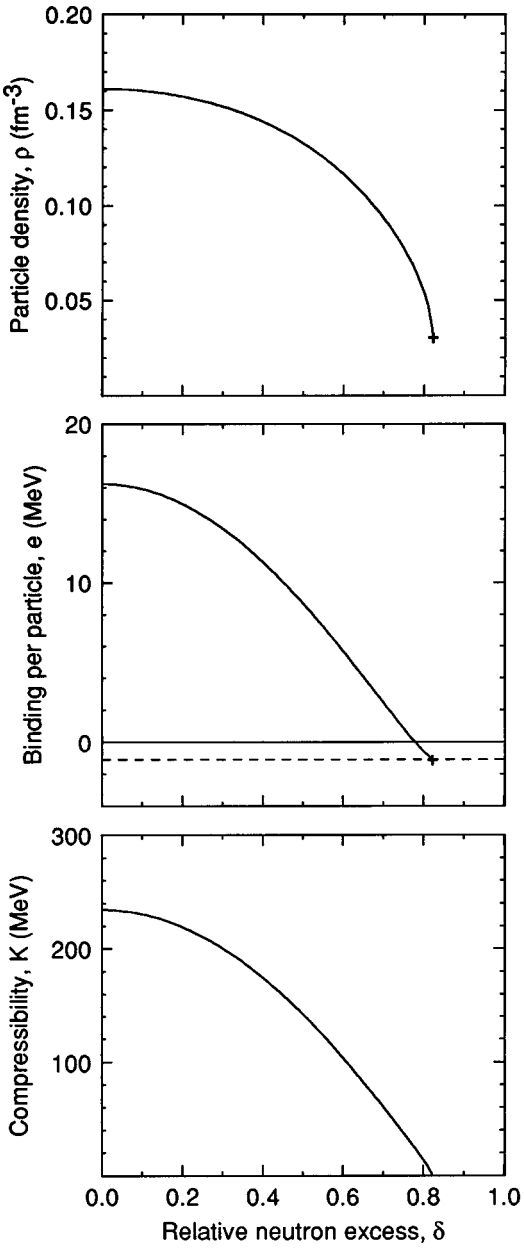

FIG. 2. The saturation density ρ, the energy per particle e and the compressibility $K_0(\delta)$ at saturation, plotted as function of the relative neutron excess δ. The compressibility vanishes at the critical point defined by $\delta_c=0.8213$, where $\rho_c=0.03039$ fm^{-3} and $e_c=1.1029$ MeV. Note the similarity of the δ dependences of K_0 and e_c-e. (The dashed line corresponds to $e=e_c$.)

TABLE I. Properties of the nuclear equation of state.

