α-cluster structure of ^{44}Ti in core-excited $\alpha+^{40}$Ca model

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It is shown that all the rotational bands in ^{44}Ti at low and high excitation energies can be well explained in an α-cluster model in which the α-like excitation of the core, ^{40}Ca, is incorporated by employing density-dependent M3Y (DDM3Y) double folding potentials. [S0556-2813(98)03105-7]

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α clustering is one of the important correlations in nuclei [1]. Recently it has been shown that an α-cluster model works well even in the fp-shell region [2–13] where it had been believed that the strong spin-orbit force may break the α correlations [14]. The observation of the theoretically predicted parity-doublet $N=13$ $K=0_1^-$ α-cluster band in ^{44}Ti [5,6] and ^{40}Ca [7,8] has confirmed the validity of the α-cluster model. Also in the fp-shell region the coexistence of cluster and shell structure is clearly seen as was shown in ^{42}Ca [9] and ^{40}Ca [10].

However, there still remain unsolved problems about the α-cluster structure in the typical nucleus ^{44}Ti at highly excited energies above $E_x=10$ MeV. The states 0^+ (11.19 MeV), 1^- (11.69 MeV), 2^+ (12.18 MeV), 3^- (12.78 MeV), and 4^+ (13.18 MeV) seen in a high-resolution $\alpha+^{40}$Ca scattering [15,16] were once interpreted as members of the $N=12$ and $N=13$ ''mixed-parity'' α-cluster band [16–18]. This interpretation was questioned theoretically from the viewpoint of unified description of bound and scattering states of the $\alpha+^{40}$Ca system [2,3]. However, recently this ''mixed-parity'' α-cluster theory has been emphasized again based on the coincidence experiment of α-transfer reactions [19], which observed the 1^- (11.80 MeV), 2^+ (12.28 MeV), 3^- (12.88 MeV), 4^+ (13.42 MeV), and 5^- (14.7 MeV) states and the authors have questioned the theory of Refs. [2–4]. Independently another 0^+ state with large α-spectroscopic factor has been observed at 10.86 MeV in the α-transfer reactions [20]. Also the 0^+ (11.08 MeV), 1^- (11.73 MeV), 2^+ (12.12 MeV), 3^- (12.57 MeV), and 4^+ (12.86 MeV) states have been reported from the analysis of elastic α scattering [21]. It is important to understand these states comprehensively in an α-cluster model.

On the other hand, in the low-energy region below 10 MeV the rotational $K=0_2^+$ band [0_2^+ (1.90 MeV), 2_2^+ (2.52 MeV), and 4_2^+ (3.35 MeV)] and the $K=2^+$ band [2_3^+ (2.88 MeV), 3_1^+ (3.44 MeV), and 4_3^+ (3.98 MeV)] [22,23] have not been understood yet in a frozen α-cluster model. A soft asymmetric rotor model was successfully applied to the energy levels and the strong $B(E2)$ values [23], however, it cannot explain the low-lying 0_3^+ (4.85 MeV) state. The 0_3^+ (4.85 MeV) state, which had been ever interpreted as the $N=14$ α-cluster state in Ref. [24], is too low to be the case. In the beginning of the fp-shell region systematic appearance of the low-lying intruder states in the shell model suggests the importance of the excitation of the core. By using a schematic α-cluster model Yamada and one of the present authors (S.O.) discussed preliminarily that these bands can be interpreted to have an $\alpha+^{40}$Ca* structure with core excitation [25].

The purpose of this paper is to show that all the above-mentioned low-lying rotational bands and high-lying bands below 20 MeV can be understood in a unified way in the α-cluster model in which the α-like excitation of the core is incorporated by employing a double folding model.

