New evaluation of the Baldin sum rule

D. Babusci, G. Giordano, and G. Matone
Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Rome, Italy
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The Baldin sum rule for the nucleon has been recalculated at the light of the most recent photoabsorption cross section measurements. The proton value $(\alpha+\beta)_p=13.69\pm0.14$ is smaller but consistent with the one usually quoted in literature. However, the value for the neutron $(\alpha+\beta)_n=14.40\pm0.66$ turns out to be three standard deviations away from the previously calculated one. [S0556-2813(98)00701-8]

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The optical theorem applied to the forward Compton amplitude, together with the low-energy theorem, leads to the once-subtracted dispersion relation worldwide known as the Baldin sum rule [1]. This equation establishes a firm connection between the integral of the ν^2-weighted nucleon unpolarized photoabsorption cross section and the sum of the electric (α) and magnetic (β) polarizabilities of the nucleon target:

$$(\alpha+\beta)_N=\frac{1}{2\pi^2}\int_{\nu_0}^{\infty}d\nu\,\frac{\sigma(\gamma N\to X)}{\nu^2}, \tag{1}$$

where ν_0 is the pion photoproduction threshold. Since the integral on the right-hand side can be numerically evaluated on the basis of the photoabsorption cross section data, Eq. (1) leads to an unavoidable bound, that, as such, is routinely used to constrain the values of the polarizabilities extracted from the low-energy Compton scattering data.

The numerical value quoted in literature for the proton[1]

$$(\alpha+\beta)_p=14.2\pm0.3, \tag{2}$$

was calculated over 25 years ago by Damashek and Gilman [2]. They used the experimental data available at that time and postulated a reasonable theoretical ansatz for the extrapolation at infinite energy whose uncertainty is what fully determines the error bar quoted in Eq. (2), without taking into account any other source of errors.

As for the neutron, the first, and still unique, complete calculation of the sum rule was made in 1979 by the authors of Ref. [3]. In this calculation the integration domain is broken down into a resonance ($\nu\leqslant1.5$ GeV) and an asymptotic ($\nu>1.5$ GeV) region. In the first region, they used a multipole analysis of the single-pion photoproduction data and assumed that the two pion contributions were dominated by the leading Δ and ρ-meson photoproduction channels. By using the parametrization given in Ref. [5] for the asymptotic regime, they finally obtained

$$(\alpha+\beta)_n=15.8\pm0.5. \tag{3}$$

[1]Hereafter the polarizability values are expressed in units of 10^{-4} fm^3.

Since today the status of the experimental data is much better defined than it was 20 years ago, it is now time to revisit the analysis for both the values of Eqs. (2) and (3). Let us discuss the two cases separately.

THE PROTON

The integration domain has been divided into the following four energy regions: the threshold region $\mathcal{A}^{(p)}$: $\nu\in[\nu_0,\ 0.2)$ GeV; the resonance region $\mathcal{B}^{(p)}$: $\nu\in[0.2,\ 2.0)$ GeV; the high-energy region $\mathcal{C}^{(p)}$: $\nu\in[2.0,\ 183.0)$ GeV; the asymptotic region $\mathcal{D}^{(p)}$: $\nu\in[183.0,\ \infty)$ GeV.

In the threshold region the total cross section has been calculated by a numerical integration of the π^0p and π^+n contributions given by the SAID program (solution SP97K) [4]. The finite spacing between the points (1 MeV) generates an uncertainty in the evaluation of the subtended area which reflects itself in the error quoted in Table I for $(\alpha+\beta)$ in this region.

In the resonance region we have used the old values for the total cross section measured at Daresbury [5] and the new data recently obtained at Mainz [6] in the interval $\nu\in(204,\ 789)$ MeV. All these data (for a total of 138 points) have been fitted using a minimizing function written as a sum of the six prominent Breit-Wigner resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $F_{15}(1680)$, $F_{37}(1950)$, and a smooth background parametrized as follows [5]:

$$\sigma_{\mathrm{B}}=\sum_{k=-2}^{2}C_k(W-W_0)^k, \tag{4}$$

where $W=M_p\sqrt{1+2\nu/M_p}$ is the center-of-mass energy and $W_0=W(\nu=\nu_0)$. However, our major interest has not been focused on the extraction of the resonance parameters but only on the determination of the most faithful mathematical description of the data. As a consequence of the ν^2 weight in Eq. (1), this description must be particularly accurate in the low-energy region. Therefore, instead of using the parametrization of Ref. [12], we have adopted Eq. (4) as a description of the nonresonant pionic background. This choice produces a lower reduced χ^2_{df} and a more accurate description of the behavior of the data on the rise of the Δ resonance. Only the statistical errors have been considered.

