Particle number fluctuations in the quasiparticle random-phase approximation and renormalized quasiparticle random-phase approximation

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Particle number fluctuations in the quasiparticle random-phase approximation (QRPA) and the renormalized quasiparticle random-phase approximation are investigated. A closed expression for the ground state wave function is used, which allows an exact evaluation of the ground state phonon number and correlation function, being valid to any order in the backward amplitudes. In realistic calculations a sudden increase in the particle number fluctuations is found in both approximations when the particle-particle interaction approaches the value at which the QRPA collapses. This behavior is strongly correlated with the ground state quasiparticle content and can be understood as a signature of the phase transition previously found in simpler exactly solvable models. [S0556-2813(98)06005-1]

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I. INTRODUCTION

Boson expansion techniques provide a useful tool to perform nuclear structure calculations [1]. Among them, the quasiboson approximation, more often called the random phase approximation (RPA) and its quasiparticle generalization (QRPA), have been widely used in the last decades to study electromagnetic transitions and beta decays in medium and heavy nuclei [1,2].

Quite recently studies analyzing the role of the proton-neutron interaction have had a revival. They were motived by novel experimental and theoretical results in $N\sim Z$ nuclei [3] and in double-beta decay. The proton-neutron (pn)-QRPA has been applied with success in the description of single- and double-beta decay processes. Nevertheless, it has some important limitations, the more remarkable being its collapse, i.e., the presence of imaginary eigenvalues for residual interaction strengths beyond a critical value [4–7]. Different improvements to the QRPA were presented [9–11] but were unable to avoid the collapse.

A whole family of extensions of the QRPA, called renormalized QRPA (RQRPA), is known that does not develop any collapse by implementing the Pauli principle in a consistent way, beyond the simplest quasiboson approximation [12–16]. However, in its simplest versions there is a violation of the non-energy-weighted Ikeda sum rule [17]. Calculations to determine the amount of the violation and some improvements to the RQRPA, in order to restore the sum rule, have been presented [18,19]. Recently, it has been shown that treating simultaneously BCS and QRPA equations one can fulfill the Ikeda sum rule in the Fermi case [20].

Using exactly solvable models a second and more profound difficulty has been found [21–24]. The calculations, which are equivalent to a complete shell model treatment in a single-j shell, show that the collapse of the QRPA correlates with the presence of an exact eigenvalue at zero energy or, in a more general view, with the clear presence of a phase transition [21,25]. Various authors have found a growing of the ground state quasiparticle content near the QRPA collapse, which could eventually produce strong fluctuations in the particle number, with important effects on the nuclear observables [26]. In this work we want to address these points by studying the expectation values of the particle and quasiparticle numbers, as well as particle number fluctuations in realistic spaces for different nuclei in the QRPA and RQRPA ground states. The paper is organized as follows: In Sec. II we review expressions for the mean particle number and fluctuations within a quasiparticle picture, for a correlated ground state. In Sec. III we evaluate these quantities for the RQRPA and its QRPA limit. The results for the case of ^{76}Ge, ^{82}Se, and ^{100}Mo for beta Fermi-type excitations are presented at Sec. IV and the conclusions in Sec. V.

II. PARTICLE NUMBER FLUCTUATION

The mean square fluctuation in the particle number is defined as

$$\Delta N^2 \equiv \langle 0|(\hat{N}-N)^2|0\rangle, \tag{1}$$

where $|0\rangle$ is the nuclear ground state, $N=\mathcal{Z},\mathcal{N}$ the actual number of protons or neutrons, and $\hat{N}=\hat{Z},\hat{N}$ the particle number operators, respectively. We make the transformation to quasiparticle creation and annihilation operators

$$\alpha^\dagger_{\mathbf{t}} = u_t a^\dagger_{\mathbf{t}} - v_t a_{\bar{\mathbf{t}}}, \tag{2}$$

where the subscripts $\mathbf{t}(t)$ stand for $\mathbf{p}(p)$ (protons) or $\mathbf{n}(n)$ (neutrons), being $\mathbf{t}\equiv t,m_t$, with $t\equiv\{n_t,l_t,j_t\}$ and $m_t\equiv m_j$, and $a_{\bar{\mathbf{t}}}=(-1)^{t+m_t}a_{t,-mt}$. The particle number $\hat{N}\equiv\Sigma_{\mathbf{t}}a^\dagger_{\mathbf{t}}a_{\mathbf{t}}$ in terms of quasiparticles reads

