Polarization phenomena for low energy $d+{}^3\mathrm{He}$ collisions

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Polarization phenomena are studied for the reaction $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ (for $E_d\leq 1$ MeV) using a formalism of partial amplitudes. The nuclei are considered as elementary particles with definite values of spin and P parity. A general parametrization of the matrix element in terms of a limited number of partial amplitudes (for $d+{}^3\mathrm{He}$ interaction in s and p states) is given and the expressions for all polarization observables are derived for the s interaction. Relations between different polarization observables are derived and the conditions for maximizing the differential cross section of the process $\vec{d}+{}^3\vec{\mathrm{He}}\to p+{}^4\mathrm{He}$ are indicated. The spin structure of the matrix elements of the processes $d+{}^3\mathrm{He}\to d+{}^3\mathrm{He}$, $d+{}^3\mathrm{He}\to n+p+{}^3\mathrm{He}$, $d+{}^3\mathrm{He}\to p+p+{}^3\mathrm{H}$, and $d+{}^3\mathrm{He}\to d+d+p$ is established in the near-threshold region. [S0556-2813(98)02106-2]

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I. INTRODUCTION

The experimental and theoretical study of the process $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ (as well as the charge symmetric process $d+{}^3\mathrm{H}\to n+{}^4\mathrm{He}$) at very low energies has always been motivated by questions in fundamental physics and by interesting possible applications.

The structure of light nuclear systems [1,2] is investigated in systematic experimental studies, which represent a good test of theoretical calculations of reaction mechanisms involving deuterons and three-body systems [3,4].

Concerning applications we will quote in particular the interest in astrophysics and in energy production. The S factors, which determine the threshold behavior of cross sections, are very important parameters in models of big-bang nuclear synthesis, stellar hydrogen burning, deuterium in low mass protostars, solution of the Sun-neutrino puzzle, etc.

Recently a conceptual design of a $d-{}^3\mathrm{He}$ fueled field reverse configuration reactor (ARTEMIS) [5–7], has been stimulated by the discovery of a large quantity of minable ${}^3\mathrm{He}$ on the lunar surface. The main advantage of using reactions based on ${}^3\mathrm{He}$ instead of ${}^3\mathrm{H}$ for energy production is that it is possible in principle to decrease the production of neutrons or long-lived radioactive nuclei.

Largely esotermic $d+{}^3\mathrm{He}$ and $d+{}^3\mathrm{H}$ reactions pass through the excitation of a fusion resonance, characterized, in the first case, by $\mathcal{J}^\pi=\frac{3}{2}^+$ and peaked at $E_d\simeq 450$ keV ($\mathcal{J}$ and π are the total angular momentum and the P parity of the channel). The details of the reaction mechanisms can be clarified by the measurement of polarization observables. Experimental results have been obtained recently for the reactions $\vec{d}+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ and $\vec{d}+{}^3\mathrm{H}\to n+{}^4\mathrm{He}$ [8,9], for $E_d\leq 1$ MeV. The measured values of vector and tensor analyzing powers and the isotropic differential cross section confirm the hypothesis of the $\mathcal{J}^\pi=\frac{3}{2}^+$ excitation.

An attractive property of the low-energy $d+{}^3\mathrm{He}$ and $d+{}^3\mathrm{H}$ collisions is the possibility to study the energy level structure of such five-nucleon systems as ${}^5\mathrm{He}$ and ${}^5\mathrm{Li}$, which are relatively simple nuclear systems with few excited (well-separated) states. Microscopic calculations predicting six positive parity states and two negative parity states at excitation energy of 20 MeV [10] can be fully tested by a comparison with experimental data if polarization observables are measured.

The comparison of different observables for the processes $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ and $d+{}^3\mathrm{H}\to n+{}^4\mathrm{He}$ is important also to test the charge symmetry (isotopic invariance) of nuclear interactions. For a five-nucleon system the behavior of the polarization observables can show possible effects of charge symmetry violation. In the framework of QCD the charge symmetry breaking (CSB) is connected mainly with the mass difference between the u and d quarks [11]. From the existing data there is some evidence of CSB: for example the position of the $\mathcal{J}^\pi=\frac{3}{2}^+$ resonance in $d+{}^3\mathrm{He}$ ($E_x=16.87$ MeV, $E_d=430$ keV) and $d+{}^3\mathrm{H}$ ($E_x=16.84$ MeV, $E_d=107$ keV) are the same but the tensor analyzing powers are definitely different [8].

For completeness it is necessary to mention also that the process $\vec{d}+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ can be considered a stable beam polarization monitor, at low energies. In this respect the advantages of this reaction are well known: the high Q value (Q value=18.353 MeV) allows an easy detection of the final proton, the large cross section ($\sigma\simeq 695\pm 14$ mb) results in rapid measurements and the tensor analyzing power is generally large and varies smoothly with the deuteron energy.

Note that the measurement of the cross section and of a single polarization observable (such as T_{20}) is not sufficient for a complete reconstruction of the two possible amplitudes of the low energy $d+{}^3\mathrm{He}$ interaction in s state, corresponding to the possible quantum numbers $\mathcal{J}^\pi=\frac{1}{2}^+$ and $\frac{3}{2}^+$ in the entrance channel. Shell model calculations [10] predict also the presence of $\mathcal{J}^\pi=\frac{1}{2}^+$ state of ${}^5\mathrm{Li}$ and ${}^5\mathrm{He}$ above the first $\frac{1}{2}^-$ excited state, but the strength of this resonance is strongly dispersed, due to a large coupling to the $\alpha+N$ channel. Polarization phenomena could bring new information, on this point, in a model independent way.

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Polarization observables are also important to understand the mechanisms of electromagnetic processes such as $d+{}^3\mathrm{He}\to{}^5\mathrm{Li}+\gamma$ and $d+{}^3\mathrm{H}\to{}^5\mathrm{He}+\gamma$ [12]. The energy dependence of the cross section for $d+{}^3\mathrm{He}\to{}^5\mathrm{Li}+\gamma$ shows a large contribution of the $\frac{3}{2}^+$ resonance. The measure of the tensor analyzing powers allowed a multipole analysis of the data which gave two solutions (with the same χ^2) for the amplitudes of the electric dipole ($E1$) radiation: one corresponding to a single $E1$ transition from $\mathcal{J}^\pi=\frac{3}{2}^+$ and another to two possible $E1$ transitions (of equal intensity) from $\mathcal{J}^\pi=\frac{1}{2}^+$ and $\frac{3}{2}^+$. This ambiguity can be solved by further polarization measurements.

We propose here a formalism for the description of polarization phenomena for the $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ process in a model independent way, based on general symmetry properties of the strong interaction, such as the P invariance and the conservation of the total angular momentum. The relevant parameters for this analysis are the quantum numbers (spin and P parity) of the participating nuclei: the nuclei are considered as elementary particles, with definite values of spin and parity. Our formalism is well adapted to the analysis of polarization phenomena in binary reactions at low energies, where only s and p interactions are important.