δ	a	b	c	Ω	η	ρ	e	K_0	ratio
0.00	-0.08203	0.97342	0.61687	1.0000	-0.43859	0.16114	-16.237	234.44	1.000
0.01	-0.08195	0.97339	0.61684	1.0000	-0.43850	0.16113	-16.233	234.40	1.000
0.02	-0.08170	0.97331	0.61677	0.9999	-0.43823	0.16110	-16.224	234.28	1.000
0.03	-0.08128	0.97317	0.61666	0.9998	-0.43779	0.16105	-16.207	234.09	1.000
0.04	-0.08070	0.97298	0.61650	0.9997	-0.43718	0.16098	-16.185	233.81	1.000
0.05	-0.07995	0.97274	0.61629	0.9995	-0.43639	0.16088	-16.156	233.46	0.999
0.06	-0.07903	0.97244	0.61604	0.9992	-0.43543	0.16077	-16.120	233.04	0.999
0.07	-0.07795	0.97209	0.61574	0.9990	-0.43430	0.16064	-16.078	232.53	0.999
0.08	-0.07671	0.97168	0.61540	0.9986	-0.43299	0.16049	-16.030	231.95	0.999
0.09	-0.07530	0.97121	0.61501	0.9983	-0.43152	0.16031	-15.975	231.29	0.998
0.10	-0.07373	0.97070	0.61457	0.9979	-0.42987	0.16012	-15.914	230.56	0.998
0.11	-0.07200	0.97013	0.61409	0.9974	-0.42805	0.15990	-15.847	229.75	0.997
0.12	-0.07011	0.96950	0.61357	0.9969	-0.42607	0.15967	-15.773	228.86	0.997
0.13	-0.06805	0.96882	0.61300	0.9964	-0.42391	0.15941	-15.693	227.90	0.996
0.14	-0.06583	0.96808	0.61238	0.9958	-0.42159	0.15914	-15.607	226.86	0.996
0.15	-0.06345	0.96729	0.61172	0.9952	-0.41910	0.15884	-15.515	225.75	0.995
0.16	-0.06091	0.96645	0.61101	0.9945	-0.41644	0.15852	-15.417	224.56	0.995
0.17	-0.05821	0.96555	0.61026	0.9938	-0.41362	0.15818	-15.313	223.30	0.994
0.18	-0.05535	0.96460	0.60946	0.9931	-0.41064	0.15782	-15.202	221.97	0.993
0.19	-0.05234	0.96359	0.60862	0.9923	-0.40749	0.15743	-15.085	220.56	0.992
0.20	-0.04916	0.96253	0.60774	0.9914	-0.40417	0.15703	-14.963	219.07	0.992
0.21	-0.04582	0.96141	0.60681	0.9905	-0.40070	0.15660	-14.834	217.52	0.991
0.22	-0.04233	0.96024	0.60583	0.9896	-0.39706	0.15615	-14.699	215.89	0.990
0.23	-0.03867	0.95901	0.60482	0.9886	-0.39327	0.15567	-14.559	214.19	0.989
0.24	-0.03486	0.95773	0.60376	0.9875	-0.38931	0.15518	-14.413	212.41	0.988
0.25	-0.03090	0.95640	0.60265	0.9864	-0.38520	0.15465	-14.260	210.57	0.986
0.26	-0.02677	0.95501	0.60150	0.9852	-0.38093	0.15411	-14.102	208.65	0.985
0.27	-0.02249	0.95357	0.60031	0.9840	-0.37650	0.15354	-13.938	206.66	0.984
0.28	-0.01805	0.95207	0.59908	0.9828	-0.37192	0.15294	-13.769	204.59	0.983
0.29	-0.01345	0.95052	0.59780	0.9814	-0.36718	0.15232	-13.593	202.46	0.981
0.30	-0.00870	0.94891	0.59648	0.9800	-0.36229	0.15168	-13.412	200.26	0.980
0.31	-0.00380	0.94725	0.59512	0.9786	-0.35725	0.15101	-13.226	197.98	0.979
0.32	0.00127	0.94553	0.59372	0.9771	-0.35206	0.15031	-13.034	195.64	0.977
0.33	0.00649	0.94376	0.59227	0.9755	-0.34672	0.14958	-12.836	193.22	0.975
0.34	0.01186	0.94194	0.59079	0.9738	-0.34123	0.14883	-12.633	190.74	0.974
0.35	0.01739	0.94006	0.58926	0.9721	-0.33560	0.14804	-12.424	188.19	0.972
0.36	0.02307	0.93813	0.58769	0.9704	-0.32982	0.14723	-12.210	185.57	0.970
0.37	0.02891	0.93614	0.58609	0.9685	-0.32390	0.14639	-11.991	182.88	0.968
0.38	0.03490	0.93410	0.58444	0.9666	-0.31784	0.14551	-11.767	180.12	0.966
0.39	0.04105	0.93200	0.58275	0.9646	-0.31164	0.14461	-11.537	177.30	0.964
0.40	0.04735	0.92985	0.58103	0.9625	-0.30530	0.14367	-11.302	174.40	0.962
0.41	0.05380	0.92764	0.57926	0.9603	-0.29882	0.14270	-11.062	171.45	0.959
0.42	0.06041	0.92538	0.57746	0.9580	-0.29221	0.14169	-10.818	168.42	0.957
0.43	0.06718	0.92307	0.57562	0.9557	-0.28547	0.14065	-10.568	165.34	0.954
0.44	0.07409	0.92070	0.57374	0.9532	-0.27859	0.13957	-10.314	162.18	0.952
0.45	0.08116	0.91827	0.57182	0.9507	-0.27159	0.13846	-10.055	158.96	0.949
0.46	0.08839	0.91580	0.56987	0.9480	-0.26447	0.13730	-9.791	155.68	0.946
0.47	0.09576	0.91326	0.56788	0.9453	-0.25722	0.13611	-9.522	152.34	0.943
0.48	0.10329	0.91068	0.56585	0.9424	-0.24985	0.13488	-9.250	148.93	0.940
0.49	0.11098	0.90804	0.56379	0.9394	-0.24237	0.13360	-8.973	145.47	0.936
0.50	0.11882	0.90534	0.56170	0.9363	-0.23477	0.13228	-8.691	141.94	0.933