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$$\hat{N}=N+\sum_t \{2\Omega_t(u_t^2-v_t^2)\hat{\mathcal{N}}_t-u_tv_t[\hat{\mathcal{A}}_{tt}(0)+\hat{\mathcal{A}}_{tt}^\dagger(0)]\}, \tag{3}$$

with

$$\hat{\mathcal{N}}_t\equiv\frac{[\alpha_t^\dagger\alpha_{\bar{t}}]^0}{\sqrt{2\Omega_t}}, \quad \hat{\mathcal{A}}_{tt'}^\dagger(J)\equiv[\alpha_t^\dagger\alpha_{t'}^\dagger]^J, \quad \Omega_t\equiv\frac{(2j_t+1)}{2}, \tag{4}$$

and where we have assumed the BCS particle number condition $N=\Sigma_t 2\Omega_t v_t^2$. The particle number operator depends on the quasiparticle number operator $\hat{\mathcal{N}}_t$ and the two quasiparticle creation and destruction operators $\hat{\mathcal{A}}_{tt}^\dagger(0)$ and $\hat{\mathcal{A}}_{tt}(0)$, respectively.

Now from Eq. (1)

$$\Delta N^2=\langle N^2\rangle-2N\langle N\rangle+N^2, \tag{5}$$

with

$$\langle N\rangle\equiv\langle 0|\hat{N}|0\rangle=N+2\sum_t \Omega_t(u_t^2-v_t^2)\langle 0|\hat{\mathcal{N}}_t|0\rangle \tag{6}$$

and

$$\begin{aligned}\langle N^2\rangle\equiv\langle 0|\hat{N}^2|0\rangle=&N^2+4\sum_t \Omega_t^2u_t^2v_t^2+2\sum_t \Omega_t\\ &\times[2(u_t^2-v_t^2)N+u_t^4+v_t^4-6u_t^2v_t^2]\langle 0|\hat{\mathcal{N}}_t|0\rangle\\ &+4\sum_{t,t'}\sqrt{\Omega_t\Omega_{t'}}u_tv_tu_{t'}v_{t'}\langle 0|\hat{\mathcal{A}}^\dagger(0)_{tt}\hat{\mathcal{A}}_{t't'}(0)|0\rangle\\ &+\sum_{t,t'}(u_t^2-v_t^2)(u_{t'}^2-v_{t'}^2)\sum_J\sqrt{(2J+1)}\\ &\times\langle 0|[\hat{\mathcal{A}}^\dagger(J)_{tt'}\hat{\mathcal{A}}_{tt'}(\bar{J})]^0|0\rangle,\end{aligned} \tag{7}$$

where we have assumed that $|0\rangle$ has an equal number of proton and neutron quasiparticles, which is true for all the QRPA-type models we are interested in. Putting both Eqs. (6) and (7) into Eq. (5) we finally obtain

$$\begin{aligned}\Delta N^2=&4\sum_t \Omega_t^2u_t^2v_t^2+2\sum_t \Omega_t[u_t^4+v_t^4-6u_t^2v_t^2]\langle 0|\hat{\mathcal{N}}_t|0\rangle\\ &+4\sum_{t,t'}\sqrt{\Omega_t\Omega_{t'}}u_tv_tu_{t'}v_{t'}\langle 0|\hat{\mathcal{A}}^\dagger(0)_{tt}\hat{\mathcal{A}}_{t't'}(0)|0\rangle\\ &+\sum_{t,t'}(u_t^2-v_t^2)(u_{t'}^2-v_{t'}^2)\sum_J\sqrt{(2J+1)}\\ &\times\langle 0|[\hat{\mathcal{A}}^\dagger(J)_{tt'}\hat{\mathcal{A}}_{tt'}(\bar{J})]^0|0\rangle,\end{aligned} \tag{8}$$