Note that the most general analysis of polarization phenomena for the reaction $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ (or $d+{}^3\mathrm{H}\to n+{}^4\mathrm{He}$) has been done in [13], in a model independent way, when the spin structure of the corresponding matrix element and the properties of polarization phenomena have been established using only symmetries of the strong interaction. Such a formalism has been developed by Csonka, Moravcsik, and Scadron [14] and can be considered as a generalization of the well-known Wolfenstein analysis of elastic nucleon-nucleon scattering [15]. However we would like to note here that threshold conditions are very specific and can not be considered as a limiting case of such general formalism: the parametrization of the $\mathcal{M}$ matrix in terms of six independent spin structures [13], does not permit a direct transition to threshold. At threshold, for s-wave interaction, we have only one three-momentum for the final particles, therefore it is not possible to define a standard coordinate system (on the basis of two independent three-momenta), with $\vec{p}$, $\vec{n}$, and $\vec{k}$ direction (from [13]), as is the case, in general, for any binary reaction. The parametrization of the spin structure, as well as the analysis of polarization phenomena must be done in the framework of a special formalism, which takes into account only one three-momentum direction.

This paper is organized as follows. In Sec. II we parametrize the spin structure of the matrix element of the $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ process for the $d+{}^3\mathrm{He}$ interaction in s state and analyze the possible polarization observables (including three-spin correlations). Section III contains the derivation of polarization phenomena in the $s+p$ approximation. In Sec. IV we discuss the spin structure of the threshold amplitudes for different possible processes induced by low energy $d+{}^3\mathrm{He}$ collisions, namely $d+{}^3\mathrm{He}\to d+{}^3\mathrm{He}$ (elastic scattering), $d+{}^3\mathrm{He}\to n+p+{}^3\mathrm{He}$, $d+{}^3\mathrm{He}\to p+p+{}^3\mathrm{H}$, and $d+{}^3\mathrm{He}\to d+d+p$.

II. POLARIZATION PHENOMENA FOR THE *s*-STATE INTERACTION

We consider here the reaction $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ ($d+{}^3\mathrm{H}\to n+{}^4\mathrm{He}$) at small incident energy, where the configuration of the colliding particles is s state only. The analysis of polarization phenomena is essentially simplified and can be done in a very general form. The P invariance of the strong interaction and the conservation of total angular momentum allow the following transitions:

$$S_i=\frac{1}{2}\to\mathcal{J}^\pi=\frac{1}{2}^+\to\ell_f=0,$$

$$S_i=\frac{3}{2}\to\mathcal{J}^\pi=\frac{3}{2}^+\to\ell_f=2,$$

where S_i is the total spin of the $d+{}^3\mathrm{He}$ system and ℓ_f is the orbital angular momentum of the final proton. The spin structure of the corresponding matrix element can be parametrized in the form

$$\mathcal{M}=\chi_2^\dagger\mathcal{F}_{th}\chi_1,$$

$$\mathcal{F}_{th}=g_s\vec{\sigma}\cdot\vec{D}+g_d(3\vec{k}\cdot\vec{D}\vec{\sigma}\cdot\vec{k}-\vec{\sigma}\cdot\vec{D}), \quad (1)$$

where χ_1 and χ_2 are the two-component spinors of the initial ${}^3\mathrm{He}$ and final p, $\vec{D}$ is the three-vector of the deuteron polarization (more exactly, $\vec{D}$ is the axial vector due to the positive parity of the deuteron), $\vec{k}$ is the unit vector along the three-momenta of the proton (in the CMS of the considered reaction) and g_s and g_d are the amplitudes of the s and d production of the final particles. In the general case g_s and g_d are complex functions of the excitation energy.

We can notice that, at any energy, the spin structure of the matrix element for the process $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ in collinear kinematics ($\theta=0°$ or π) is the same as in Eq. (1), but the physical interpretation of these amplitudes is different: in this case two independent transitions characterize the conservation of helicity.

It follows from Eq. (1) that, in principle, the full reconstruction of the spin structure of the threshold matrix element requires, in the general case, at least three independent experiments: in addition to the differential unpolarized cross section, it is necessary to measure two polarization observables. The recent developments of polarized beams and targets makes possible experiments where both colliding particles are polarized. Depending on the reaction kinematics the study of the collision between two polarized particles may be preferable (from an experimental point of view) to the measurement of the polarization of an outgoing particle with the help of a polarimeter (which requires a secondary scattering), and brings the same physical information.

The general parametrization of the differential cross section in terms of the polarizations of the colliding particles (in s state) is given by

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}})=\left(\frac{d\sigma}{d\Omega}\right)_0[1+\mathcal{A}_1(Q_{ab}k_ak_b)+\mathcal{A}_2\vec{S}\cdot\vec{P}+\mathcal{A}_3\vec{k}\cdot\vec{P}\vec{k}\cdot\vec{S}+\mathcal{A}_4\vec{k}\cdot\vec{P}\times\vec{Q}], \quad Q_a=Q_{ab}k_b, \quad (2)$$

where $(d\sigma/d\Omega)_0$ is the differential cross section with unpolarized particles, $\vec{P}$ is the axial vector of the target polarization, $\vec{S}$ and Q_{ab} are the vector and tensor deuteron polarizations. The density matrix of the deuteron can be written as

$$\overline{D_a D_b^*} = \frac{1}{3}\left(\delta_{ab} - \frac{3}{2} i \epsilon_{abc} S_c - Q_{ab}\right),$$

$$Q_{aa} = 0, \quad Q_{ab} = Q_{ba}. \tag{3}$$

The coefficient $\mathcal{A}_1$ in Eq. (2) is real and represents the tensor analyzing power in $\vec{d} + {}^3\mathrm{He} \to p + {}^4\mathrm{He}$; the coefficients $\mathcal{A}_2$ and $\mathcal{A}_3$ determine the spin correlation coefficients in $\vec{d} + {}^3\vec{\mathrm{He}}$ collisions (due to the vector polarization of the colliding particles):

$$C_{xx} = C_{yy} = \mathcal{A}_2, \quad C_{zz} = \mathcal{A}_2 + \mathcal{A}_3, \tag{4}$$

if the z axis is along the three-momenta $\vec{k}$. The coefficient $\mathcal{A}_4$ characterizes the simplest T-odd polarization effect for this process, induced by a definite combination of the tensor deuteron polarization and the vector polarization of ^{3}He.

