σ fields and chiral scalars in nuclear three-body potentials

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The scalar-isoscalar field of an effective chiral Lagrangian transforms differently in either linear or nonlinear frameworks: in the former case it is the counterpart of the pion whereas in the latter it is chiral invariant on its own. We compare the predictions from these two models for nucleon interactions and find results which are identical for two-body and rather different for three-body potentials. Some qualitative features of three-body interactions are discussed. [S0556-2813(98)07105-2]

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I. INTRODUCTION

Since the fundamental work of Yukawa, the meson exchange theory has been successfully applied to the description of the nucleon-nucleon force. Nowadays it is well established that the one-pion exchange potential (OPEP) is dominant at large distances, yielding a strong force with spin and isospin dependences.

The next layer of the interaction is associated with the exchange of two pions. This system has the lightest mass beyond the OPEP and involves an intermediate pion-nucleon (πN) amplitude in a kinematical region which is not directly accessible to experiment. Therefore a proper theoretical treatment of this component of the potential requires the use of information based on dispersion relations, as pointed out long ago by Cottinghan and Vinh Mau [1]. The implementation of this idea led to the Paris potential [2], which is rather successful in describing experimental data.

An important feature of the πN amplitude is chiral symmetry, as emphasized more than 25 years ago by Brown and Durso [3]. Recently the application of chiral symmetry to the NN interaction has deserved much attention, initially in the restricted framework of pion and nucleon degrees of freedom [4], and nowadays it is fair to say that the theoretical formulation of this sector of the interaction is free of ambiguities. However, a Lagrangian containing only pions and nucleons cannot reproduce low-energy πN data and hence this very reliable part of the model has to be complemented [5,6]. The combination of chiral symmetry and experimental information about the intermediate πN amplitude, based on dispersion relations, was brought into this problem in the last two years [6], with a successful description of asymptotic NN scattering data [7,8].

In contrast with the OPEP, the two-pion exchange potential (TPEP) produces an attractive interaction that depends little on spin and isospin, because it is dominated by the exchange of a scalar-isoscalar system. In many phenomenological potentials, the actual two-pion dynamics is simulated by the exchange of an effective scalar-isoscalar meson, with a mass around 550 MeV. In the framework of chiral symmetry, this meson is usually identified with the counterpart of the pion in the linear σ model and one obtains a prediction for its coupling constant to the nucleon. The existence of a scalar-isoscalar meson is controversial, the main candidate being the f_0 (400–1200), which may be present in $\pi\pi$ scattering [9]. In the case of NN scattering, the TPEP is quite well accounted for by nonlinear chiral dynamics, constrained by experimental information, and the scalar-isoscalar meson is unnecessary. According to the unwritten law of quantum mechanics, stating that processes which are not forbidden are compulsory, the exchange of two pions must be considered in any realistic description of the NN interaction and the use of the σ field to simulate the actual TPEP gives rise to shortcomings. In particular, a scalar field with mass m_s yields a central NN potential proportional to $e^{-m_s r}/r$, whereas the spatial dependence of TPEP is closer to $(e^{-\mu r}/r)^2$, μ being the pion mass [6,8]. Moreover, the TPEP is proportional to g^4, where g is the πN coupling constant, whereas the exchange of a σ is proportional to g^2. In spite of all these problems, the use of an effective scalar field may be useful in problems where simplicity is more important than precision. As far as the former is concerned an effective scalar field allows calculations at tree level, whereas the exchange of two pions involves loop integrations.

If one is willing to use an effective scalar field in a calculation, there are two possibilities at hand. The first one consists in employing the usual σ field of the linear model, which is the chiral partner of the pion. The other possibility is to use a scalar field in the framework of nonlinear Lagrangians, which is chiral invariant and appears naturally when the nonlinear fields are obtained from the linear ones [10]. These two scalars fields couple differently to pions and nucleons and hence lead to predictions which do not overlap for some specific processes. The main purpose of this work is to explore these predictions in the case of three-body forces. Our presentation is divided as follows: in Sec. II we compare linear and nonlinear Lagrangians and in Sec. III we motivate the physical predictions of the latter. In Sec. IV we

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derive the kernel of the three-body force, which is fully developed in Sec. V. Finally, in Sec. VI we discuss qualitatively some features of the force associated with σ's and chiral scalars.

II. LAGRANGIANS

In this section we introduce the Lagrangians describing both the σ and the chiral scalar. In the framework of linear dynamics the field σ is the chiral partner of the pion field $\boldsymbol{\pi}$ since, for an axial transformation δ^A, we have $\delta^A \boldsymbol{\pi} \rightarrow \sigma$ and $\delta^A \sigma \rightarrow -\boldsymbol{\pi}$. In the case of nonlinear dynamics, the pion field is represented by $\boldsymbol{\phi}$ and we also consider a new field S, which corresponds to a generalization of the field σ', introduced long ago by Weinberg [10]. The corresponding axial transformation are $\delta^A \boldsymbol{\phi} = F(\boldsymbol{\phi}^2)$, where $F(\boldsymbol{\phi}^2)$ is a function of $\boldsymbol{\phi}^2$ and $\delta^A S = 0$. The last transformation implies that S is a chiral scalar.