TABLE I. Contributions to $(\alpha+\beta)$ of the proton from the four energy regions defined in the text. The two values for the region $\mathcal{D}$ are obtained using Eq. (6) (upper) and Eq. (7) (lower), respectively.

Energy region	$\mathcal{A}^{(p)}$	$\mathcal{B}^{(p)}$	$\mathcal{C}^{(p)}$	$\mathcal{D}^{(p)}$	Total
$(\alpha+\beta)_p$	1.25 ± 0.02	11.71 ± 0.13	0.72 ± 0.03	$(7.0\pm0.3)\times10^{-3}$ $(6.8\pm2.1)\times10^{-3}$	13.69 ± 0.14

The complete collection of the data in the resonance region together with our fitting curve are shown in Fig. 1. Since below about 400 MeV, the absorption cross section is completely dominated by single pion photoproduction, an independent measurement of the total cross section can also be deduced from the multipole analysis of $\gamma N\rightarrow\pi N$. However, pion production experiments between 1970 and 1980 display an unusual dichotomy near the peak of Δ resonance. For photon energies either below 280 MeV or above 360 MeV, π^+ and π^0 data taken at Bonn [7], Tokyo [8], and Lund [9] are quite in good agreement. Instead, within this energy range the Tokyo π^+ data and the Lund π^0 data are consistently higher than their Bonn counterparts. Since the recent Mainz absorption measurements [6] are in good agreement with integrations of the Bonn π^+ and π^0 cross sections [10], the Tokyo and Lund data have fallen into general disfavor. However, very recent π^+ and π^0 cross sections measured by the LEGS collaboration at BNL [11] in the interval $\nu\in(210,\ 333)$ MeV are in fact in good agreement with the Tokyo and Lund data sets. Evidently the dichotomy at the Δ resonance still persists.

To examine the consequences of the higher Tokyo/Lund/BNL cross sections we have repeated the fit in the resonance region, using the total cross sections from the multipole analysis of the BNL data in place of the Daresbury and Mainz data below 340 MeV (in the following we shall refer to this as fit II). This fit departs from the one displayed in Fig. 1 only at the top of the Δ resonance, where the total cross section turns out to be approximately 6% higher. The χ^2_{df} is slightly worse but the parameters of all the resonances involved are well reproduced within the errors.

According to Ref. [5] in the region between 2 and 3 GeV, the cross section can be parametrized in the following way:

$$\sigma(\gamma p\rightarrow X)=a_1+\frac{a_2}{\sqrt{\nu}}$$

with

$$a_1=91.0\pm5.6\ \ \mu\text{b},$$

$$a_2=71.4\pm9.6\ \ \mu\text{b}\ \ \text{GeV}^{1/2}. \tag{5}$$

An accurate fit of all the data avalaible in the remaining part of the region $\mathcal{C}$ can be found in Ref. [13] where the following parametrization is used (ν in GeV):

$$\sigma(\gamma p\rightarrow X)=A+B\ln^2\nu+C\ln\nu$$

with

$$A=147\pm1\ \ \mu\text{b},$$

$$B=2.2\pm0.1\ \ \mu\text{b},$$

$$C=-17.0\pm0.7\ \ \mu\text{b}. \tag{6}$$

This parametrization has been assumed valid also in the asymptotic region $\mathcal{D}$ and its result compared to the one given by the model of Donnachie and Landshoff where, for $\nu\geqslant12$ GeV, the total cross section is parametrized in this other way [14] ($s=W^2$ in GeV^2):

$$\sigma(\gamma p\rightarrow X)=Xs^{\varepsilon}+Ys^{\eta}$$

with

$$X=71\pm18\ \ \mu\text{b},\quad Y=120\pm40\ \ \mu\text{b},$$

$$\varepsilon=0.075\pm0.030,\quad \eta=-0.46\pm0.25. \tag{7}$$

The contributions to $(\alpha+\beta)$ coming from the four regions defined above are reported in Table I, where for the asymptotic region the two values are the results obtained from Eq. (6) (upper) and Eq. (7) (lower), respectively. By summing up these four contributions one has