In the general case, for a binary process $A + B \to C + D$ the deuteron tensor polarization, Q_{ab}, contributes to the cross section through three combinations $Q_{ab} k_a k_b$, $Q_{ab} q_a q_b$, and $Q_{ab} k_a q_b$, where $\vec{q}$ is the unit vector along the three-momentum of the colliding particles. In the particular cases of collinear kinematics and threshold conditions, the only meaningful combination is $Q_{ab} k_a k_b$ (which is equivalent to T_{20}, if the z axis is along $\vec{k}$). Equation (2) is the first example of a particular structure of polarization observables, with evident simplification in comparison with the general case. This result is obtained directly, taking into account the P invariance of strong interaction, and the presence of a single three-momentum, $\vec{k}$, for threshold conditions.

After summing over the final proton polarizations one can find the following expressions for the coefficients $\mathcal{A}_i$:

$$\mathcal{A}_1 \left(\frac{d\sigma}{d\Omega}\right)_0 = -2\,\mathrm{Re}\, g_s g_d^* - |g_d|^2,$$

$$\mathcal{A}_2 \left(\frac{d\sigma}{d\Omega}\right)_0 = -|g_s|^2 - \mathrm{Re}\, g_s g_d^* + 2|g_d|^2, \tag{5}$$

$$\mathcal{A}_3 \left(\frac{d\sigma}{d\Omega}\right)_0 = 3\,\mathrm{Re}\, g_s g_d^* - 3|g_d|^2,$$

$$\mathcal{A}_4 \left(\frac{d\sigma}{d\Omega}\right)_0 = -2\,\mathrm{Im}\, g_s g_d^*.$$

In a particular normalization of the amplitudes g_s and g_d the differential cross section can be represented in the following way:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = |g_s|^2 + 2|g_d|^2. \tag{6}$$

One can see from Eqs. (5) and (6) that the polarization observables discussed above are not independent, as the following linear relation holds among the coefficients $\mathcal{A}_i$:

$$\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 = -1 \tag{7}$$

independently on the values of the amplitudes g_s and g_d.

So the C_{zz}-spin correlation coefficient, which is connected with the longitudinal polarization of both colliding particles, can be derived from this relation and the complete experiment for the s-wave interaction in $d + {}^3\mathrm{He} \to p + {}^4\mathrm{He}$ can be realized measuring the following quantities: (a) the differential cross section $(d\sigma/d\Omega)_0$; (b) the tensor analyzing power, characterized by the coefficient $\mathcal{A}_1$; (c) the spin correlation coefficient $C_{xx} = \mathcal{A}_2$.

Using these quantities, one can find the following relations:

$$9|g_s|^2 = (5 + 2\mathcal{A}_1 - 4\mathcal{A}_2)\left(\frac{d\sigma}{d\Omega}\right)_0,$$

$$9|g_d|^2 = (2 - \mathcal{A}_1 + 2\mathcal{A}_2)\left(\frac{d\sigma}{d\Omega}\right)_0, \tag{8}$$

$$-9\,\mathrm{Re}\, g_s g_d^* = (1 + 4\mathcal{A}_1 + \mathcal{A}_2)\left(\frac{d\sigma}{d\Omega}\right)_0.$$

In addition to the linear relation (7) between the coefficients $\mathcal{A}_i$, one can obtain also the following quadratic relation which connects the T-odd polarization observable $\mathcal{A}_4$ with the T-even ones:

$$\frac{9}{4}\mathcal{A}_4^2 = -1 - \mathcal{A}_1 - 2\mathcal{A}_1^2 - \mathcal{A}_2^2. \tag{9}$$

As the discussed reaction is characterized by a large Q value, the produced proton can be easily identified, and its polarization measured. We derive here the polarization properties of the produced protons. Due to the s-wave mechanism, if the initial particles are unpolarized, the polarization of the proton is zero for any value of the amplitudes g_s and g_d, but the polarization of one particle in the entrance channel can produce nonzero polarization of the final proton. For example, the dependence of the polarization $\vec{P}_1$ of the proton on the polarization of $\vec{P}$ of the ^{3}He target can be parametrized in the following form:

$$\vec{P}_1 = p_1 \vec{P} + p_2 \vec{k} \vec{k} \cdot \vec{P}, \tag{10}$$

where the real coefficients p_i characterize the spin transfer coefficients, namely:

$$K_x^{x'} = K_y^{y'} = p_1, \quad K_z^{z'} = p_1 + p_2. \tag{11}$$

Averaging over the polarizations of the initial deuteron we obtain the following expressions for the coefficients p_1 and p_2, in terms of the partial amplitudes g_s and g_d:

$$p_1 \left(\frac{d\sigma}{d\Omega}\right)_0 = -\frac{1}{3}(|g_s|^2 + 4\,\mathrm{Re}\, g_s g_d^* + 4|g_d|^2),$$

$$p_2 \left(\frac{d\sigma}{d\Omega}\right)_0 = 4\,\mathrm{Re}\, g_s g_d^* + 2|g_d|^2. \tag{12}$$

Comparing Eqs. (5) and (12), one can find

$$3p_1 = -1 + 2\mathcal{A}_1, \quad p_2 = -2\mathcal{A}_1, \quad 3p_1 + p_2 = -1. \tag{13}$$

In the considered energy region, the polarization properties of the proton can be predicted exactly, knowing only the tensor analyzing power. The test of all these relations is interesting to confirm the range of validity of the s-wave nature of the low-energy $d+{}^3$He interaction.

The dependence of the polarization $\vec{P}_1$ of the proton on the polarization states of the initial deuterons (the ^{3}He target is unpolarized) can be parametrized by the following general formula:

$$\vec{P}_1 = s_1\vec{S} + s_2\vec{k}\vec{k}\cdot\vec{S} + s_3\vec{k}\times\vec{Q}, \tag{14}$$

where the real coefficients s_i characterize the polarization transfer coefficients from the initial deuteron to the final proton. Averaging over the polarizations of the ^{3}He, one can find the following relations between both sets s_i and $\mathcal{A}_i$ of polarization observables:

$$s_1 = -\mathcal{A}_2, \quad s_2 = -\mathcal{A}_3, \quad s_3 = \mathcal{A}_4. \tag{15}$$

The above relations are correct for s-wave $d+{}^3$He interaction.

Let us consider now the most general polarization correlations for this process, i.e., the triple spin correlations. The dependence of the axial vector $\vec{P}_1$ on the polarization properties of the colliding particles can be parametrized as follows:

$$\vec{P}_1 = t_1\vec{P}\times\vec{S} + t_2\vec{k}\vec{k}\cdot\vec{P}\times\vec{S} + t_3\vec{k}\times\vec{P}\vec{k}\cdot\vec{S} + t_4\vec{k}\vec{Q}\cdot\vec{P} + t_5\vec{Q}\vec{k}\cdot\vec{P}$$
$$+ t_6\vec{P}\vec{Q}\cdot\vec{k} + t_7\vec{Q}(P) + t_8\vec{k}\vec{P}\cdot\vec{k}\vec{Q}\cdot\vec{k}, \tag{16}$$

where we used the notation $Q_a(P) = Q_{ab}P_b$. So the deuteron vector polarization together with the ^{3}He polarization induces at least three polarization correlations (of T-odd nature) and the tensor polarization is involved in five independent correlations.