The linear Lagrangian $\mathcal{L}_\sigma$ has the usual form

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) - \frac{\mu^2}{2}(\sigma^2 + \boldsymbol{\pi}^2) + \frac{1}{2} f_\pi \mu^2 \sigma + U(\sigma^2 + \boldsymbol{\pi}^2) + \bar{N} i \partial\!\!\!/ N - g \bar{N}(\sigma + i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \gamma_5) N, \tag{1}$$

where N is the nucleon field that transforms linearly, f_π is the pion decay constant, μ is the pion mass, g is the πN coupling constant, and the term $U(\sigma^2 + \boldsymbol{\pi}^2)$ represents the self-interactions of the mesonic fields. In the framework of the linear σ model the scalar fluctuations are associated with a field ϵ, related to σ by

$$\sigma = f_\pi + \epsilon. \tag{2}$$

Replacing this in the Lagrangian (1), we obtain

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu \epsilon \partial^\mu \epsilon - m_\epsilon^2 \epsilon^2) + \frac{1}{2}(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \mu^2 \boldsymbol{\pi}^2) + U[(f_\pi + \epsilon)^2 + \boldsymbol{\pi}^2] + \bar{N} i \partial\!\!\!/ N - g \bar{N}(f_\pi + \epsilon + i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \gamma_5) N. \tag{3}$$

In the nonlinear approach, the Lagrangian $\mathcal{L}_S$ is written as

$$\mathcal{L}_S = \left[\frac{1}{2}(\partial_\mu S \partial^\mu S - m_s^2 S^2) - U(S)\right] + \left[\frac{1}{2}(\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} + \partial_\mu f \partial^\mu f) + f_\pi \mu^2 f\right] + \mathcal{L}_N, \tag{4}$$

where f corresponds to the function $f = \sqrt{f_\pi^2 - \boldsymbol{\phi}^2}$. Formally, $U(S)$ describes the self-interactions of the scalar field and $\mathcal{L}_N$ represents both the nucleon sector and its interactions with bosonic fields. This part of Lagrangian may be cast in many different forms, two of which are widely employed in the literature. In one of them, the pion-nucleon coupling is pseudovector (PV) and, in the other, it is pseudoscalar (PS). For PV coupling, one has

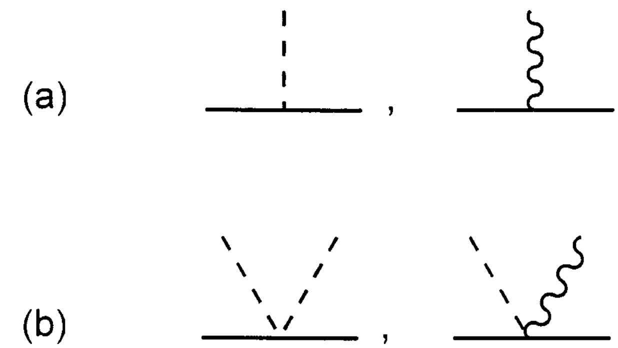

FIG. 1. Interactions of nucleons (full lines), pions (broken lines) and scalar-isoscalar mesons (wavy lines) present in both linear and nonlinear models (a) and only in the latter (b).

$$\mathcal{L}_N^{\mathrm{PV}} = \bar{\psi} i \gamma_\mu D^\mu \psi - m \bar{\psi} \psi + \frac{g}{2m} \bar{\psi} \gamma_\mu \gamma_5 \boldsymbol{\tau} \psi \cdot D^\mu \boldsymbol{\phi} - g_s S \bar{\psi} \psi, \tag{5}$$

where ψ is the nucleon field that transforms nonlinearly, m is its mass, and g_s represents the coupling of the nucleon to the scalar. In this expression the pion and nucleon covariant derivatives are given by [11]

$$D^\mu \boldsymbol{\phi} = \partial_\mu \boldsymbol{\phi} - \frac{1}{f + f_\pi} \partial^\mu f \boldsymbol{\phi}, \tag{6}$$

$$D^\mu \psi = \left[\partial^\mu + i \frac{1}{f_\pi (f + f_\pi)} \frac{\boldsymbol{\tau}}{2} \cdot (\boldsymbol{\phi} \times \partial^\mu \boldsymbol{\phi})\right] \psi. \tag{7}$$

This Lagrangian has been used recently in the study of πN form factors in constituent quarks models [12] and it is worth noting that its last term describes a chiral invariant coupling between the scalar and the nucleon.

In the case of PS coupling, one has

$$\mathcal{L}_N^{\mathrm{PS}} = \bar{N} i \partial\!\!\!/ N - g \bar{N}(f + i \boldsymbol{\tau} \cdot \boldsymbol{\phi} \gamma_5) N - \frac{g_s}{f_\pi} S \bar{N}(f + i \boldsymbol{\tau} \cdot \boldsymbol{\phi} \gamma_5) N, \tag{8}$$

where N is the nucleon field that transforms linearly. The last term of this expression has the same meaning as the corresponding one in Eq. (5).