We take the model wave function for the $\alpha+^{40}$Ca(I^π) system as follows:

$$\Psi_{JM}=\sum_{I,L}[\varphi(\alpha)\varphi_I(^{40}\mathrm{Ca})\otimes i^L Y_L(\hat{\boldsymbol{R}})]_{JM}\chi_{IL}^J(R) \qquad (1)$$

where $\varphi(\alpha)$ is the intrinsic wave function of the α-particle, $\varphi_I(^{40}$Ca) is that of the core with spin I, and $\chi_{IL}^J(R)$ is the relative wave function between α and ^{40}Ca with L being the orbital angular momentum of the relative motion. The form factor in the coupled channel equation is given by

$$F_{ILI'L'}^J(R)=\langle[\varphi(\alpha)\varphi_I(^{40}\mathrm{Ca})\otimes i^L Y_L(\hat{\boldsymbol{R}})]_{JM}|V| \times[\varphi(\alpha)\varphi_{I'}(^{40}\mathrm{Ca})\otimes i^{L'} Y_{L'}(\hat{\boldsymbol{R}})]_{JM}\rangle \qquad (2)$$

with

$$V=\sum_{\substack{i\in\alpha\\ j\in ^{40}\mathrm{Ca}}} v_{NN}(\boldsymbol{r}_i+\boldsymbol{R}-\boldsymbol{r}_j). \qquad (3)$$

We take the DDM3Y [26] for a two-body effective interaction v_{NN} which was also useful in the heavier $\alpha+^{90}$Zr and $\alpha+^{208}$Pb systems [27]. For $\varphi_I(^{40}$Ca), we take the wave functions calculated in the orthogonality condition model (OCM) of ^{40}Ca [10], which reproduce the energy levels of cluster

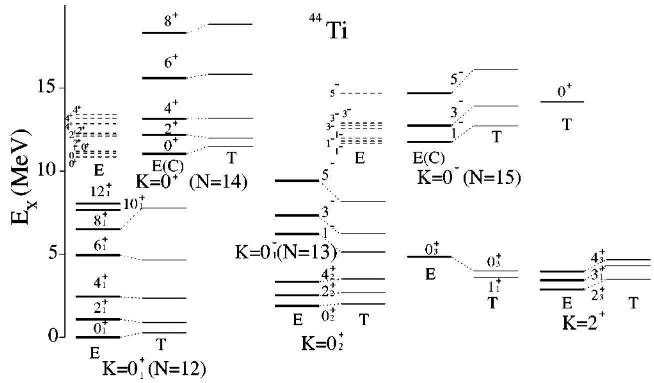

FIG. 1. Theoretical energy levels (thin solid lines and labeled T) of ^{44}Ti are compared with the experimental data (thick solid lines and labeled E). The fragmented experimental energy levels (E) are displayed by the dashed lines and their centroid is indicated by the thick solid lines with label $E(C)$.

states and shell-like states. As for the excited states of ^{40}Ca, we take the 0^+(3.35 MeV) and 2^+(3.90 MeV) states, which are the members of the rotational band with the well-developed α-cluster structure. The form factor $F^J_{ILI'L'}(R)$, whose diagonal parts are the folding potentials, is multiplied by a normalization factor λ. The form factors for the Coulomb interaction are also calculated in the same way.

By using the folding potentials, we investigate the structure of ^{44}Ti. The renormalization factors are kept at 1.2, which is consistent with the values widely used in the DDM3Y folding model calculations [27], except finely tuned $\lambda=1.185$ is used for the g.s. channel so that the ground state energy with respect to the α-threshold corresponds to the experiment. In the bound state calculations the harmonic oscillator basis functions with $N\geqslant 12$ (size parameter $\nu=0.14$ fm^{-2}) are spanned in order to take into account the Wildermuth condition due to the Pauli principle. To take into account the Pauli principle in the coupling interactions phenomenologically, the tensor coupling interactions, which are too strong in the double folding calculations, are weakened so that the calculated energy levels correspond to experiment.