In the mean number of particles, Eq. (6), the second term gives the contributions from ground state quasiparticle correlations and comes from the spurious state $(\hat{N}-N)|0\rangle$. On the other hand, the contributions to Eq. (8) come from the $(\hat{N}-N)^2|0\rangle$ spurious state. The first term is the number fluctuation within the BCS approximation. The other terms also come from ground state quasiparticle correlations. The second and third ones depend on the one- and two-quasiparticle occupations, while the fourth one depends on the two-quasiparticle (2qp) correlation functions.

III. RQRPA

We are interested in the evaluation of the particle number fluctuations in the pn-QRPA and pn-RQRPA ground states. Within the RQRPA the nuclear excited states are constructed as [14,19]

$$|\lambda JM\rangle\equiv\Omega^\dagger(\lambda JM)|0\rangle, \tag{9}$$

$$\Omega^\dagger(\lambda JM)=\sum_{pn}[X_{pn}(\lambda J)A_{pn}^\dagger(JM)-Y_{pn}(\lambda J)A_{pn}(\overline{JM})], \tag{10}$$

where

$$\hat{A}_{pn}^\dagger(JM)\equiv\hat{\mathcal{A}}_{pn}^\dagger(JM)D_{pn}^{-1/2}, \quad D_{pn}\equiv(1-\langle 0|\hat{\mathcal{N}}_p+\hat{\mathcal{N}}_n|0\rangle) \tag{11}$$

are the renormalized two-quasiparticle proton-neutron creation operators, which satisfy

$$\langle 0|[A_{pn}(JM),A_{p'n'}^\dagger(J'M')]|0\rangle=\delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'}. \tag{12}$$

Here $|0\rangle$ is the RQRPA correlated ground state, which must fulfil the condition

$$\Omega(\lambda JM)|0\rangle=0. \tag{13}$$

The amplitudes X and Y and the eigenvalues ω_λ satisfy the equations

$$\begin{pmatrix}A(J) & B(J)\\ B^*(J) & A^*(J)\end{pmatrix}\begin{pmatrix}X(\lambda J)\\ Y(\lambda J)\end{pmatrix}=\omega_{\lambda J}\begin{pmatrix}X(\lambda J)\\ -Y(\lambda J)\end{pmatrix}, \tag{14}$$

where

$$\begin{aligned}A_{pn,p'n'}(J)&=(\epsilon_p+\epsilon_n)\delta_{pp'}\delta_{nn'}+D_{pn}^{1/2}U_{pn,p'n'}^{\mathcal{F}}(J)D_{p'n'}^{1/2},\\ B_{pn,p'n'}(J)&=D_{pn}^{1/2}U_{pn,p'n'}^{\mathcal{B}}(J)D_{p'n'}^{1/2},\end{aligned} \tag{15}$$

with

$$\begin{aligned}U_{pn,p'n'}^{\mathcal{F}}(J)=&G(pn,p'n',JM)(u_pu_nu_{p'}u_{n'}+v_pv_nv_{p'}v_{n'})\\ &+F(pn,p'n',JM)(u_pv_nu_{p'}v_{n'}\\ &+v_pu_nv_{p'}u_{n'}),\\ U_{pn,p'n'}^{\mathcal{B}}(J)=&-G(pn,p'n',JM)(u_pu_nv_{p'}v_{n'}\\ &+v_pv_nu_{p'}u_{n'})+F(pn,p'n',JM)\\ &\times(v_pu_nu_{p'}v_{n'}+u_pv_nv_{p'}u_{n'}),\end{aligned} \tag{16}$$

F and G being the usual particle-hole (PH) and particle-particle (PP) coupled two-particle matrix elements. In order to evaluate the mean particle number and fluctuation from Eqs. (6) and (8), we need some information on the structure of the RQRPA ground state to get the quasiparticle occupations and correlation functions.