TABLE I. (*Continued*).

δ	a	b	c	Ω	η	ρ	e	K_0	ratio
0.51	0.12681	0.90259	0.55957	0.9331	-0.22706	0.13091	-8.406	138.35	0.929
0.52	0.13495	0.89978	0.55741	0.9297	-0.21925	0.12950	-8.117	134.70	0.925
0.53	0.14325	0.89692	0.55521	0.9262	-0.21133	0.12803	-7.823	131.00	0.921
0.54	0.15170	0.89401	0.55298	0.9225	-0.20331	0.12652	-7.527	127.24	0.917
0.55	0.16030	0.89104	0.55072	0.9187	-0.19520	0.12495	-7.226	123.42	0.912
0.56	0.16906	0.88802	0.54843	0.9147	-0.18699	0.12333	-6.923	119.55	0.908
0.57	0.17797	0.88494	0.54611	0.9106	-0.17870	0.12165	-6.616	115.62	0.903
0.58	0.18704	0.88181	0.54376	0.9062	-0.17033	0.11991	-6.306	111.64	0.897
0.59	0.19626	0.87862	0.54137	0.9016	-0.16188	0.11811	-5.993	107.61	0.892
0.60	0.20563	0.87538	0.53896	0.8969	-0.15336	0.11625	-5.677	103.53	0.885
0.61	0.21516	0.87209	0.53652	0.8919	-0.14478	0.11431	-5.360	99.40	0.879
0.62	0.22484	0.86874	0.53406	0.8866	-0.13614	0.11230	-5.040	95.23	0.872
0.63	0.23467	0.86533	0.53157	0.8811	-0.12745	0.11022	-4.718	91.01	0.865
0.64	0.24466	0.86188	0.52905	0.8753	-0.11872	0.10806	-4.395	86.74	0.857
0.65	0.25480	0.85836	0.52651	0.8692	-0.10995	0.10581	-4.070	82.44	0.848
0.66	0.26510	0.85480	0.52395	0.8627	-0.10116	0.10347	-3.745	78.09	0.839
0.67	0.27556	0.85117	0.52136	0.8559	-0.09235	0.10103	-3.419	73.70	0.830
0.68	0.28617	0.84750	0.51875	0.8487	-0.08354	0.09849	-3.093	69.28	0.819
0.69	0.29693	0.84377	0.51612	0.8410	-0.07474	0.09584	-2.767	64.83	0.807
0.70	0.30785	0.83998	0.51347	0.8328	-0.06596	0.09308	-2.442	60.34	0.794
0.71	0.31893	0.83614	0.51080	0.8241	-0.05721	0.09017	-2.118	55.83	0.780
0.72	0.33017	0.83225	0.50812	0.8147	-0.04852	0.08713	-1.796	51.28	0.764
0.73	0.34156	0.82830	0.50542	0.8046	-0.03990	0.08392	-1.477	46.71	0.747
0.74	0.35311	0.82430	0.50270	0.7936	-0.03136	0.08054	-1.161	42.11	0.727
0.75	0.36482	0.82024	0.49997	0.7816	-0.02294	0.07695	-0.849	37.49	0.704
0.76	0.37669	0.81613	0.49723	0.7684	-0.01466	0.07311	-0.543	32.85	0.677
0.77	0.38871	0.81196	0.49447	0.7537	-0.00656	0.06898	-0.243	28.17	0.646
0.78	0.40090	0.80774	0.49171	0.7369	0.00132	0.06449	0.049	23.45	0.608
0.79	0.41325	0.80346	0.48894	0.7175	0.00894	0.05951	0.331	18.66	0.559
0.80	0.42576	0.79913	0.48617	0.6938	0.01621	0.05381	0.600	13.76	0.494
0.81	0.43843	0.79475	0.48339	0.6624	0.02303	0.04683	0.852	8.58	0.395
0.82	0.45126	0.79031	0.48061	0.6040	0.02912	0.03551	1.078	2.12	0.159
0.83	0.46426	0.78582	0.47783						
0.84	0.47743	0.78127	0.47505						
0.85	0.49076	0.77667	0.47228						
0.86	0.50426	0.77201	0.46951						
0.87	0.51793	0.76730	0.46675						
0.88	0.53177	0.76253	0.46401						
0.89	0.54578	0.75771	0.46128						
0.90	0.55997	0.75284	0.45858						
0.91	0.57434	0.74791	0.45590						
0.92	0.58888	0.74293	0.45325						
0.93	0.60361	0.73789	0.45063						
0.94	0.61853	0.73279	0.44805						
0.95	0.63364	0.72765	0.44552						
0.96	0.64895	0.72245	0.44306						
0.97	0.66447	0.71719	0.44066						
0.98	0.68021	0.71188	0.43835						
0.99	0.69618	0.70652	0.43615						
1.00	0.71244	0.70110	0.43413						