On general grounds one knows that, in the framework of chiral symmetry, results should not depend on the choice of $\mathcal{L}_N$ [13,14]. The equivalence between the PS and PV Lagrangians was verified explicitly in the case of TPEP [6] and of pion-nucleon interactions in constituent quark models [12]. Because of this equivalence and of the similarity with the linear Lagrangian, our discussions are set in the PS case. Thus our complete nonlinear Lagrangian is written as

$$\mathcal{L}_S^{\mathrm{PS}} = \frac{1}{2}(\partial_\mu S \partial^\mu S - m_S^2 S^2) + \frac{1}{2}(\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} + \partial_\mu f \partial^\mu f) + f_\pi \mu^2 f + U(S) + \bar{N} i \partial\!\!\!/ N - \left(g + \frac{g_s}{f_\pi} S\right) \bar{N}(f + i \boldsymbol{\tau} \cdot \boldsymbol{\phi} \gamma_5) N. \tag{9}$$

The interaction terms in Eqs. (9) and (3) have the same scalar-nucleon and pion-nucleon vertices depicted in Fig. 1(a). However, the nonlinear approach also yields extra ver-

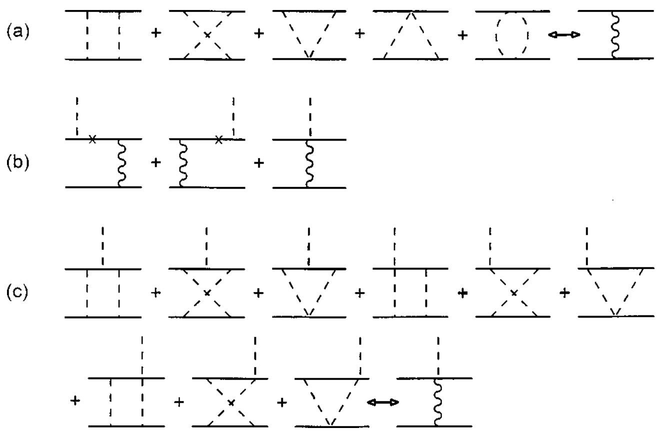

FIG. 2. Dynamical content of the effective scalar-isoscalar field in (a) nucleon-nucleon interactions, (b) the πNN kernel, and (c) the seagull term; conventions are the same as in Fig. 1 and the crosses in nucleon propagators indicate that they do not contain positive frequency components.

tices, representing seagull scalar-pion-nucleon and pion-pion-nucleon interactions, among others, displayed in Fig. 1(b). These results indicate that, at tree level, both Lagrangians produce the same NN potential. On the other hand, the extra scalar-pion-nucleon vertex represents a genuine difference between the two approaches and has consequences in processes such as pion absorption by a two-nucleon system or three-body forces.

III. EFFECTIVE SEAGULL

The purpose of this section is to discuss the meaning of the effective seagull interaction in terms of more basic processes. In order to do this, we note that, in the framework of nonlinear Lagrangians, the TPEP in the pure nucleon sector is given by the five diagrams of Fig. 2(a) [6], which we may want to associate with an effective scalar exchange. When an external pion is attached to these processes, we have the three possibilities indicated in Fig. 2(b), the last one corresponding to an effective seagull, whose dynamical meaning is given in Fig. 2(c).

The nine diagrams associated with the effective seagull interaction can be understood as arising from the product of the amplitudes T^{ba} and P^{bca}, where the former describes the pion-nucleon scattering $\pi^a(k)N(p)\to\pi^b(k')N(p')$ and latter represents the contribution from pion production, $\pi^a(k)N(p)\to\pi^b(k')\pi^c(q)N(p')$, as indicated in Fig. 3. The composite amplitude is denoted by A and given by

$$A=-i\int\frac{d^4Q}{(2\pi)^4}T^{ba}P^{bca}\frac{1}{k^2-\mu^2}\frac{1}{k'^2-\mu^2}, \qquad (10)$$

where $Q=\frac{1}{2}(k-k')$.

Using the πN vertices obtained from Eq. (9), we have

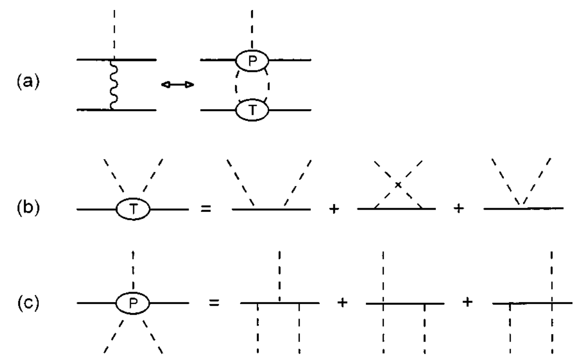

FIG. 3. The effective seagull (a) as composed by the $\pi N\to\pi N$ (b), and $\pi N\to\pi\pi N$ (c) amplitudes.