We write the ground state in the form

$$|0\rangle = N_0 e^{\hat{S}}|\mathrm{BCS}\rangle, \tag{17}$$

where

$$\hat{S} = \frac{1}{2}\sum_{pnp'n'J}(2J+1)C(J)_{pnp'n'}[A^{\dagger}_{pn}(J)A^{\dagger}_{p'n'}(J)]^0. \tag{18}$$

The coefficients $C(J)_{pnp'n'}$ can be estimated through Eq. (13), which leads to the set of equations

$$Y(\lambda J)^{*}_{p'n'} = \sum_{pn} C(J)_{pnp'n'}X(\lambda J)^{*}_{pn}. \tag{19}$$

While in previous works iterative expressions were developed for the matrix $C(J)$ [27,28], in this work we will manage the above expression as a matrix product, $\mathcal{Y}(J) = C(J)\mathcal{X}(J)$, with the matrices $\mathcal{X}(J)$ and $\mathcal{Y}(J)$ defined as

$$\mathcal{X}(J)_{pn,\lambda} \equiv X_{pn}(\lambda J), \quad \mathcal{Y}(J)_{pn,\lambda} \equiv Y_{pn}(\lambda J), \tag{20}$$

where $\mathcal{X}(J)$ [$\mathcal{Y}(J)$] have one column for each eigenvector $X(\lambda,J)$ [$Y(\lambda,J)$], and their rows are formed by the 2qp components pn. Thus introducing the *inverse matrix* $\mathcal{X}(J)^{-1}$ it is possible to obtain the matrix $C(J)$ explicitly [1] as

$$C(J)_{pnp'n'} = \sum_{\lambda} \mathcal{Y}(J)^{*}_{pn,\lambda}\mathcal{X}(J)^{*^{-1}}_{\lambda,p'n'}. \tag{21}$$

Now through the relation

$$\langle 0|\hat{O}|0\rangle = \langle 0|\hat{S}(\hat{O}+[\hat{O},\hat{S}]+1/2[[\hat{O},\hat{S}],\hat{S}]+\cdots)|\mathrm{BCS}\rangle \tag{22}$$

valid for any operator $\hat{O}$, and making the quasiboson approximation

$$[A_{pn}(JM),A^{\dagger}_{p'n'}(J'M')] \approx \langle 0|[A_{pn}(JM),A^{\dagger}_{p'n'}(J'M')]|0\rangle = \delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'}, \tag{23}$$

in the evaluation of the commutators involved in Eq. (22), we get for the case of protons

$$\langle 0|\hat{\mathcal{N}}_p|0\rangle = \sum_{\lambda Jn'}\frac{(2J+1)}{2\Omega_p}|\mathcal{Y}(J)_{pn',\lambda}|^2,$$

$$\langle 0|\hat{\mathcal{A}}^{\dagger}(0)_{pp}\hat{\mathcal{A}}_{p'p'}(0)|0\rangle = \sum_{\lambda\lambda'JJ'}\sum_{n,n'}\delta_{j_nj_{n'}}\frac{(2J+1)(2J'+1)}{\sqrt{\Omega_p\Omega_{p'}\Omega_n\Omega_{n'}}} \times \mathcal{Y}(J)_{pn,\lambda}\mathcal{Y}(J')^{*}_{p'n,\lambda'}\mathcal{X}(J)^{*}_{pn',\lambda}\mathcal{X}(J')^{*^{-1}}_{\lambda',p'n'} + \delta_{p,p'}\langle 0|\hat{\mathcal{N}}_p|0\rangle^2,$$