$$K_0(\delta)=9\left[\rho^2\frac{\partial^2 e}{\partial\rho^2}\right]_{\rho=\rho_{\min}}=T_0(2a\Omega^2-6b\Omega^3+20c\Omega^5), \tag{11}$$

with Ω given by Eq. (7).

As can be seen from Fig. 2 and Table I, $K_0(\delta)$ starts at 234.44 MeV at $\delta=0$ and decreases to zero at the critical point δ_c. Figure 2 shows also that the behavior of $K_0(\delta)$ parallels the behavior of the depth of the binding energy minimum taken with respect to the energy e_c at δ_c. This is an extension to large values of δ of the parallelism between $K(N,Z)$ and $e(N,Z)$ for finite nuclei, discovered in Refs. [4,5]. To illustrate this near proportionality of compressibility to binding energy, the last column in Table I gives the ratio $(e_c-e)/K_0$ normalized to 1 at $\delta=0$. It will be seen that this ratio changed by only 7.5% for $\delta=0.52$, where e has been halved to -8.12 MeV from its original value of -16.24 MeV.

For small deviations of ρ and δ from standard nuclear matter (specified by $\rho=\rho_0,\delta=0$), the energy per particle may be expanded as follows (we use the notation of Refs. [6–8]):

$$e=-a_1+J\delta^2+\tfrac{1}{2}(K\epsilon^2-2L\epsilon\delta^2+M\delta^4)+\cdots, \tag{12}$$

where ϵ is related to the deviation of ρ from ρ_0 by

$$\epsilon=1-\left(\frac{\rho}{\rho_0}\right)^{1/3}, \tag{13}$$

so that for small $\rho-\rho_0$ we have

$$\frac{\rho}{\rho_0}=1-3\epsilon+\cdots. \tag{14}$$

The values of the coefficients in Eq. (12), as given by our model [2], are $a_1=16.24$ MeV, $J=32.65$ MeV, $K=234.44$ MeV, $L=49.9$ MeV, $M=7.2$ MeV. According to Eq. (12), $\frac{1}{2}M$ contributes to the initial deviation of the symmetry energy from a parabolic dependence on δ, and the coefficient L controls the decrease of the saturation density with increasing δ according to $\epsilon=(L/K)\delta^2$, i.e.,

$$\frac{\rho}{\rho_0}=1-(3L/K)\delta^2+\cdots =1-0.6385\delta^2+\cdots. \tag{15}$$