$$\begin{aligned}P^{bca}=ig^3\Bigg\{&(\delta_{ab}\tau_c-\delta_{bc}\tau_a-\delta_{ac}\tau_b+i\epsilon_{bac})\\&\times\bar{u}\left[\frac{\not{k}'}{(p'+k')^2-m^2}\gamma_5\frac{\not{k}}{(p+k)^2-m^2}\right]u\\&+\delta_{ac}\tau_b\bar{u}\left[\gamma_5\frac{\not{k}'}{(p'+k')^2-m^2}\frac{1}{gf_\pi}\right]u\\&+\delta_{bc}\tau_a\bar{u}\left[\frac{1}{gf_\pi}\frac{\not{k}}{(p+k)^2-m^2}\gamma_5\right]u\Bigg\}. \qquad (11)\end{aligned}$$

The part of A corresponding to a scalar-isoscalar exchange is associated with the factor δ_{ab} in the πN amplitude and hence it is proportional to

$$\begin{aligned}\delta_{ab}P^{bac}=ig^3\tau_c\bar{u}\Bigg[&\frac{\not{k}'}{(p'+k')^2-m^2}\gamma_5\frac{\not{k}}{(p+k)^2-m^2}\\&-\gamma_5\frac{\not{k}'}{(p'+k')^2-m^2}\frac{1}{gf_\pi}\\&-\frac{1}{gf_\pi}\frac{\not{k}}{(p+k)^2-m^2}\gamma_5\Bigg]u. \qquad (12)\end{aligned}$$

In order to identify the main contribution to this amplitude, we go to the soft pion limit, by using $k=k'=(k_0,0)$ and then take the limit $k_0\to 0$. We thus obtain

$$\delta_{ab}P^{bca}\xrightarrow{\text{soft}}-\frac{ig^3}{4m^2}\tau_c\bar{u}\gamma_5u, \qquad (13)$$

which has the same structure as the effective scalar-nucleon seagull vertex predicted by the nonlinear Lagrangian given by Eq. (9).

IV. EFFECTIVE πNN VERTEX

As far as NN interactions are concerned both the linear and nonlinear Lagrangians, given by Eqs. (3) and (9), yield

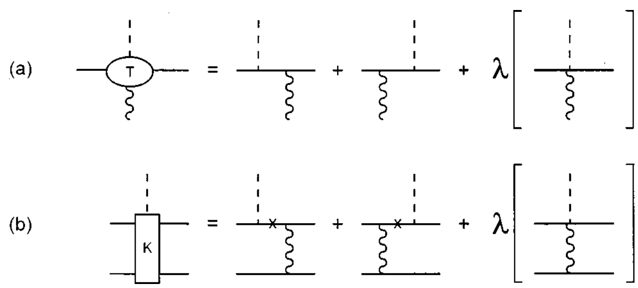

FIG. 4. (a) Amplitudes for the processes $\pi N \to \sigma N(\lambda=0)$ and $\pi N \to SN(\lambda=1)$; (b) the corresponding πNN kernel.

potentials at tree level, which are due to the exchanges of one-pion and one-scalar meson associated with the diagrams of Fig. 1(a). In principle, the scalar coupling constants predicted by these Lagrangians could be different. However, we are interested only in dynamical distinctions between the σ and the chiral scalar and, from now on, we set $g_s = g$ in order to force both models to simulate exactly the same two-body potential.

On the other hand, these models produce different predictions for the effective πNN vertex and in this section we consider the form of this interaction. The basic ingredients are the amplitudes T_σ and T_s, that describe the processes $\pi N \to \sigma N$ and $\pi N \to SN$, respectively. The amplitude T_σ is determined by just the first two diagrams of Fig. 4(a) whereas T_s also includes the seagull term. As the calculations of T_σ and T_s are quite similar, we denote both amplitudes by T and identify the seagull contribution by a parameter λ, such that $T_\sigma = T(\lambda=0)$ and $T_s = T(\lambda=1)$. At tree level, T is given by the diagrams depicted in Fig. 4(a), and we have, for a pion with isospin index c,

$$T(\lambda) = -ig^2 \tau_c \overline{u}(\boldsymbol{p}') \left[\frac{\not p_d + m}{p_d^2 - m^2} \gamma_5 + \gamma_5 \frac{\not p_x + m}{p_x^2 - m^2} + \frac{\lambda}{g f_\pi} \gamma_5 \right] u(\boldsymbol{p}). \tag{14}$$

with $p_d = p + k$, $p_x = p' - k$. Using Dirac equation, we rewrite Eq. (14) as

$$T(\lambda) = -ig^2 \tau_c \overline{u}(\boldsymbol{p}') \left[\frac{\not k}{p_d^2 - m^2} + \frac{\not k}{p_x^2 - m^2} + \frac{\lambda}{g f_\pi} \right] \gamma_5 u(\boldsymbol{p}). \tag{15}$$

It is worth pointing out that, as shown in [12], this result does not depend on our choice of the PS coupling in the nonlinear Lagrangian and it would be the same if the PV scheme were adopted.