$$\langle 0|[\hat{\mathcal{A}}^{\dagger}(J)_{pp'}\hat{\mathcal{A}}_{pp'}(\overline{J})]^0|0\rangle = \sqrt{2J+1}\sum_{\lambda\lambda'}\sum_{n,n'}\mathcal{Y}(J)_{pn,\lambda}\mathcal{Y}(J)^{*}_{pn,\lambda'}\mathcal{X}(J)^{*}_{p'n',\lambda} \times \mathcal{X}(J)^{*^{-1}}_{\lambda',p'n'} + \frac{4\Omega_p\Omega_{p'}}{\sqrt{(2J+1)^3}}\langle 0|\hat{\mathcal{N}}_p|0\rangle\langle 0|\hat{\mathcal{N}}_{p'}|0\rangle. \tag{24}$$

Note that the quasiparticle occupations and correlation function are evaluated as functions of the RQRPA forward and backward matrix amplitudes $\mathcal{X}$ and $\mathcal{Y}$, respectively, and must be obtained together with the solution of the eigenvalue problem (14). The same quantities for neutrons are obtained from Eqs. (24) changing p and p' on the left hand side by n and n', and doing the same change in the sum indices. Finally the usual QRPA expressions can be obtained by taking the limit $D_{pn}=1$ in Eqs. (15).

IV. RESULTS

Now we present the calculus of the Fermi-type β excitations, and also analyze the evolution of the total mean quasiparticle number $\langle N_{\mathrm{qp}}\rangle \equiv \Sigma_t\langle 0|\hat{\mathcal{N}}_t|0\rangle$, the mean particle number $\langle N\rangle$, and the fluctuation ΔN as a function of the residual interaction parameter in the PP channel.

The Hamiltonian that leads to the eigenvalue problem in Eqs. (14) and (15) has the form

$$H = H_p + H_n + H_{p,n}. \tag{25}$$

The first two terms refer to the single-particle plus pairing Hamiltonians

$$H_t = \sum_{\mathbf{t}} e_t a^{\dagger}_{\mathbf{t}}a_{\mathbf{t}} + \sum_{\mathbf{t}'s}\langle \mathbf{t}_1\mathbf{t}_2|V|\mathbf{t}_3\mathbf{t}_4\rangle a^{\dagger}_{\mathbf{t}_1}a^{\dagger}_{\mathbf{t}_2}a_{\mathbf{t}_4}a_{\mathbf{t}_3}, \tag{26}$$

where the single particle energies are denoted by $e_{\mathbf{t}}$, and the last term to the proton-neutron interaction

$$H_{p,n} = \sum_{\mathbf{p},\mathbf{p}',\mathbf{n},\mathbf{n}'}\langle \mathbf{p},\mathbf{n}|V|\mathbf{p}',\mathbf{n}'\rangle a^{\dagger}_{\mathbf{p}}a^{\dagger}_{\mathbf{n}}a_{\mathbf{n}'}a_{\mathbf{p}'}. \tag{27}$$

After performing the quasiparticle transformation (2) we get [8]

$$H \approx \sum_{\mathbf{p}}\epsilon_p\alpha^{\dagger}_{\mathbf{p}}\alpha_{\mathbf{p}} + \sum_{\mathbf{n}}\epsilon_n\alpha^{\dagger}_{\mathbf{n}}\alpha_{\mathbf{n}} + H_{22} + H_{40} + H_{04}, \tag{28}$$

with

$$H_{22}=\sum_{p,n,p',n',JM} U^{\mathcal{F}}_{pn,p'n'}(J)A^{\dagger}_{pn}(JM)A_{p'n'}(\overline{JM}), \tag{29}$$

$$H_{04}=H^{\dagger}_{04}=-\sum_{p,n,p',n',JM} U^{\mathcal{B}}_{pn,p'n'}(J)A^{\dagger}_{pn}(JM)A^{\dagger}_{p'n'}(\overline{JM}). \tag{30}$$

The quasiparticle energies $\epsilon_{\mathbf{t}}$ explicitly appear in the matrix elements (15) while $U^{\mathcal{F}}$ and $U^{\mathcal{B}}$ have been defined in Eq. (16), being $J^{\pi}=0^{+}$ for the Fermi case.