For the real coefficients t_i the following expressions hold:

$$t_1\left(\frac{d\sigma}{d\Omega}\right)_0 = -\frac{1}{2}t_2\left(\frac{d\sigma}{d\Omega}\right)_0 = t_3\left(\frac{d\sigma}{d\Omega}\right)_0 = 3\,\mathrm{Im}\,g_s g_d^*,$$

$$t_4\left(\frac{d\sigma}{d\Omega}\right)_0 = t_5\left(\frac{d\sigma}{d\Omega}\right)_0 = -2\,\mathrm{Re}(g_s - g_d)g_d^*,$$

$$3t_6\left(\frac{d\sigma}{d\Omega}\right)_0 = 2\,\mathrm{Re}\,g_s g_d^* + |g_d|^2, \tag{17}$$

$$t_7\left(\frac{d\sigma}{d\Omega}\right)_0 = -\frac{2}{3}|g_s - g_d|^2,$$

$$t_8\left(\frac{d\sigma}{d\Omega}\right)_0 = -6|g_d|^2.$$

These triple polarization observables are related to the coefficients $\mathcal{A}_i$:

$$t_1 = -\frac{1}{2}t_2 = t_3 = \frac{1}{2}\mathcal{A}_4,$$

$$t_4 = t_5 = -\frac{2}{3}(3 + 2\mathcal{A}_1 - \mathcal{A}_2),$$

$$t_6 = -\frac{1}{3}\mathcal{A}_1, \tag{18}$$

$$t_7 = -\frac{2}{3}(1 + \mathcal{A}_1),$$

$$t_8 = -\frac{2}{3}(2 - \mathcal{A}_1 + 2\mathcal{A}_2).$$

Note that Eqs. (10), (14), and (16), which give the polarization transfer dependence, are correct not only for the particular case of threshold conditions, but they apply also to collinear kinematics (without any constraint on the energy of colliding particles). However, in the general case, the corresponding formulas are more complicated [13].

As stated above, all these formulas are correct for the s state interaction of the $d+{}^3$He system, with both possible values of $\mathcal{J}^\pi$, $\mathcal{J}^\pi = \frac{1}{2}^+$, and $\mathcal{J}^\pi = \frac{3}{2}^+$. But in the region of the fusion resonance, at $E_d \simeq 450$ keV, only the amplitude g_d is important. Therefore, neglecting the amplitude g_s, it is possible to predict definite numerical values for all polarization observables, in a model independent way: all the T-odd polarization observables vanish and the T-even polarization observables take the following values:

$$\mathcal{A}_1 = -\frac{1}{2}, \quad \mathcal{A}_2 = 1, \quad \mathcal{A}_3 = -\frac{3}{2},$$

$$t_4 = t_5 = -2, \quad t_6 = -\frac{1}{3}, \quad \text{and} \quad t_8 = -3. \tag{19}$$

Even in the near threshold region of the considered reaction the polarization observables take sizable values which can be measured.

Using these values for the coefficients $\mathcal{A}_i$, the differential cross section, for collisions of polarized particles, can be written as

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}}) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{1}{2}Q_{ab}k_a k_b + \vec{P}\cdot\vec{S} - \frac{3}{2}\vec{k}\cdot\vec{P}\vec{k}\cdot\vec{S}\right). \tag{20}$$

Equation (20) shows the large dependence of the differential cross section in the region of the fusion resonance with $\mathcal{J}^\pi = \frac{3}{2}^+$ to polarization characteristics of colliding particles.

One can see from Eq. (20) that, if a fully tensor polarized deuteron beam were available ($Q_{zz} = -2$ and $\vec{S} = 0$), it would be possible to increase the cross section by a factor of two, in the region of the fusion resonance, using an unpolarized ^{3}He target.[1] This result is correct for production of the

[1]After $\vec{k}$ integration, in Eq. (20), the total cross section becomes

$$\sigma(\vec{d},{}^3\vec{\mathrm{He}}) = \sigma(\vec{d},{}^3\mathrm{He})_0\left(1 + \frac{1}{2}\vec{P}\cdot\vec{S}\right),$$

which shows an increase of a factor of 1.5 with respect to the unpolarized case, in agreement with [16].

final particles along the magnetic field ($\theta=0$, where θ is the angle between the three-vector $\vec{k}$ and the direction of the polarizing magnetic field).

To understand this result, let us calculate, using Eq. (1) for the matrix element, the helicity amplitudes for the g_d contribution, which characterizes the $\mathcal{J}^\pi=\frac{3}{2}^+$ excitation. Only helicity conserving amplitudes are different from zero (for s-state interaction):

$$f_{0+,+}=-f_{0-,-}=2g_d\,,$$
$$f_{+-,+}=-f_{-+,+}=\sqrt{2}g_d\,, \tag{21}$$

where we used the following notation: $f_{\lambda_d\lambda_{\rm He},\lambda_p}\equiv f_{\lambda_1\lambda_2,\lambda_3}$. The differential unpolarized cross section can be also written as

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\frac{1}{6}\sum_{\lambda_i}|f_{\lambda_1\lambda_2,\lambda_3}|^2=2|g_d|^2, \tag{22}$$

in agreement with Eq. (6). On the other hand, the cross section for the collision of particles with definite helicities may be larger, as in case of longitudinally polarized deuterons (with $\lambda_d=\lambda_1=0$) scattered by unpolarized ^{3}He target, where

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\mathrm{He})=\frac{1}{2}(|f_{0+,+}|^2+|f_{0-,-}|^2)=4|g_d|^2$$
$$=2\left(\frac{d\sigma}{d\Omega}\right)_0,\quad \theta=0. \tag{23}$$

In our notations deuterons with $\lambda_d=0$ correspond to $Q_{zz}=-2$ and $\vec{S}=0$. We would obtain the same result for the scattering of longitudinally polarized deuterons by longitudinally polarized ^{3}He target. In case of nonzero deuteron vector polarization, the cross section is not increasing: for example, if $Q_{zz}=1$ and $\vec{k}\cdot\vec{S}=\pm1$, only one helicity amplitude is present, $f_{+-,+}$ or $f_{-+,-}$, with the following result:

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}})=|f_{+-,+}|^2=2|g_d|^2=\left(\frac{d\sigma}{d\Omega}\right)_0,\quad \theta=0,$$

i.e., the same value as in the case of unpolarized particles.

Polarization phenomena in conditions of magnetic fusion reactors were first discussed in [16], and later it was pointed out that polarized fuels can decrease the size of a reactor such as ARTEMIS [17], as well as the neutron flux.

