Charged current weak electroproduction of the Δ resonance

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We study the weak production of Δ (i.e., $e^- + p \rightarrow \Delta^0 + \nu_e$ and $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$) in the intermediate energy range corresponding to the Mainz and TJNAF electron accelerators. The differential cross sections $\sigma(\theta)$ are found to be of the order of 10^{-39} cm^2/sr, over a range of angles which increases with energy. The possibility of observing these reactions with the high luminosities available at these accelerators, and studying the weak N-Δ transition form factors through these reactions is discussed. The production cross section of $N^*(1440)$ in the same kinematic region is also estimated and found to be smaller than the Δ production cross section. [S0556-2813(98)04005-9]

I. INTRODUCTION

The study of the nucleon and its excitation spectrum has been pursued for a long time both experimentally and theoretically, especially in the region of the Δ resonance. In this region, the QCD inspired quark models for baryon structure provide theoretical insight into the nonperturbative regime of QCD at low and intermediate energies. At these energies, the study of various electromagnetic excitation processes, induced by electrons and photons, has been made at many research facilities around the world, and there exists extensive literature on electromagnetic transition form factors [1]. On the other hand, similar studies on the corresponding weak form factors have been few and far between. In a series of experiments done with intermediate energy neutrinos at the ANL, BNL, and CERN laboratories, attempts have been made to study the transition form factors for the charge changing weak current, and there exists a fair amount of data to determine these form factors [2–6]. This is not the case with the neutral current processes, where there are very few experiments [7,8] in the intermediate energy range and no serious analysis has been made of the transition form factors. The main interest in the neutral current sector has been to study the parity violating asymmetry in the polarized electron scattering with nucleons and nuclei in order to explore the nonzero strangeness content of the nucleon [9,10].

Now, the availability of continuous wave electron accelerators with 100% duty cycle in the energy range of a few GeV, and the possibility of achieving very high luminosities at these accelerators, has led to the feasibility of performing electron scattering experiments in the resonance region with very good statistics [11,12]. These experimental studies, in principle, can be extended to explore weak interaction physics in the Δ resonance region. In this paper, we explore the possibility of doing such experiments and present a quantitative analysis of the charged current reaction in which Δ's are produced. Similar theoretical studies in the neutral current sector have been performed earlier [13] and most re-

cently by Mukhopadhyay et al. [14].

The aim of the present paper is to give a general analysis of the weak production of Δ through the processes

$$e^- + p \rightarrow \Delta^0 + \nu_e \tag{1a}$$

and

$$e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e, \tag{1b}$$

and to examine critically the feasibility of doing such an experiment at the Mainz Microtron and/or at TJNAF. These reactions were earlier studied by Hwang et al. [15], where a bag model was used to calculate the N-Δ form factors and the effect of the Δ width was not taken into account. In this work we retain all the weak vector and axial vector form factors in the matrix elements of N-Δ transitions. The present available information on these form factors from the experimental data on electromagnetic and neutrino production of Δ has been fully utilized through the application of conserved vector current (CVC) and partially conserved axial current (PCAC) hypotheses in the N-Δ sector [16–20]. In addition, the width of the Δ resonance is properly taken into account and is found to give important effects on the differential cross section $\sigma(\theta)$. The effect of various parametrizations of the N-Δ form factors, discussed recently in the literature [16,17], has been studied to explore the possibility of distinguishing between them experimentally. Finally, we have also estimated the production cross section for the Roper, $N^*(1440)$, the next higher resonance, in order to understand its effect in the kinematic region of Δ production. We find its effect to be sufficiently small and well separated from the kinematic region of present interest to allow for a clean identification of Δ through observation of the pions and nucleons produced as decay products.

In Sec. II, we describe the transition currents for the production of Δ and $N^*(1440)$, and derive expressions for the cross sections. In Sec. III, we present the numerical results for the differential cross sections $\sigma(\theta)$ for the considered reactions, and in Sec. IV we discuss the possibility of experimentally observing them.