This amplitude contains positive frequency states, which do not enter in the construction of the proper πNN kernel. In order to isolate these contributions, we write the nucleon propagator as

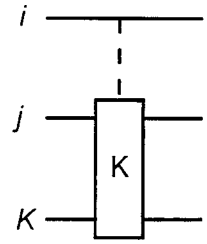

FIG. 5. Basic diagram for the three-body force.

$$\frac{\not p + m}{p^2 - m^2} = \frac{1}{2E} \left[\frac{1}{p^0 - E} \times \sum_s u^s(\boldsymbol{p}) \overline{u^s}(\boldsymbol{p}) + \frac{1}{p^0 + E} \sum_s v^s(-\boldsymbol{p}) \overline{v^s}(-\boldsymbol{p}) \right], \tag{16}$$

where $E = \sqrt{\boldsymbol{p}^2 + m^2}$. Thus

$$\left[\frac{\not p + m}{p^2 - m^2} \right]_{(+)} = \frac{1}{2E} \left[\frac{\not p + m}{p^0 - E} - \gamma^0 \right] \tag{17}$$

and the positive energy contribution is

$$T_{(+)} = -ig^2 \tau_c \overline{u}(\boldsymbol{p}') \left[\frac{\not k}{2E_d(p_d^0 - E_d)} + \frac{\not k}{2E_x(p_x^0 - E_x)} - \left(\frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 \right] \gamma_5 u(\boldsymbol{p}). \tag{18}$$

Subtracting Eq. (18) from Eq. (15) we get $\overline{T}$, the irreducible positive frequency amplitude

$$\overline{T}(\lambda) = -ig^2 \tau_c \overline{u}(\boldsymbol{p}') \left[-\frac{\not k}{2E_d(p_d^0 + E_d)} - \frac{\not k}{2E_x(p_x^0 + E_x)} + \left(\frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 + \frac{\lambda}{g f_\pi} \right] \gamma_5 u(\boldsymbol{p}). \tag{19}$$

This allows the proper kernel for the process $\pi N_j N_k$, as shown in Fig. 4(b), to be written as

$$K(\lambda) = -ig^3 \left\{ \tau_c \overline{u}(\boldsymbol{p}') \left[-\frac{\not k}{2E_d(p_d^0 + E_d)} - \frac{\not k}{2E_x(p_x^0 + E_x)} + \left(\frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 + \frac{\lambda}{g f_\pi} \right] \gamma_5 u(\boldsymbol{p}) \right\}^{(j)} \times \frac{1}{q^2 - m_s^2} [\overline{u}(\boldsymbol{p}') u(\boldsymbol{p})]^{(k)}, \tag{20}$$

where $q = p_k' - p_k$.

V. THREE-BODY FORCE

In this section we derive the three-nucleon force due to the simultaneous exchanges of one pion and one effective scalar meson and the basic diagram is shown in Fig. 5. It produces the following three-nucleon amplitude:

$$T^{ijk}(\lambda)=g^4\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}[\overline{u}(\boldsymbol{p}')\gamma_5 u(\boldsymbol{p})]^{(i)}\frac{1}{k^2-\mu^2}\times\left\{\overline{u}(\boldsymbol{p}')\left[-\frac{\not k}{2E_d(p_d^0+E_d)}-\frac{\not k}{2E_x(p_x^0+E_x)}+\left(\frac{1}{2E_d}-\frac{1}{2E_x}\right)\gamma^0+\frac{\lambda}{gf_\pi}\right]\gamma_5 u(\boldsymbol{p})\right\}^{(j)}\times\frac{1}{q^2-m_s^2}[\overline{u}(\boldsymbol{p}')u(\boldsymbol{p})]^{(k)}. \tag{21}$$

In order to obtain the nonrelativistic limit of $T^{ijk}(\lambda)$, denoted by $t^{ijk}(\lambda)$, we use

$$\frac{1}{2E_d}-\frac{1}{2E_x}\sim\frac{\boldsymbol{p}^2}{m^3},$$

$$\frac{1}{2E_d(p_d^0+E_d)}+\frac{1}{2E_x(p_x^0+E_x)}\sim\frac{1}{2m^2},$$

$$\overline{u}(\boldsymbol{p}')u(\boldsymbol{p})\rightarrow I,$$

$$\overline{u}(\boldsymbol{p}')\gamma_5 u(\boldsymbol{p})\rightarrow\frac{1}{2m}\boldsymbol{\sigma}\cdot(\boldsymbol{p}-\boldsymbol{p}'),$$

$$\overline{u}(\boldsymbol{p}')\gamma^0\gamma_5 u(\boldsymbol{p})\rightarrow\frac{1}{2m}\boldsymbol{\sigma}\cdot(\boldsymbol{p}+\boldsymbol{p}'),$$

$$\overline{u}(\boldsymbol{p}')\not k\gamma_5 u(\boldsymbol{p})\rightarrow-\boldsymbol{\sigma}\cdot\boldsymbol{k},$$