III. POLARIZATION PHENOMENA FOR $s+p$ CONTRIBUTIONS

We consider here the p-wave contributions to the matrix element of the process $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$. This analysis is justified by the existence of a ^{5}Li excited state with $\mathcal{J}^\pi=\frac{3}{2}^-$ at $E_x=19.28$ MeV with large width ($\Gamma\simeq1$ MeV), just above the level with $\mathcal{J}^\pi=\frac{3}{2}^+$. In the region between these two levels the interference of s and p contributions have to be important.

Let us first enumerate all possible transitions in the process $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ induced by the $d+{}^3\mathrm{He}$ interaction in a state with orbital angular momentum $\ell_i=1$. For this aim it is suitable to compose the orbital momentum and the deuteron spin in the total angular momentum j of the deuteron, so that $j=\vec{1}+\vec{\ell}_i\to0,$ 1, and 2, with the following transitions (in CMS of the considered reaction):

$$j=0\to\mathcal{J}^\pi=\frac{1}{2}^-\to\ell_f=1,$$
$$j=1\to\mathcal{J}^\pi=\frac{1}{2}^-\to\ell_f=1,$$
$$j=1\to\mathcal{J}^\pi=\frac{3}{2}^-\to\ell_f=1, \tag{24}$$
$$j=2\to\mathcal{J}^\pi=\frac{3}{2}^-2\to\ell_f=1,$$
$$j=2\to\mathcal{J}^\pi=\frac{5}{2}^-\to\ell_f=3,$$

where ℓ_f is the orbital momentum of the produced proton.

The spin structure of these transitions can be written as

$$\chi_2^\dagger\vec{\sigma}\cdot\vec{k}\chi_1\vec{q}\cdot\vec{D}\quad j=0,\ p_{01}\,,$$

$$\left.\begin{array}{l}\chi_2^\dagger\chi_1\vec{k}\cdot\vec{q}\times\vec{D}\\ \chi_2^\dagger\vec{\sigma}\cdot\vec{q}\times\vec{D}\vec{\sigma}\cdot\vec{k}\chi_1\end{array}\right\}\quad j=1,\ p_{11}\ \text{ and }\ p_{13}\,, \tag{25}$$

$$\chi_2^\dagger\left(\vec{\sigma}\cdot\vec{q}\vec{k}\cdot\vec{D}+\cos\,\theta\vec{\sigma}\cdot\vec{D}-\frac{2}{3}\vec{\sigma}\cdot\vec{k}\vec{q}\cdot\vec{D}\right)\chi_1,\quad j=2,\quad p_{23}\,,$$

where $\vec{q}$ is the unit vector along the three-momenta of the deuteron and for the p-wave partial amplitudes we used the notation $p_{j,2J}$. The simple structures in Eq. (25), corresponding to $j=1$, are a mixture of the transitions for $\mathcal{J}^\pi=\frac{1}{2}^-$ and $\frac{3}{2}^-$. To obtain the spin structure for the partial amplitudes p_{11} and p_{13} it is necessary to use the following forms:

$$\vec{\phi}_{1/2}=[i\vec{q}\times\vec{D}+\vec{\sigma}(\vec{q}\times\vec{D})]\chi_1,$$
$$\vec{\phi}_{3/2}=[2i\vec{q}\times\vec{D}-\vec{\sigma}\times(\vec{q}\times\vec{D})]\chi_1, \tag{26}$$

where $\vec{\phi}_{1/2}$ and $\vec{\phi}_{3/2}$ are the wave functions of the $d+{}^3\mathrm{He}$ system with $j=1$ and $\mathcal{J}^\pi=\frac{1}{2}^-$ and $\mathcal{J}^\pi=\frac{3}{2}^-$, respectively. Finally we can write

$$\chi_2^\dagger(i\vec{k}\cdot\vec{q}\times\vec{D}+\cos\,\theta\vec{\sigma}\cdot\vec{D}-\vec{\sigma}\cdot\vec{q}\vec{k}\cdot\vec{D})\chi_1, \quad p_{11},$$
$$\chi_2^\dagger(2i\vec{k}\cdot\vec{q}\times\vec{D}\times\cos\,\theta\vec{\sigma}\cdot\vec{D}+\vec{\sigma}\cdot\vec{q}\vec{k}\cdot\vec{D})\chi_1, \quad p_{13}, \tag{27}$$

where θ is the proton production angle in CMS, $\cos\,\theta=\vec{k}\cdot\vec{q}$.

The spin structure of the matrix element in $s+p$ approximation is quite complex. We derive here a general parametrization of the matrix element for the process $d+{}^3\mathrm{He}\rightarrow p+{}^4\mathrm{He}$, which is valid for any energy of the interacting particles. Such parametrization is given in terms of six independent spin structures, which can be chosen as follows:

$$\mathcal{M}=\chi_2^\dagger\mathcal{F}\chi_1,$$
$$\mathcal{F}=\vec{m}\cdot\vec{D}(g_1\vec{\sigma}\cdot\vec{m}+g_2\vec{\sigma}\cdot\vec{k})+\vec{n}\cdot\vec{D}(ig_3+g_4\vec{\sigma}\cdot\vec{n})$$
$$+\vec{k}\cdot\vec{D}(g_5\vec{\sigma}\cdot\vec{m}+g_6\vec{\sigma}\cdot\vec{k}), \tag{28}$$

where the unit vectors $\vec{m}$ and $\vec{n}$ are defined by the formulas

$$\vec{n}=\vec{k}\times\vec{q}/|\vec{k}\times\vec{q}|, \quad \vec{m}=\vec{n}\times\vec{k}.$$

The scalar amplitudes g_i are complex functions of two kinematic variables: E_d and $\cos\,\theta$. This parametrization is especially adapted to the analysis of the polarization phenomena in the considered process and to define a complete experiment.

Using expressions (1) and (25) for the s- and p-wave contributions, the general amplitudes g_i, $i=1-6$, can be written as

$$g_1=g_4=g_s-g_d+\cos\,\theta(p_{11}-p_{13}+p_{23}),$$
$$g_2=\sin\,\theta\left(p_{01}-\frac{2}{3}p_{23}\right),$$
$$g_3=\sin\,\theta(p_{11}+2p_{13}), \tag{29}$$
$$g_5=\sin\,\theta(p_{11}-p_{13}+p_{23}),$$
$$g_6=g_s+2g_d+\cos\,\theta\left(p_{01}+\frac{4}{3}p_{23}\right).$$

The $s-p$ interference results in a rich set of one spin polarization observables: instead of a single tensor analyzing power, which is different from zero in the case of s interaction, we have two vector and three tensor analyzing powers (for the collision of a polarized beam on a polarized target):

$$\frac{d\sigma}{d\Omega}(d,{}^3\vec{\mathrm{He}})=(1+\mathcal{A}^{(t)}\vec{P}\cdot\vec{k}\times\vec{q})\left(\frac{d\sigma}{d\Omega}\right)_0,$$

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}})=[1+\mathcal{A}^{(b)}\vec{S}\cdot\vec{k}\times\vec{q}+\mathcal{A}_{kk}(Q_{ab}k_ak_b)$$
$$+\mathcal{A}_{qq}(Q_{ab}q_aq_b)+\mathcal{A}_{kq}(Q_{ab}k_aq_b)]\left(\frac{d\sigma}{d\Omega}\right)_0, \tag{30}$$

where $\mathcal{A}^{(b)}$ and $\mathcal{A}^{(t)}$ are the vector analyzing powers due to polarized beam and target, respectively, $\mathcal{A}_{kk}$, $\mathcal{A}_{qq}$, and $\mathcal{A}_{kq}$ are the possible tensor analyzing powers.