We adopt a δ-type residual interaction already used previously [7,8],

$$V_{\alpha}=-4\pi v_{\alpha}\delta(\vec{r}) \;\; \text{MeV fm}^3, \tag{31}$$

with four different strength constants $v_{\alpha}=v^{\text{pair}}_{\mathbf{pp}},v^{\text{pair}}_{\mathbf{nn}},v^{pp}_{\mathbf{pn}}$, and $v^{ph}_{\mathbf{pn}}$, where ''pair'' refers to the $J=0$ PP channels for like nucleons and pp and ph to the $J=0$ PP and PH channels, respectively, in the proton-neutron case. Within $U^{\mathcal{F}}$ and $U^{\mathcal{B}}$, the PP matrix elements $G(pn,p'n',0)$ depend only on $v^{pp}_{\mathbf{pn}}$ and the particle-hole matrix elements $F(pn,p'n',0)$ only on $v^{ph}_{\mathbf{pn}}$.

Our Hilbert space has 11 single-particle energy levels, including all the single-particle orbitals from oscillator shells $3\hbar\omega$ and $4\hbar\omega$ plus $0h_{9/2}$ and $0h_{11/2}$ from the $5\hbar\omega$ oscillator shell. They were obtained using a Coulomb-corrected Woods-Saxon potential. Their numerical values for ^{76}Ge and ^{82}Se nuclei are tabulated in Table 1 of Ref. [8]. The pairing constants $v^{\text{pair}}_{\mathbf{pp}}$ and $v^{\text{pair}}_{\mathbf{nn}}$ were adjusted to reproduce the experimental pairing gaps for each nuclei, being their numerical values being close to 24 (see Table 2, Ref. [8]).

The particle-hole constant $v^{ph}_{\mathbf{pn}}$ mainly defines the energy of the isobaric analog state (IAS), which is experimentally found to be very close to the Coulomb displacement energy. It is fixed to reproduce this value. The only remaining parameter is $v^{pp}_{\mathbf{pn}}$. It has a very limited effect in the spectra of the odd-odd neighbor nuclei and its β^{-} decays, affecting mainly the lower energy states. But it is crucial in the determination of the β^{+} decay strength and the two-neutrino double-beta ($\beta\beta_{2\nu}$) decay amplitudes. It has been argued that $v^{pp}_{\mathbf{pn}}$ can be determined invoking maximal restoration of the isospin symmetry [7,8]. Other authors used some β^{+} decays in semimagic nuclei. It is by far the least controlled parameter in the formalism, and many studies have been devoted to the dependence of nuclear observables on it [4–6,14]. This interaction strength is responsible for the pn-QRPA collapse and our results will be given as a function of it. To this end we define the parameter

$$s=\frac{v^{pp}_{\mathbf{pn}}}{v^{\text{pair}}},$$

which is the ratio between the $T=1$, $S=0$ coupling constants in the PP channels and the pairing force constant $v^{\text{pair}}=(v^{\text{pair}}_{\mathbf{pp}}+v^{\text{pair}}_{\mathbf{nn}})/2$.

We compare the results obtained within the usual QRPA with those coming from the RQRPA. For simplicity we keep only states with $J^{\pi}=0^{+}$ in the summations in Eqs. (8) and (24), which corresponds to the SI1 approximation of Ref. [19].