$$\frac{1}{k^2-\mu^2}\cong-\frac{1}{\boldsymbol{k}^2+\mu^2},$$

$$\frac{1}{q^2-m_s^2}\cong-\frac{1}{\boldsymbol{q}^2+m_s^2},$$

where the arrows indicate that the normalization of the spinors were also changed. Using these results and keeping only terms of the order $\mathbf{p}/m$, we get the nonrelativistic amplitude

$$t^{ijk}(\lambda)=\frac{g^4}{(2m)^2}\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\boldsymbol{\sigma}^{(i)}\cdot\mathbf{k}\frac{1}{\mathbf{k}^2+\mu^2}\boldsymbol{\sigma}^{(j)}\cdot\left[\left(\frac{1}{m}-\frac{\lambda}{gf_\pi}\right)\mathbf{k}+\frac{\lambda}{gf_\pi}\mathbf{q}\right]\frac{1}{\mathbf{q}^2+m_s^2}, \tag{22}$$

where $\mathbf{k}=\mathbf{p}_i-\mathbf{p}'_i$ and $\mathbf{q}=\mathbf{p}'_k-\mathbf{p}_k$. The full nonrelativistic amplitude t_{3N} is given by the permutation over all indices ijk.

In momentum space the three-body potential W is defined by

$$\langle\boldsymbol{p}_1'\boldsymbol{p}_2'\boldsymbol{p}_3'|W|\boldsymbol{p}_1\boldsymbol{p}_2\boldsymbol{p}_3\rangle=-(2\pi)^3\delta^3(\boldsymbol{p}_1'+\boldsymbol{p}_2'+\boldsymbol{p}_3'-\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3)t_{3N}. \tag{23}$$

We apply Fourier transform to Eq. (23) in order to obtain the potential in configuration space and we have

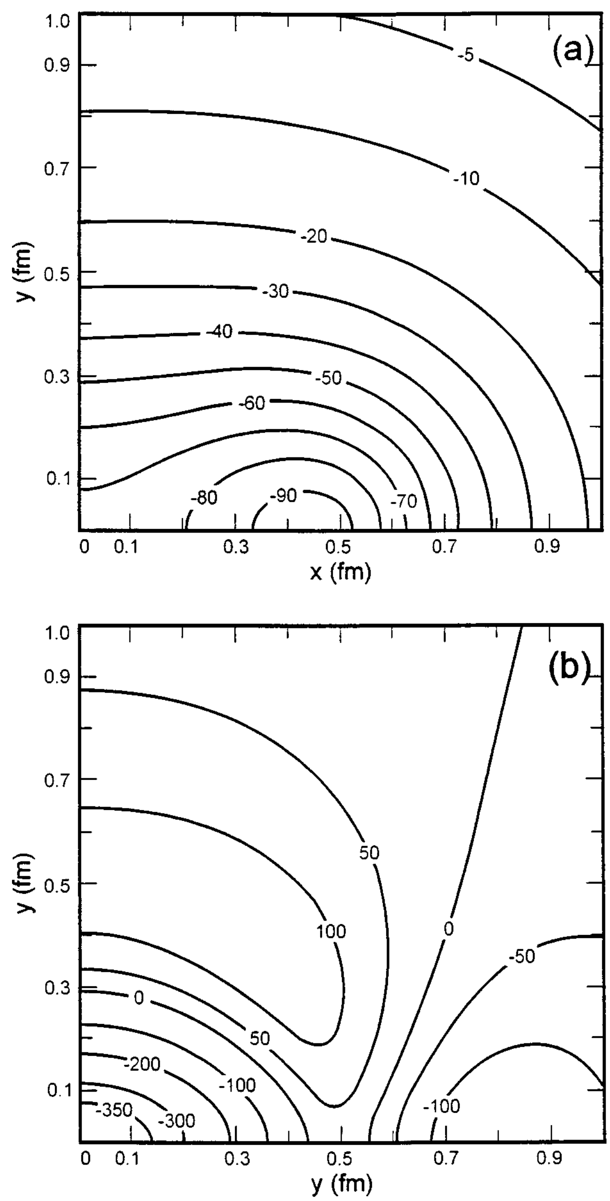

FIG. 6. Equipotential plots for the expectation value of the three-body force in the trinucleon ground state, calculated using Eq. (38) with $\Lambda=1.5$ GeV for the linear (a) and nonlinear (b) models. One of the nucleons is fixed at $x=0.5$ fm, another at $x=-0.5$ fm and the position of the third one is varied.

$$\langle \boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'|W|\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\rangle = \delta(\boldsymbol{r}_1'-\boldsymbol{r}_1)\,\delta(\boldsymbol{r}_2'-\boldsymbol{r}_2)\,\delta(\boldsymbol{r}_3'-\boldsymbol{r}_3)\,W, \tag{24}$$

and the component W^{ijk} of the local potential W is given by

$$W^{ijk} = \frac{g^4}{(4\pi)^2(2m)^2}\frac{\mu m_s}{m}\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\Bigg[\left(1-\frac{\lambda m}{g f_\pi}\right)\times U(m_s r_{kj})\,\boldsymbol{\sigma}^{(i)}\cdot\nabla_{ji}\boldsymbol{\sigma}^{(j)}\cdot\nabla_{ji}U(\mu r_{ji}) + \frac{\lambda m}{g f_\pi}\boldsymbol{\sigma}^{(i)}\cdot\nabla_{ji}U(\mu r_{ji})\,\boldsymbol{\sigma}^{(j)}\cdot\nabla_{kj}U(m_s r_{kj})\Bigg], \tag{25}$$

where $U(\mu r)=\exp(-\mu r)/\mu r$ and $r_{kj}=r_k-r_j$.