With the help of Eqs. (1) and (25) for the spin structures of s- and p-wave transitions, one can obtain the following expressions for these polarization observables (taking into account s^2 and sp-interference contributions only):

$$\mathcal{A}^{(t)}\left(\frac{d\sigma}{d\Omega}\right)_0=2\,\mathrm{Im}\,[-g_s(p_{01}-3p_{13})^*+g_d(p_{01}-3p_{11})^*],$$
$$\mathcal{A}^{(b)}\left(\frac{d\sigma}{d\Omega}\right)_0=\mathrm{Im}\,[g_s(p_{01}+p_{11}-p_{13})^*$$
$$+g_d(2p_{01}-p_{11}+p_{13})^*], \tag{31}$$
$$\mathcal{A}_{kq}\left(\frac{d\sigma}{d\Omega}\right)_0=-\frac{2}{3}\,\mathrm{Re}\,[g_s(p_{01}-p_{11}+p_{13})^*$$
$$+g_d(2p_{01}+p_{11}-p_{13})^*],$$
$$\mathcal{A}_{qq}=0, \quad \mathcal{A}_{kk}=\mathcal{A}_1.$$

The differential cross section with unpolarized particles contains a linear $\cos\,\theta$-contribution:

$$\left(\frac{d\sigma}{d\Omega}\right)_0=a+b\,\cos\,\theta, \quad a=|g_s|^2+2|g_d|^2,$$
$$b=\frac{2}{3}\,\mathrm{Re}\,g_s(p_{01}+2p_{11}-2p_{13})^*$$
$$+\frac{4}{3}\,\mathrm{Re}\,g_d(p_{01}-p_{11}+p_{13})^*. \tag{32}$$

The excitation of the resonance with $\mathcal{J}^\pi=\frac{3}{2}^-$ is characterized by two p-wave amplitudes p_{13} and p_{23}. Each one produces different polarization effects. For example, for the tensor analyzing powers one can find

p_{13} amplitude:

$$\mathcal{A}_{kk}=\frac{3}{5-3\cos^2\theta}, \quad \mathcal{A}_{qq}=\frac{4}{5-3\cos^2\theta},$$
$$\mathcal{A}_{kq}=-\frac{6\cos\theta}{5-3\cos^2\theta}, \tag{33}$$

p_{23} amplitude:

$$\mathcal{A}_{kk}=\frac{9}{13+21\cos^2\theta},\quad \mathcal{A}_{qq}=-\frac{4}{13+21\cos^2\theta},$$

$$\mathcal{A}_{kq}=\frac{18\cos\,\theta}{13+21\cos^2\theta}. \tag{34}$$

There are several possible two-spin correlations, in the $s+p$ approximation. For example, the dependence of the differential cross section on the polarizations of both colliding particles can be written in the following form:

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}})=\left(\frac{d\sigma}{d\Omega}\right)_0[&1+v_1\vec{P}\cdot\vec{S}+v_2\vec{P}\cdot\vec{k}\vec{S}\cdot\vec{k}+v_3\vec{P}\cdot\vec{q}\vec{S}\cdot\vec{q}\\&+v_4\vec{P}\cdot\vec{k}\vec{S}\cdot\vec{q}+v_5\vec{P}\cdot\vec{q}\vec{S}\cdot\vec{k}+\vec{n}\cdot\vec{P}(q_1Q_{ab}k_ak_b\\&+q_2Q_{ab}q_aq_b+q_3Q_{ab}k_aq_b)\\&+\vec{k}\cdot\vec{P}(q_4Q_{ab}k_an_b+q_5Q_{ab}q_an_b)\\&+\vec{q}\cdot\vec{P}(q_6Q_{ab}k_an_b+q_7Q_{ab}q_an_b)],\\&\vec{n}=\vec{k}\times\vec{q},\end{aligned} \tag{35}$$

where the real coefficients v_i, $i=1-5$, describe correlations of vector polarizations of both colliding particles and the real coefficients q_i, $i=1-7$, describe all possible P-even correlations due to the tensor deuteron polarization and to the vector polarization of ^{3}He. Using these formulas it is possible to calculate any polarization observable for the considered reaction (in the framework of a definite model for the reaction mechanism).

IV. THE $d+{}^3$He INTERACTION AT LOW ENERGIES

Let us discuss now, for completeness, other possible processes of the $d+{}^3$He interactions in the region of ^{5}Li-excitation with $\mathcal{J}^\pi=\frac{3}{2}^+$:

$$\begin{aligned}&d+{}^3\mathrm{He}\to d+{}^3\mathrm{He}\quad\text{(elastic scattering)},\\&d+{}^3\mathrm{He}\to p+p+{}^3\mathrm{H}\quad(Q_m=-1.461\ \mathrm{MeV}),\\&d+{}^3\mathrm{He}\to n+p+{}^3\mathrm{He}\quad(Q_m=-2.224\ \mathrm{MeV}),\\&d+{}^3\mathrm{He}\to d+d+p\quad(Q_m=-5.494\ \mathrm{MeV}).\end{aligned} \tag{36}$$

A. Elastic $d+{}^3$He scattering

The following transitions corresponding to $\mathcal{J}^\pi=\frac{3}{2}^+$ are allowed:

$$\begin{aligned}&\ell_i=0,\quad S_i=\frac{3}{2}\to\ell_f=0,\quad S_f=\frac{3}{2},\\&\ell_i=2,\quad S_i=\frac{1}{2}\to\ell_f=2,\quad S_f=\frac{1}{2},\\&\ell_i=2,\quad S_i=\frac{3}{2}\to\ell_f=2,\quad S_f=\frac{3}{2},\\&\ell_i=2,\quad S_i=\frac{3}{2}\to\ell_f=0,\quad S_f=\frac{3}{2},\\&\ell_i=2,\quad S_i=\frac{1}{2}\to\ell_f=2,\quad S_f=\frac{3}{2}.\end{aligned} \tag{37}$$