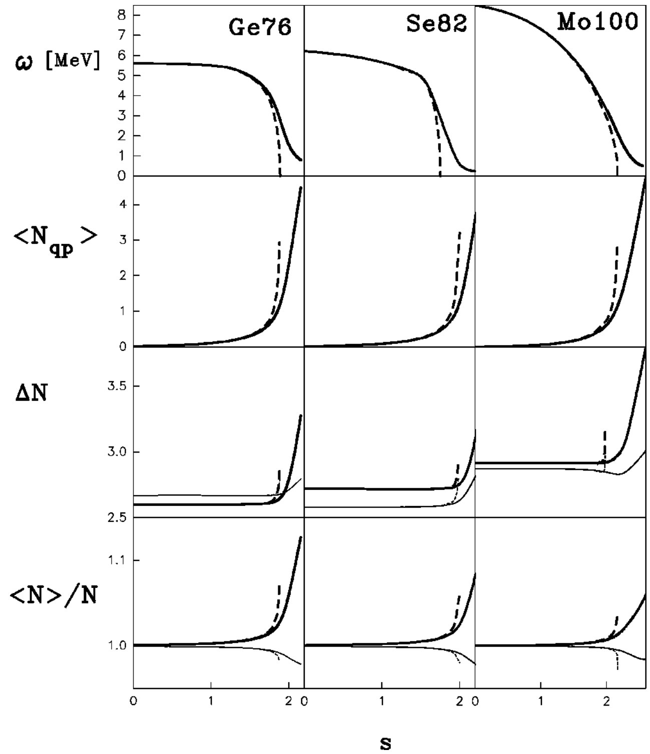

FIG. 1. Lowest eigenvalues ω, mean quasiparticle numbers $\langle N_{\text{qp}}\rangle$, particle number fluctuations ΔN and fractional particle numbers $\langle N\rangle/N$ are shown for ^{76}Ge, ^{82}Se, and ^{100}Mo nuclei. QRPA results are plotted as dashed lines and RQRPA ones as solid lines. In the last two rows thick lines are used for protons and thin lines for neutrons.

In the first row of Fig. 1 we show the lowest $J^{\pi}=0^{+}$ energies ω (in MeV) for ^{76}Ge, ^{82}Se, and ^{100}Mo for the QRPA (dashed lines) and RQRPA (solid lines) approximations. It is clear that the collapse in ω for the QRPA, which occurs around $s\approx 2$ in the different nuclei, is avoided for the RQRPA case. In the other rows in Fig. 1 we show the total mean quasiparticle number $\langle N_{\text{qp}}\rangle$ (equal for protons and neutrons), the fluctuations ΔN, and the fractional mean number of particles, $\langle N\rangle/N$, for protons and neutrons, respectively. The dashed thick (protons) and dashed thin lines (neutrons) correspond to QRPA results, and the solid-thick (protons) and solid-thin lines (neutrons) to the RQRPA approximation. As we can see near the collapse the total number of quasiparticles grows suddenly, and as the quasiparticle occupations are proportional to $|Y|^2$, it is an indication that backward-going amplitudes become dominant. It invalidates the simplest quasiboson approximation assumed to derive the QRPA and causes its collapse, producing a complex eigenvalue. The RQRPA is able to have a real lowest energy because it satisfies a quasiboson approximation with operators that are renormalized while ground state correlations grow. This renormalization process acts as an effective reduction of the residual interaction and the RQRPA eigenvalue problem remains stable. Nevertheless, if we observe the behavior of quasiparticle number and the mean number of particles, we note an important departure from the BCS values for these quantities above the collapse. The clear change in the particle number fluctuations in these realistic calculations confirms that the phase transition found in exactly solvable models [21,25] is present and affects these calculations. It could be also an indication that in the RQRPA spurious states have a significant weight in the ground state wave function beyond the QRPA collapse.

In the last row in Fig. 1 the fractional mean number of particles is presented. It increases for protons and decreases for neutrons. This behavior is controlled by the second term in Eq. (6), and in all the three examples the proton valence shell is more than half empty ($v_p^2 < u_p^2$), while the opposite is true for neutrons.