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II. TRANSITION CURRENTS AND CROSS SECTIONS

A. $e^- + p \to \Delta^0 + \nu_e$ and $e^+ + p \to \Delta^{++} + \bar{\nu}_e$

The matrix element for the process $e^-(k) + p(p) \to \Delta^0(p') + \nu_e(k')$ is written as [21]

$$\mathcal{M} = \frac{G}{\sqrt{2}} \cos\theta_c l_\alpha J^\alpha, \qquad (2)$$

with

$$l_\alpha = \bar{u}(k')\gamma_\alpha(1-\gamma_5)u(k), \qquad (3)$$

and

$$J^\alpha = \bar{\psi}_\mu(p')\left\{ \left[\frac{C_3^V}{M}(g^{\mu\alpha}\slashed{q} - q^\mu\gamma^\alpha) + \frac{C_4^V}{M^2}(g^{\mu\alpha}q\cdot p' - q^\mu p'^\alpha) \right.\right.$$
$$\left. + \frac{C_5^V}{M^2}(g^{\mu\alpha}q\cdot p - q^\mu p^\alpha) \right]\gamma_5 + \frac{C_3^A}{M}(g^{\mu\alpha}\slashed{q} - q^\mu\gamma^\alpha)$$
$$\left. + \frac{C_4^A}{M^2}(g^{\mu\alpha}q\cdot p' - q^\mu p'^\alpha) + C_5^A g^{\mu\alpha} + \frac{C_6^A}{M^2}q^\mu q^\alpha \right\}u(p), \qquad (4)$$

where M is the nucleon mass, $\psi_\mu(p')$ and $u(p)$ are the Rarita Schwinger and Dirac spinors for Δ and nucleon of momentum p' and p, $q = p' - p = k - k'$ is the momentum transfer, C_i^V and C_i^A ($i = 3,4,5,6$) are the vector and axial vector transition form factors as defined by Llewellyn Smith [20] and are discussed in detail in Sec. II B. It is relevant to mention here that $C_6^V(q^2) \equiv 0$, assuming CVC. With the matrix element given in Eqs. (2)–(4), the differential cross section $d\sigma/d\Omega_\Delta$ is calculated to be

$$\frac{d\sigma}{d\Omega_\Delta} = \frac{1}{16\pi^3}G^2\cos^2\theta_c \int d|\mathbf{p}'|\frac{|\mathbf{p}'|^2}{E_e E_\nu}$$
$$\times \frac{\Gamma/2}{(W-M')^2 + \Gamma^2/4}L_{\alpha\beta}J^{\alpha\beta}, \qquad (5)$$

with

$$L_{\alpha\beta} = k_\alpha k'_\beta + k'_\alpha k_\beta - g_{\alpha\beta}k\cdot k' + i\epsilon_{\alpha\beta\gamma\delta}k^\gamma k'^\delta, \qquad (6)$$

and

$$J_{\alpha\beta} = \bar{\Sigma}\Sigma J_\alpha^\dagger J_\beta, \qquad (7)$$

where the summation is performed over the hadronic spins using a spin 3/2 projection operator $P_{\mu\nu}$ given by

$$P_{\mu\nu} = -\frac{\slashed{p}' + M'}{2M'}\left(g_{\mu\nu} - \frac{2}{3}\frac{p'_\mu p'_\nu}{M'^2} \right.$$
$$\left. + \frac{1}{3}\frac{p'_\mu\gamma_\nu - p'_\nu\gamma_\mu}{M'} - \frac{1}{3}\gamma_\mu\gamma_\nu \right). \qquad (8)$$

In Eq. (5), W is the invariant mass of the Δ given by $W^2 = p'^2$, M' is the Δ mass and Γ is its decay width given by

$$\Gamma = \frac{1}{6\pi}\left(\frac{f^*}{m_\pi}\right)^2\frac{M}{W}|\mathbf{q}_{cm}|^3\theta(W - M - m_\pi), \qquad (9)$$

where m_π is the pion mass, $\mathbf{q}_{cm}$ is the pion momentum in the rest frame of the resonance and $f^* = 2.13$.

We now turn to the process $e^+ + p \to \Delta^{++} + \bar{\nu}_e$. The matrix element is written in the same way as Eqs. (2)–(4) with the following replacements.

(i) The leptonic current l_α in Eq. (3), now involves antiparticles and is written in terms of v spinors instead of u spinors.

(ii) The matrix element of the hadronic current J_α in Eq. (4) is now evaluated between initial proton and final Δ^{++} states, with the relation

$$\langle\Delta^{++}|J_\alpha|p\rangle = \sqrt{3}\langle\Delta^0|J_\alpha|p\rangle. \qquad (10)$$

With these two changes the differential cross section is effectively given by Eq. (5), with $L_{\alpha\beta}(k,k') \to L_{\alpha\beta}(k',k)$ and $J_{\alpha\beta}(p,p') \to 3J_{\alpha\beta}(p,p')$.