The analysis of the existing data about the process $d+{}^3\mathrm{He}\to p+{}^4\mathrm{He}$ in this region shows that the contribution of transitions with $\ell_i\neq 0$ is negligible. In this case we have a single s-wave elastic amplitude with the following expression for the matrix element:

$$\mathcal{M}^{(+)}=g_3^{(+)}\chi_2^\dagger(2i\vec{D}_2^*+\vec{\sigma}\times\vec{D}_2^*)\cdot(2i\vec{D}_1-\vec{\sigma}\times\vec{D}_1)\chi_1, \tag{38}$$

where $\vec{D}_1(\vec{D}_2)$ is the three-vector of polarization of initial (final) deuteron and $\chi_1(\chi_2)$ is the two-component spinor of the initial (final) nucleus ^{3}He and $g_3^{(+)}$ is the partial amplitude for $\ell_i\to\ell_f$ transitions. Equation (38) can be rewritten in the form

$$\mathcal{M}^{(+)}=g_3^{(+)}\chi_2^\dagger(\vec{D}_1\cdot\vec{D}_2^*+i\vec{\sigma}\cdot\vec{D}_1\times\vec{D}_2^*)\chi_1. \tag{39}$$

The presence of a single amplitude in Eq. (38) allows one to derive precise values for all the polarization observables induced by this matrix element. For example, the dependence of the differential cross section of the elastic $\vec{d}+{}^3\vec{\mathrm{He}}$ scattering on the polarization states of the colliding particles has a simple form:

$$\frac{d\sigma}{d\Omega}(\vec{d},{}^3\vec{\mathrm{He}})=\left(1+\frac{1}{2}\vec{P}\cdot\vec{S}\right)\left(\frac{d\sigma}{d\Omega}\right)_0. \tag{40}$$

Comparing this expression with the general formula, Eq. (2), one finds

$$\mathcal{A}_2=\frac{1}{2}\quad\text{and}\quad\mathcal{A}_1=\mathcal{A}_3=\mathcal{A}_4=0. \tag{41}$$

In the region of the ^{5}Li resonance with $\mathcal{J}^\pi=\frac{3}{2}^-$, the following transitions are allowed (taking into account only p-wave contributions):

$$\begin{aligned}&j_i=1\to j_f=1,\quad\ell_i=\ell_f=1,\\&j_i=2\to j_f=2,\quad\ell_i=\ell_f=1,\\&j_i=1\leftrightarrow j_f=2,\quad\ell_i=\ell_f=1.\end{aligned} \tag{42}$$

The T invariance of the strong interaction allows one to prove that the amplitudes of the last two nondiagonal transitions, $j_i=1\leftrightarrow j_f=2$, are equal, so we have here at least three different p-wave resonant amplitudes.

As an example let us consider only the first diagonal transition with $j=1$, for which the following matrix element can be written:

$$\begin{aligned}\mathcal{M}^{(-)}=g_3^{(-)}\chi_2^\dagger[&\vec{D}_1\times\vec{q}\cdot\vec{D}_2^*\times\vec{k}+i\vec{\sigma}\cdot(\vec{D}_1\times\vec{q})\\&\times(\vec{D}_2^*\times\vec{q})]\chi_1.\end{aligned} \tag{43}$$

After averaging over the polarizations of the initial particles and summing over the polarizations of the final particles, one can find for the θ-dependence of the elastic $d+{}^3$He differential cross section at the $\mathcal{J}^\pi=\frac{3}{2}^-$ resonance:

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\frac{2}{3}|g_3^{(-)}|^2(1+\cos^2\theta), \tag{44}$$

and for the tensor analyzing powers:

$$\mathcal{A}_{kk}=0, \quad \mathcal{A}_{qq}=\frac{1}{1+\cos^2\theta} \quad \text{and} \quad A_{kq}=-\frac{3\cos\theta}{2(1+\cos^2\theta)}. \tag{45}$$

On the other hand, in the case of $\mathcal{J}^\pi=\frac{3}{2}^+$ all these polarization observables are zero.

Let us now consider the processes of $d+{}^3\mathrm{He}$ interaction with production of three particles at threshold.

B. $d+{}^3\mathrm{He}\to p+p+{}^3\mathrm{H}$

The Pauli principle allows the production of the final $p+p$ system in an s state with zero value of total angular momentum, so the entrance channel is in a $\mathcal{J}^\pi=\frac{1}{2}^+$ state. Then the conservation of the P parity and of the angular momentum allows only two transitions:

$$S_i=\frac{1}{2}, \quad \ell_i=0\to\mathcal{J}^\pi=\frac{1}{2}^+,$$

$$S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{1}{2}.$$

The threshold matrix element, corresponding to these transitions, can be parametrized in the form

$$\mathcal{M}=\chi_2^\dagger(f_1\vec{\sigma}\cdot\vec{D}+f_2\vec{\sigma}\cdot\vec{q}\vec{q}\cdot\vec{D})\chi_1(\chi_3^\dagger\sigma_2\tilde{\chi}_4^\dagger), \tag{46}$$

where χ_1 and χ_2 are the two component spinors of the ${}^3\mathrm{He}$ and ${}^3\mathrm{H}$ nuclei, χ_3 and χ_4 are the two component spinors of the produced protons, $\vec{q}$ is the unit vector along the three-momenta of the deuteron (in the CMS of the considered reaction), f_1 and f_2 are the corresponding amplitudes.

C. $d+{}^3\mathrm{He}\to n+p+{}^3\mathrm{He}$

The situation here is more complicated, as nonidentical particles produced in the final state can be at threshold in singlet and triplet s states. The singlet production of the $n+p$ system is described by Eq. (46). For the triplet $n+p$ production the following transitions are allowed:

$$S_i=\frac{1}{2}, \quad \ell_i=0\to\mathcal{J}^\pi=\frac{1}{2}^+\to S_f=\frac{1}{2},$$

$$S_i=\frac{3}{2}, \quad \ell_i=0\to\mathcal{J}^\pi=\frac{3}{2}^+\to S_f=\frac{3}{2},$$

$$S_i=\frac{1}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{3}{2}^+\to S_f=\frac{3}{2}, \tag{47}$$

$$S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{1}{2}^+\to S_f=\frac{1}{2},$$

$$S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{3}{2}^+\to S_f=\frac{3}{2}.$$

The spin structure of the amplitudes for these transitions can be written as

$$\begin{gathered} I\otimes\vec{\sigma}\cdot\vec{D}, \\ I\otimes\vec{\sigma}\cdot\vec{k}\vec{k}\cdot\vec{D}, \\ (\vec{\sigma}\times\vec{D})_a\otimes\sigma_a, \\ ([\vec{\sigma}\times\vec{D}]\times\vec{k})_a\otimes(\vec{\sigma}\times\vec{k})_a, \\ (\vec{\sigma}\cdot\vec{k})\otimes(\vec{\sigma}\cdot\vec{D}\times\vec{k}) \end{gathered} \tag{48}$$

where we use the following notations:

$$A\otimes B=(\chi_2^\dagger A\chi_1)(\chi_3^\dagger B\sigma_2\tilde{\chi}_4^\dagger). \tag{49}$$