We assume the approximate identity $m/gf_\pi \simeq 1$ and, similarly to the procedure used in the two-pion exchange three-nucleon force [15], we regularize the mesonic exchanges with the following dipole form factor:

$$G(k^2)=\left(\frac{\Lambda^2-\mu^2}{\Lambda^2-k^2}\right)^2, \tag{26}$$

where Λ is a cutoff parameter.

The regularization changes the Yukawa-type function into

$$U(\mu r,\Lambda)=\frac{e^{-\mu r}}{\mu r}-\frac{\Lambda}{\mu}\frac{e^{-\Lambda r}}{\Lambda r}-\frac{1}{2}\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2}-1\right)e^{-\Lambda r}, \tag{27}$$

and its derivatives are given by

$$\frac{\partial U(\mu r,\Lambda)}{\partial r_\alpha}=\mu\frac{r_\alpha}{r}U_1(\mu r,\Lambda), \tag{28}$$

$$\frac{\partial^2 U(\mu r,\Lambda)}{\partial r_\alpha \partial r_\beta}=\frac{\mu^2}{3}\Bigg[\delta_{\alpha\beta}(U(\mu r,\Lambda)-G(r)) + \left(\frac{3r_\alpha r_\beta}{r^2}-\delta_{\alpha\beta}\right)U_2(\mu r,\Lambda)\Bigg], \tag{29}$$

where

$$U_1(\mu r,\Lambda)=-\left(\frac{1}{\mu r}+1\right)\frac{e^{-\mu r}}{\mu r}+\frac{\Lambda^2}{\mu^2}\left(1+\frac{1}{\Lambda r}\right)\frac{e^{-\Lambda r}}{\Lambda r} + \frac{1}{2}\left(\frac{\Lambda^2}{\mu^2}-1\right)e^{-\Lambda r}, \tag{30}$$

$$U_2(\mu r,\Lambda)=\left(1+\frac{3}{\mu r}+\frac{3}{\mu^2 r^2}\right)\frac{e^{-\mu r}}{\mu r}-\frac{\Lambda^3}{\mu^3}\times\left(1+\frac{3}{\Lambda r}+\frac{3}{\Lambda^2 r^2}\right)\frac{e^{-\Lambda r}}{\Lambda r}-\frac{1}{2}\frac{\Lambda}{\mu}\left(\frac{\Lambda^2}{\mu^2}-1\right)\times\left(1+\frac{1}{\Lambda r}\right)e^{-\Lambda r}, \tag{31}$$

$$G(r)=\frac{1}{2}\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2}-1\right)^2 e^{-\Lambda r}. \tag{32}$$

Substituting these results into the potential, we have

$$W^{ijk}=\frac{4}{3}\left(\frac{g\mu}{2m}\right)^4\frac{m}{(4\pi)^2}\frac{m_s}{\mu}\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\Big\{(1-\lambda)[\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}U(m_s r_{kj},\Lambda)(U(\mu r_{ji},\Lambda)-G(r_{ji},\Lambda)) + \mathbf{S}_{ij}(\hat{\boldsymbol{r}}_{ji},\hat{\boldsymbol{r}}_{ji})U(m_s r_{kj},\Lambda)U_2(\mu r_{ji},\Lambda)]+\lambda\frac{m_s}{\mu}[\mathbf{S}_{ij}(\hat{\boldsymbol{r}}_{ji},\hat{\boldsymbol{r}}_{kj})+\hat{\boldsymbol{r}}_{ji}\cdot\hat{\boldsymbol{r}}_{kj}\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}]U_1(\mu r_{ji},\Lambda)U_1(m_s r_{kj},\Lambda)\Big\}, \tag{33}$$

where

$$\mathbf{S}_{ij}(\hat{\boldsymbol{r}}_{ji},\hat{\boldsymbol{r}}_{kj})=3\,\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{r}}_{ji}\boldsymbol{\sigma}^{(j)}\cdot\hat{\boldsymbol{r}}_{kj}-\hat{\mathbf{r}}_{ji}\cdot\hat{\boldsymbol{r}}_{kj}\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}. \tag{34}$$

VI. DISCUSSION

Initially, we discuss the role of chiral symmetry in the results of the previous section. Inspecting Eq. (25), one notes that all the terms of the potential contain two gradients, reflecting the fact that they come from Eq. (22), which is a uniform second-order polynomial in meson momenta, as expected from a calculation based on chiral symmetry. This feature of the problem is independent of λ. On the other hand, the detailed form of the potential is quite sensitive to this parameter. The value $\lambda=0$ corresponds to a scalar field, denoted by σ, which is the chiral partner of the pion. The value $\lambda=1$, in turn, indicates processes based on a chiral invariant field S. It is worth pointing out that results with $\lambda=1$ are, as they should be, independent of the representation adopted for the pion field and, in particular, of the use of either PS or PV pion-nucleon couplings. So the parameter λ describes dynamical processes which are genuinely different.