Calculated energy levels of ^{44}Ti are shown in Fig. 1 in comparison with the experimental data [5,6,15,16,19–23]. The calculated four rotational bands at low excitation energies correspond well with the experimental $N=12$ $K=0^+_1$, $N=13$ $K=0^-_1$, $K=0^+_2$, and $K=2^+$ bands. Although it is difficult to get the 0^+_2 state as low as 1.90 MeV in the conventional shell model, the present model gives the 0^+_2 state at such a low energy because of α clustering. The inspection of the wave functions shows that the $K=0^+_2$ band has dominantly $[\alpha+{}^{40}\mathrm{Ca}^*(0^+{:}3.35\ \mathrm{MeV})]_{J=L}$ structure, while the $K=2^+$ band has dominantly $[\alpha+{}^{40}\mathrm{Ca}^*(2^+{:}3.90\ \mathrm{MeV})]_J$ structure. The mixing of the two structures is rather strong for the 4^+ states in contrast to the cases of the 0^+ and 2^+ states. As we see in Table I, this different mixing character gives good agreement of the calculated intraband transitions from each 4^+ state with experiment. The 0^+_3 state and the unobserved 1^+ (3.44 MeV) state are members of the $[L=2\otimes I=2]_J$ weak coupling quintet states ($0^+_3, 1^+_1, 2^+_3, 3^+_1, 4^+_3$ in Fig. 1). In Table I calculated $B(E2)$ values are compared with the experimental data. In the calculations the theoretical electric transition amplitude in ^{40}Ca from the 2^+ (3.90 MeV) state to the ground state in Ref. [10], which is too small compared with experiment [10], is adjusted by multiplying a factor of 10 to reproduce the experimental $B(E2)$ value of ^{40}Ca. Although the intraband transitions of ^{44}Ti are hardly effected by this, the intraband transitions are greatly enhanced about factor 10 improving the agreement with experiment. We see that the strong experimental intraband transitions are well reproduced by the present calculations without effective charges. The large transitions are brought about by the collectivity due to the relative motion of α clustering. The interband transitions are also reproduced giving weak transitions to be weak.

As for the highly excited energy region our model gives the higher nodal $N=14$ $K=0^+$ band starting at 11.7 MeV. Yamaya *et al.* [20] claimed that the 10.86 MeV 0^+ state is the $N=14$ higher nodal state because it has a very large α-spectroscopic factor ($S=1.06$) deduced from the α-transfer reactions. Guazzoni *et al.* [6] claimed that the 0^+ state at 9.32 MeV is the higher nodal state, however, it has not been observed in the same (^{6}Li,d) transfer reactions at $E_L=37$ MeV and 50 MeV [5,20]. On the other hand, the 11.19 MeV 0^+ state was claimed by Frekers *et al.* [15–18] to be the band head state of the $N=12$ and $N=13$ ‘‘mixed-parity’’ band whose negative parity partner 11.69 MeV 1^- state is , as was discussed by Michel *et al.* [2], difficult to be

TABLE I. Theoretical and experimental $B(E2)$ values [22,23] for the $J^\pi_i \rightarrow J^\pi_f$ transitions (in W.u.) of ^{44}Ti are compared.