V. CONCLUSIONS

Inverting a matrix built with all the QRPA eigenvectors $X(\lambda J)$ we were able to use an exact expression for the QRPA and RQRPA ground state wave functions and evaluate the two-quasiparticle occupations and correlation functions. With these results particle number fluctuations were calculated in the QRPA and RQRPA ground states in realistic calculations of Fermi-type beta transitions. It was shown that these fluctuations in the RQRPA have a sudden increase for residual interaction strengths larger than those which produce the QRPA collapse. This phenomenon can be understood as a signature of the phase transition previously found in simpler exactly solvable models.

Self-consistent calculations can cure the problems with $\langle N \rangle$ [20], but the $J^\pi = 1^+$ Gamow-Teller Ikeda sum rule will be still violated due to the lack of scattering terms [19]. In both cases we would expect that the particle number fluctuation will remain exploding near the phase transition.

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[1] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
[2] D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970).
[3] S. Pittel, Rev. Mex. Fis. **43**, Suppl. 1, 92 (1997) and references therein.
[4] P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57**, 731 (1986).
[5] O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. B **194**, 11 (1987).
[6] J. Engel, P. Vogel, and M. R. Zirnbauer, Phys. Rev. C **37**, 731 (1988).
[7] J. Hirsch and F. Krmpotić, Phys. Rev. C **41**, 792 (1990).
[8] J. Hirsch, E. Bauer, and F. Krmpotić, Nucl. Phys. **A516**, 304 (1990).
[9] A. A. Raduta, A. Faessler, S. Stoica, and W. A. Kaminski, Phys. Lett. B **254**, 7 (1991).
[10] A. A. Raduta, A. Faessler, and D. S. Delion, Nucl. Phys. **A564**, 185 (1993).
[11] F. Krmpotić, A. Mariano, T. T. S. Kuo, and K. Nakayama, Phys. Lett. B **319**, 393 (1993).
[12] D. J. Rowe, Phys. Rev. **175**, 1283 (1968); Rev. Mod. Phys. **40**, 153 (1968).
[13] F. Catara, N. Dinh Dang, and M. Sambataro, Nucl. Phys. **A579**, 1 (1994).
[14] J. Toivanen and J. Suhonen, Phys. Rev. Lett. **75**, 410 (1995).
[15] J. Schwieger, F. Šimkovic, and A. Faessler, Nucl. Phys. **A600**, 179 (1996).
[16] J. Dukelsky and P. Schuck, Phys. Lett. B **387**, 233 (1996).
[17] J. G. Hirsch, P. O. Hess, and O. Civitarese, Phys. Rev. C **54**, 1976 (1996).
[18] F. Krmpotić, A. Mariano, E. J. V. de Passos, A. F. R. de Toledo Piza, and T. T. S. Kuo, Fizika B **5**, 93 (1996).
[19] F. Krmpotić, T. T. S. Kuo, A. Mariano, E. J. V. de Passos, and A. F. R. de Toledo Piza, Nucl. Phys. **A612**, 223 (1996).
[20] D. S. Delion, J. Dukelsky, and P. Schuck, Phys. Rev. C **55**, 2340 (1997).
[21] J. Engel, S. Pittel, M. Stoitsov, P. Vogel, and J. Dukelsky, Phys. Rev. C **55**, 1781 (1997).
[22] J. G. Hirsch, P. O. Hess, and O. Civitarese, Phys. Lett. B **390**, 36 (1997).
[23] J. G. Hirsch, P. O. Hess, and O. Civitarese, Phys. Rev. C **56**, 199 (1997).
[24] J. G. Hirsch, P. O. Hess, and O. Civitarese, Rev. Mex. Fis. **43**, Suppl. 1, 78 (1997).
[25] O. Civitarese, P. O. Hess, J. G. Hirsch, Phys. Lett. B **412**, 1 (1997).
[26] M. Sambarato and J. Suhonen, Phys. Rev. C **56**, 782 (1997).
[27] J. Da Providencia, Phys. Lett. **21**, 668 (1966).
[28] E. A. Sanderson, Phys. Lett. **19**, 141 (1965).