D. $d+{}^3\mathrm{He}\to d+d+p$

The identity of the two produced deuterons results in two values of the total spin for the $d+d$ system: $s_{dd}=0$ and 2. The following transitions are allowed:

$$\begin{gathered} S_i=\frac{1}{2}, \quad \ell_i=0\to\mathcal{J}^\pi=\frac{1}{2}^+\to s_{dd}=0, \\ S_i=\frac{3}{2}, \quad \ell_i=0\to\mathcal{J}^\pi=\frac{3}{2}^+\to s_{dd}=2, \\ S_i=\frac{1}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{3}{2}^+\to s_{dd}=2, \\ S_i=\frac{1}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{5}{2}^+\to s_{dd}=2, \\ S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{1}{2}^+\to s_{dd}=0, \\ S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{3}{2}^+\to s_{dd}=2, \\ S_i=\frac{3}{2}, \quad \ell_i=2\to\mathcal{J}^\pi=\frac{5}{2}^+\to s_{dd}=2, \\ S_i=\frac{3}{2}, \quad \ell_i=4\to\mathcal{J}^\pi=\frac{5}{2}^+\to s_{dd}=2. \end{gathered} \tag{50}$$

The corresponding spin structure for these transitions (with $\mathcal{J}\leqslant\frac{3}{2}$) can be written as

$$\begin{gathered} \chi_2^\dagger\vec{\sigma}\cdot\vec{D}\chi_1\vec{D}_1^*\cdot\vec{D}_2^*, \\ \chi_2^\dagger(\vec{\sigma}\cdot\vec{D}_1^*\vec{D}\cdot\vec{D}_2^*+\vec{\sigma}\cdot\vec{D}_2^*\vec{D}\cdot\vec{D}_1^*)\chi_1, \\ (\chi_2^\dagger\vec{\sigma}\cdot\vec{D}\chi_1)\vec{k}\cdot\vec{D}_1^*\vec{k}\cdot\vec{D}_2^*, \\ (\chi_2^\dagger\vec{\sigma}\cdot\vec{k}\chi_1)(\vec{k}\cdot\vec{D}_1^*\vec{D}\cdot\vec{D}_2^*+\vec{k}\cdot\vec{D}_2^*\vec{D}\cdot\vec{D}_1^*), \\ (\chi_2^\dagger\vec{\sigma}\cdot\vec{k}\chi_1)\vec{k}\cdot\vec{D}\vec{D}_1^*\cdot\vec{D}_2^*, \end{gathered} \tag{51}$$

where $\vec{D}_1$ and $\vec{D}_2$ are the three-vector polarizations of the produced deuterons. Using all these parametrizations it is possible to calculate the polarization observables of interest, in terms of the corresponding partial threshold amplitudes.

V. CONCLUSIONS

Let us summarize here the main results of our analysis of polarization phenomena in $d+{}^3$He collisions at very low energy of colliding particles.

We have suggested a new formalism well adapted to the description of the spin structure of the matrix elements and the polarization observables for light nuclei collisions at threshold energies. We applied this formalism to the analysis of some processes induced by $\vec{d}+{}^3\vec{\mathrm{He}}$.

In the framework of s-wave $d+{}^3$He interaction, at $E_d \leqslant 1$ MeV, we have derived the spin structure of the matrix element for the process $d+{}^3\mathrm{He}\rightarrow p+{}^4$He in terms of two complex partial amplitudes characterizing proton production in s and d states. The d-wave amplitude, which results from the tensor component of the NN interaction, corresponds to the excitation of the $\mathcal{J}^\pi=\frac{3}{2}^+$ state of ^{5}Li. This amplitude determines, in particular, the large absolute value (with negative sign) of the tensor analyzing power in $\vec{d}+{}^3\mathrm{He}\rightarrow p+{}^4$He [8].

We derive the general dependence of the differential cross section for $\vec{d}+{}^3\hat{\mathrm{He}}\rightarrow p+{}^4$He on the polarizations of the colliding particles. For s-wave $d+{}^3$He interaction all four possible asymmetries are different from zero. T-even asymmetries satisfy a linear relation and the square of the T-odd spin correlation coefficient can be represented as a definite quadratic combination of T-even asymmetries.

To determine the moduli of both s amplitudes for the process $d+{}^3\mathrm{He}\rightarrow p+{}^4$He and its relative phase (low energy ''complete experiment'') it is necessary to measure in addition to the unpolarized differential cross section, the tensor analyzing power and the spin correlation coefficient C_{xx} (induced by collisions of deuterons and ^{3}He with transversal vector polarizations).

We found expressions for all the triple-spin polarization observables, i.e., for the most general polarization correlations which are possible for the process $\vec{d}+{}^3\vec{\mathrm{He}}\rightarrow\vec{p}+{}^4$He.

For the excitation of the fusion resonance with $\mathcal{J}^\pi=\frac{3}{2}^+$, we can derive numerical predictions for all polarization observables, without using any particular dynamical model. In specific polarization conditions (longitudinally tensor polarized deuterons scattered on unpolarized target) one can obtain a twofold increase of the cross section with respect to the unpolarized one for the process $\vec{d}+{}^3\mathrm{He}\rightarrow p+{}^4$He.

Possible p-wave contributions for the low-energy $d+{}^3$He collisions are taken into account. The resulting spin structure of the matrix element for $\vec{d}+{}^3\mathrm{He}\rightarrow p+{}^4$He is quite complex and all the general scalar amplitudes are different from zero. The most general parametrization of the matrix element can be done in terms of six transversal amplitudes. We give the expressions of these amplitudes in terms of partial amplitudes corresponding to s and p interactions in the $d+{}^3$He system.

We establish the spin structure of the matrix element for other processes of the $d+{}^3$He interactions (at low energies):

$$d+{}^3\mathrm{He}\rightarrow d+{}^3\mathrm{He},$$

$$d+{}^3\mathrm{He}\rightarrow p+p+{}^3\mathrm{H},$$

$$d+{}^3\mathrm{He}\rightarrow n+p+{}^3\mathrm{He},$$

$$d+{}^3\mathrm{He}\rightarrow d+d+p.$$

The given parametrization is interesting for the calculation of different polarization observables.

We can conclude (without any model for the threshold amplitudes) that the polarization phenomena for different processes of low energy $d+{}^3$He interactions are very interesting and the corresponding polarization observables are large in absolute value. This kind of analysis allows one to optimize the experimental strategy for a complete experiment in the threshold region.

The derived formulas can be applied also to the polarization properties of neutrons produced in the $d+{}^3\mathrm{H}\rightarrow n+{}^4$He reaction, which is the basic process for fusion reactor plasma.

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