$$(\alpha+\beta)_p=13.69\pm0.14. \tag{8}$$

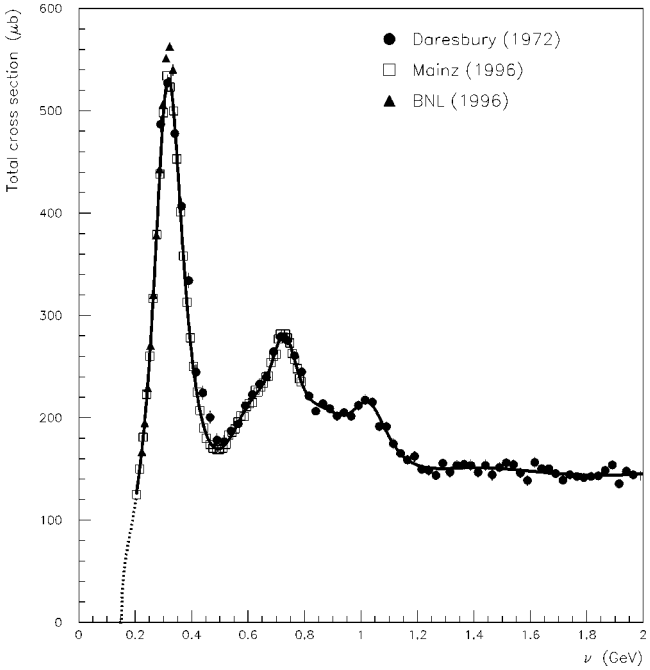

FIG. 1. Photoabsorption cross section for the proton as a function of the energy of the incoming photon. The dotted line is the result obtained from the SAID program in the threshold region.

The use of the fit II in the resonance region would increase this value up to 13.76, well within the quoted error in Eq. (8). Therefore the debate on the value of the total cross section at the top of the Δ resonance does not seem to be relevant for our present purpose.

THE NEUTRON

The neutron case can be calculated by assuming that the photoabsorption cross section on the free neutron can be simply obtained by the ''difference'' between the deuteron and proton data. The way to perform this difference is not firmly established and can drive to evident inconsistencies. As an example, in the region of the Δ resonance the sum of the one-pion photoproduction cross sections [4] alone is about 150 μb larger than the total absorption cross section published in Ref. [15]. Since the photoabsorption cross section on the deuteron measured at Daresbury has been recently confirmed by the Mainz data [6], the discrepancy has to arise from the procedure followed to extract the neutron cross section from the deuteron data. This implies that further assumptions will be necessary with the consequence that the resulting value for $(\alpha+\beta)_n$ is much more procedure dependent than that for $(\alpha+\beta)_p$.

Also in the deuteron case the energy range is divided in the four following regions: the threshold region $\mathcal{A}^{(n)}$: $\nu \in [\nu_0,\ 0.2)$ GeV; the resonance region $\mathcal{B}^{(n)}$: $\nu \in [0.2,\ 2.0)$ GeV; the high-energy region $\mathcal{C}^{(n)}$: $\nu \in [2.0,\ 18.0)$ GeV; the asymptotic region $\mathcal{D}^{(p)}$: $\nu \in [18.0,\ \infty)$ GeV. Similarly to the proton case, the total photoabsorption cross section in the threshold region results from the sum of the $\pi^- p$ and $\pi^0 n$ channels as given by the SAID program.

In the resonance region the avalaible data [6,15] for the deuteron target have been fitted using the same procedure followed in Ref. [15]. The minimizing function is the same as that for the proton with a nonresonant pionic background constrained to be twice the one found for the proton. Furthermore, following Ref. [12] we have added a deuteron photodisintegration background which gives a non-negligible contribution mainly to the Δ region [16]. The result of this fit together with the experimental data are shown in Fig. 2. The neutron cross section has been derived under the assumptions that the positions W and widths Γ of the resonances are the same for both the proton and the neutron and the coupling constants are related by

$$I_r^n = \frac{1}{\Gamma_r^p}(I_r^D \Gamma_r^D - I_r^p \Gamma_r^p).$$

The validity of these assumptions are discussed at length in Ref. [12].