At tree level, these effects cannot be distinguished in NN interactions, but they manifest themselves in the reactions $NN\rightarrow\pi NN$, $\pi d\leftrightarrow NN$, in axial form factors of nuclei and in three-body forces. In the present work we have considered only the last kind of application.

Our results with $\lambda=1$ coincide with that presented by Coon, Peña, and Riska [16]. These authors also obtained a contact interaction, employing a PV pion-nucleon coupling and a scalar-meson coupled to nucleons. In the framework of chiral symmetry, this combination means that they have tacitly used a Lagrangian equivalent of our Eq. (5) and hence their σ field corresponds to our S.

In this work we are concerned mainly with the differences between the cases $\lambda=0$ and $\lambda=1$ for three-body forces. The full exploration of this aspect of the problem would require a precise calculation of trinucleon observables, especially the binding energy. In order to produce a preliminary qualitative indication of the role of the force considered in this work, we evaluate the expectation value $\langle S|W|S\rangle$, where $|S\rangle$ represents the principal S state of the trinucleon, which is the basic component of its ground-state wave function.

This state is written as [15]

$$|S\rangle = S(\hat{\boldsymbol{r}}_{ji})\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a}), \tag{35}$$

where $S(\boldsymbol{r}_{ji})$ is the spatial component and $\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a})$ is the antisymmetric spin-isospin wave function with z components m and t, respectively.

The action of the tensor operators S_{ij} over $|S\rangle$ results in states with orbital angular momentum different from zero. Using

$$[\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a})]^{\dagger}\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a}) = -3, \tag{36}$$

we obtain

$$\begin{aligned}[][\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a})]^{\dagger}W^{ijk}\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a}) \\ = -4\left(\frac{g\mu}{2m}\right)^4 \frac{m}{(4\pi)^2}\frac{m_s}{\mu}\bigg\{(1-\lambda)U(m_s r_{kj},\Lambda) \\ \times[U(\mu r_{ji},\Lambda)-G(\mu r_{ji},\Lambda)] + \cos\,\theta_j\lambda\frac{m_s}{\mu} \\ \times U_1(\mu r_{ji},\Lambda)U_1(m_s r_{kj},\Lambda)\bigg\},\end{aligned} \tag{37}$$

where $\cos\,\theta_j = \hat{\boldsymbol{r}}_{ji}\cdot\hat{\boldsymbol{r}}_{kj}$. Using the results of [15] and neglecting the very short-range function $G(\mu r_{ji},\Lambda)$ [17], we have

$$\begin{aligned}[][\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a})]^{\dagger}W^{ijk}\Gamma^{(1/2)m}_{(1/2)t}(\mathsf{a}) \\ = -\{(1-\lambda)C_\sigma U(m_s r_{kj},\Lambda)U(\mu r_{ji},\Lambda) \\ + \lambda C_S\,\cos\,\theta_j U_1(\mu r_{ji},\Lambda)U_1(m_s r_{kj},\Lambda)\},\end{aligned} \tag{38}$$

where C_σ and C_S are the strength coefficients of σ and chiral scalar, respectively, given by

$$C_\sigma = 4\left(\frac{g\mu}{2m}\right)^4 \frac{1}{(4\pi)^2}\frac{m_s}{\mu}m,$$

$$C_S = 4\left(\frac{g\mu}{2m}\right)^4 \frac{1}{(4\pi)^2}\left(\frac{m_s}{\mu}\right)^2 m.$$

Choosing $m_s=550$ MeV and $g=13.5$, the strength coefficients become $C_\sigma=96$ MeV and $C_S=378$ MeV.

In Fig. 6 we show equipotential plots for the choices $\lambda=0$ (graph a) and $\lambda=1$ (graph b), constructed by keeping two nucleons 1 fm apart and varying the position of the third one. As the plots are symmetric under rotation around the x axes the specification of a single quadrant describes the spatial energy distribution. Inspecting this figure one learns that the predictions from both models are rather different, indicating that the effective seagull is very important. In the linear approach, the interaction is attractive over a wide region, whereas the nonlinear scalar produces a repulsion around the triangular configuration and these differences should show up in observables.

One is aware that a study based on just S trinucleon waves can provide only rough indications, since it is well known that D waves do play an important role in trinucleons. Nevertheless, in the absence of a detailed study, we may assume that the trends associated with our equipotential plots would reflect in the binding energy. This assumption is supported by the results of Ref. [16] where a three-body force given by our Eq. (22) with $\lambda=1$ was shown to produce a decrease of the binding energy which is rather welcome.

As a final comment, it is important to point out that the discussion presented in Sec. III makes us biased towards the chiral scalar, but final conclusions must wait for a complete evaluation of the diagrams of Fig. 2(c), which is now in progress.

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