$J^\pi_i \rightarrow J^\pi_f$	Exp.	Theor.	$J^\pi_i \rightarrow J^\pi_f$	Exp.	Theor.
2^+_1(1.08 MeV)$\rightarrow 0^+_1$(0.00 MeV)	13	18	4^+_3(3.98 MeV)$\rightarrow 2^+_3$(2.88 MeV)	19	16
4^+_1(2.44 MeV)$\rightarrow 2^+_1$(1.08 MeV)	30	25	2^+_2(2.52 MeV)$\rightarrow 0^+_1$(0.00 MeV)	0.17	0.033
6^+_1(4.02 MeV)$\rightarrow 4^+_1$(2.44 MeV)	17	23	2^+_2(2.52 MeV)$\rightarrow 2^+_1$(1.08 MeV)	6.7	0.29
8^+_1(6.51 MeV)$\rightarrow 6^+_1$(4.02 MeV)	>1.5	20	4^+_2(3.35 MeV)$\rightarrow 2^+_1$(1.08 MeV)	2.6	0.31
10^+_1(7.67 MeV)$\rightarrow 8^+_1$(6.51 MeV)	15	15	2^+_3(2.88 MeV)$\rightarrow 0^+_1$(0.00 MeV)	0.71	0.15
12^+_1(8.04 MeV)$\rightarrow 10^+_1$(7.67 MeV)	<6.5	7.9	2^+_3(2.88 MeV)$\rightarrow 2^+_1$(1.08 MeV)	<3.2	0.093
2^+_2(2.52 MeV)$\rightarrow 0^+_2$(1.90 MeV)	24	17	2^+_3(2.88 MeV)$\rightarrow 0^+_2$(1.90 MeV)	14	0.33
4^+_2(3.35 MeV)$\rightarrow 2^+_2$(2.52 MeV)	29	29	3^+_1(3.44 MeV)$\rightarrow 2^+_1$(1.08 MeV)	0.24	0.48
3^+_1(3.44 MeV)$\rightarrow 2^+_3$(2.88 MeV)	<64	20	4^+_3(3.98 MeV)$\rightarrow 2^+_1$(1.08 MeV)	0.24	0.048
4^+_3(3.98 MeV)$\rightarrow 3^+_1$(3.44 MeV)	<85	1.2	4^+_3(3.98 MeV)$\rightarrow 4^+_1$(2.44 MeV)	<1.8	0.44

a state with the $\alpha+{}^{40}$Ca (g.s.) configuration because its excitation energy of 11.69 MeV and its very small width, 40 keV, are incompatible. Recent experiments [7,20] and theoretical calculations in ^{40}Ca [10] have shown that the α widths are fragmented over states nearby, therefore it seems reasonable that the fragmentation occurs also in ^{44}Ti. In Fig. 1 the centroids [shown by thick solid lines and labeled $E(C)$] of the 0^+ states (10.86 MeV, 11.08 MeV, 11.19 MeV), 2^+ states (12.12 MeV, 12.18 MeV, 12.28 MeV), and 4^+ states (12.86 MeV, 13.18 MeV, 13.42 MeV) (shown by dashed lines and labeled E) which have been observed either in α-transfer reactions or elastic α scattering [15,16,19–21] (some of the states may happen to be the same state within the energy resolution) are displayed. In Fig. 1 the high spin members ($6^+\sim 8^+$) of the $N=14$ higher nodal states deduced from the peak position of the fusion excitation functions are also shown [28]. The position of the 6^+ state is consistent with the recent result of the coincidence α-transfer reaction [19]. The present calculations nicely reproduce the centroid of the experimental states which may be fragmented from the $N=14$ higher nodal band.

As for the negative parity states, in Fig. 1 the centroids [thick solid lines and labeled $E(C)$] of the 1^- states (11.69 MeV, 11.73 MeV, 11.80 MeV), 3^- states (12.57 MeV, 12.78 MeV, 12.88 MeV), and the 5^- state at 14.7 MeV (dashed lines and labeled E) [15,16,19,21] are displayed. The centroids correspond to the calculated $N=15$ $K=0^-$ $\alpha+{}^{40}$Ca (g.s.) band. If the α widths of the $N=15$ $K=0^-$ band are fragmented, the fact that each 1^- state has a small α width might not contradict the α-cluster picture, although we have to wait for more detailed experimental results.

Our calculation also locates the band head 0^+ state with the core-excited higher nodal $\alpha+{}^{40}$Ca*(0^+:3.35 MeV) structure at 14 MeV (Fig. 1). It is interesting to observe the band members in experiment.

To summarize, by taking account of the α-like core excitation we could reproduce all the rotational bands observed in ^{44}Ti in the α-cluster model. The strong intraband and weak interband electric transitions of the low-lying bands were also well reproduced. The fragmentation of the $N=14$ and $N=15$ higher nodal parity-doublet α-cluster bands was discussed. As a next step, to get more detailed understanding of the α-cluster structure of ^{44}Ti, a microscopic three cluster $\alpha+\alpha+{}^{36}$Ar orthogonality condition model calculation is needed.

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