In the high-energy region $\mathcal{C}$ the photoabsorption cross section on deuteron can be parametrized by the expression of Eq. (6), where [13]

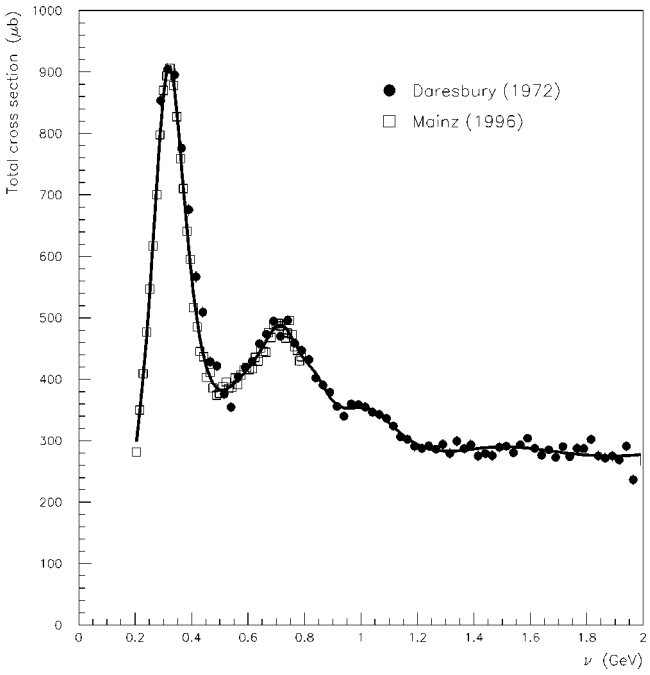

FIG. 2. As Fig. 1 in the deuterocase.

$$A_D = 300 \pm 5\ \ \mu\text{b}, \quad B_D = 9.5 \pm 2.0\ \ \mu\text{b},$$

$$C_D = -57 \pm 7\ \ \mu\text{b}.$$

According to Ref. [15] for $\nu \in (2,\ 4)$ GeV it turns out that

$$\sigma_n \simeq 1.015\sigma_D - \sigma_p\,. \tag{9}$$

Therefore, by assuming that this relationship can be extended to the whole region $\mathcal{C}$, for the neutron one has

$$A_n = 157.5 \pm 5.2\ \ \mu\text{b}, \quad B_n = 7.4 \pm 2.0\ \ \mu\text{b},$$

$$C_n = -40.9 \pm 7.1\ \ \mu\text{b}.$$

Finally, in complete analogy with the proton case, we have assumed that the parametrization in the region $\mathcal{C}$ can be successfully extended to the asymptotic region $\mathcal{D}$.

The contributions to $(\alpha+\beta)$ coming from the four regions defined above are reported in Table II and their sum is

$$(\alpha+\beta)_n = 14.40 \pm 0.66. \tag{10}$$

SUMMARY AND CONCLUSIONS

The present reevaluation of the sum rule (1) leads to the following conclusions.

(1) The new values of Eqs. (8),(10) are both smaller than the corresponding values of Eqs. (2),(3) but, within errors, they are still consistent with each other. The lower value for the proton could be due to the set of data used in the analysis of Ref. [2] that consistently exceed the Daresbury data in the

TABLE II. Same as Table I for the neutron case.

Energy region	$\mathcal{A}^{(n)}$	$\mathcal{B}^{(n)}$	$\mathcal{C}^{(n)}$	$\mathcal{D}^{(n)}$	Total
$(\alpha+\beta)_n$	1.86 ± 0.02	11.95 ± 0.66	0.52 ± 0.05	0.07 ± 0.02	14.40 ± 0.66

region of the P_{11} and D_{13} resonances.

(2) The proton and neutron values are much closer now than in the previous analysis. The present separation between these two values is within errors, whereas, before, the same separation was twice the sum of the quoted errors. This is consistent with the old claim reported in Ref. [17] that no isotopic effect has to be expected for the quantity $\alpha+\beta$.

(3) Chiral perturbation theory at $O(q^4)$ gives the following prediction for the sum rule [18]:

$$(\alpha+\beta)_p = 14.0 \pm 4.1, \quad (\alpha+\beta)_n = 21.2 \pm 3.9$$

that is consistent with Eq. (8) for the proton but appears to be ruled out by both the present and old analysis in the neutron case. Instead, our combined proton/neutron result is much more in line with the $O(q^3)$ prediction which reads [18]

$$(\alpha+\beta)_p = (\alpha+\beta)_n = 13.3.$$

As a matter of fact, this value is well consistent with our proton value and is less than two standard deviations away from the neutron value.

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