In many applications the compressibility of *nonequilibrium* nuclear matter is of interest. Defining $K(\rho,\delta)$ in the usual way as

$$K(\rho,\delta)=9\frac{\partial P}{\partial\rho}, \tag{16}$$

where P is the pressure defined by $P=\rho^2(\partial e/\partial\rho)$, we find

$$K(\rho,\delta)=T_0(10a\Omega^2-18b\Omega^3+40c\Omega^5). \tag{17}$$

This is again readily evaluated for a given δ by looking up the coefficients a, b, c in Table I.

IV. DISCUSSION

Equation (1) represents the extrapolated equation of state, as predicted by a model that gives the currently most accurate representation of measured binding energies and fission barriers [2,3,9]. For relatively modest deviations from standard density, the key quantity is the compressibility $K_0(\delta)$, whose value at $\delta=0$ we estimate as about 234 MeV. Other recent estimates of this quantity [10,11], based on an interpretation of the giant monopole resonance, suggest values near 215 MeV. At the present time there is enough uncertainty all around so that we do not regard this 8% difference between the two estimates as necessarily significant. But the relative reliability of estimates of K_0 derived from a very precise fit of a nuclear model to binding energies *and* the surface diffuseness is, perhaps, not yet generally appreciated, and we would like to comment here on this aspect of the compressibility problem.

The important thing to bear in mind is that the diffuse nuclear surface is a region where a large excursion of the density from its bulk value takes place. Consequently the compressibility plays a key role in determining both the surface energy and the surface diffuseness. The result is that, conversely, the rather well determined values of the surface energy and surface diffuseness, *when taken together* (and backed by a sound theory), place much narrower limits on the acceptable values of K_0 than a fit to binding energies alone. This was illustrated in Fig. 13 of [2].

A somewhat analogous argument may be invoked as regards extrapolations to large values of δ. Here the appearance of a neutron skin, i.e., of a shift of the neutron density profile away from the proton density profile for neutron-rich nuclei, leads to a relatively large local neutron excess, albeit only in a small region in the outer part of the surface. As discussed in Ref. [12], the crucial quantities determining the neutron skin are the symmetry energy coefficient J and the neutron skin stiffness coefficient Q. The value of J is known quite accurately and recent precise fits to binding energies have firmed up the value of Q. (This is because Q is also the controlling factor in the surface symmetry energy, which can today be extracted with some confidence from accurate fits to binding energies and fission barriers.) This means that a model which has the ability to describe correctly effects associated with the neutron skin is being supplied with empirical information on relatively large values of δ, albeit only in an indirect way. It is clear, however, that to exploit this feature of the nuclear surface, namely the sampling of conditions associated with abnormal densities and large neutron excess, one needs a sound theory and very precise fits to a *comprehensive* set of nuclear masses including saddle-point masses, the surface diffuseness and, if possible, the neutron skin.

For very large extrapolations (several times the standard density) our simple expression for $e(\rho,\delta)$ will have to be judged by whatever experimental information may become available, and by comparisons with theories that are considered to be intrinsically more reliable. (In this connection we

would refer to Ref. [13], where our $e(\rho,\delta)$ was incorporated in neutron star studies and the results compared with those based on other theoretical equations of state.) In the meantime, because of its algebraic simplicity and firm contact with relevant binding energy and diffuseness measurements, our expression for $e(\rho,\delta)$ could be used as a convenient baseline formula for the equation of state of cold nuclear matter.

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