B. N-Δ transition form factors

The N-Δ transition form factors relevant to the weak transition current have been discussed in the literature in connection with the analysis of neutrino scattering experiments [18–21] and in the context of quark model calculations [16,17]. We summarize in this section some details of these form factors, necessary for present calculations.

1. Vector form factors

As stated in Sec. II A, there are four weak vector form factors C_3^V, C_4^V, C_5^V, and C_6^V occurring in this transition. The imposition of the CVC hypothesis implies $C_6^V = 0$. The other three form factors are then given in terms of the isovector electromagnetic form factors in the p-Δ^+ electromagnetic transition. Specifically, the hadronic matrix element for the reactions (1a) and (1b) are given as

$$\langle\Delta^0|J_\alpha|p\rangle = \langle\Delta^+|J_\alpha^{em}(T=1)|p\rangle \qquad (11a)$$

and

$$\langle\Delta^{++}|J_\alpha|p\rangle = \sqrt{3}\langle\Delta^+|J_\alpha^{em}(T=1)|p\rangle, \qquad (11b)$$

where $J_\alpha^{em}(T=1)$ is the isovector electromagnetic current.

The information on the isovector electromagnetic form factors $C_i^V(q^2)$ ($i = 3,4,5$) is obtained from the analysis of photo and electroproduction data of Δ, which is done in terms of the multipole amplitudes E_{1+}, M_{1+}, and S_{1+} [22]. The present data on E_{1+} and S_{1+} amplitudes are very meager and these amplitudes are expected to be small. Assuming M_{1+} dominance of the electroproduction amplitude, which is predicted by the nonrelativistic quark model, the form factors $C_i^V(q^2)$ satisfy the relations

$$C_5^V = 0, \quad C_4^V = -\frac{M}{M'}C_3^V. \qquad (12)$$

The relations given in Eq. (12) have been used in the analysis of the electroproduction experiments and $C_3^V(q^2)$ has

been determined. The following parametrizations of $C_3^V(q^2)$, available in the literature [16,17], are used in our present calculations:

$$(1) \quad C_3^V(q^2) = \frac{2.05}{(1-q^2/0.54 \text{ GeV}^2)^2}, \quad (13)$$

$$(2) \quad C_3^V(q^2) = \frac{1.39}{\sqrt{1-q^2/1.43 \text{ GeV}^2}(1-q^2/0.71 \text{ GeV}^2)^2}. \quad (14)$$

For the purpose of comparison with a simple form factor obtained in the quark model, we also use [16]

$$(3) \quad C_3^V(q^2) = \frac{M}{\sqrt{3}m}e^{-\bar{q}^2/6}, \quad (15)$$

where $m = 330$ MeV is the quark mass and $\bar{q} = |\boldsymbol{q}|/\alpha_{HO}$, with $\alpha_{HO} = 320$ MeV, being the harmonic oscillator parameter.

2. Axial vector form factors

There are four axial vector form factors C_3^A, C_4^A, C_5^A, and C_6^A, as defined in Eq. (4). Using the pion pole dominance of the divergence of axial current (PDDAC), $C_6^A(q^2)$ can be given by the equation [19]

$$C_6^A(q^2) = \frac{g_\Delta f_\pi}{2\sqrt{3}M}\frac{M^2}{m_\pi^2 - q^2}, \quad (16)$$

where $g_\Delta = f^* 2M/m_\pi$ is the $\Delta^{++} \to p\pi^+$ coupling constant and $f_\pi = 0.97m_\pi$ is the pion decay constant. Evaluating the matrix element of the divergence of the axial current in the limit $m_\pi^2 \to 0$ and $q^2 \to 0$, gives the off diagonal Goldberger-Treiman relation

$$C_5^A(0) = \frac{g_\Delta f_\pi}{2\sqrt{3}M}, \quad C_6^A(q^2) = C_5^A\frac{M^2}{m_\pi^2 - q^2}. \quad (17)$$

In the absence of any other theoretical input, $C_3^A(q^2)$, $C_4^A(q^2)$ and $C_5^A(q^2)/C_5^A(0)$ remain undetermined. The data on neutrino scattering are analyzed using these form factors as free parameters and using Eqs. (12)–(14) for the vector form factors. The parametrizations used for the various axial form factors are given below, where dipole form factors have been modified for a better fit to the data [2–6]:

$$C_{i=3,4,5}^A(q^2) = C_i(0)\left[1 - \frac{a_i q^2}{b_i - q^2}\right]\left(1 - \frac{q^2}{M_A^2}\right)^{-2}, \quad (18)$$

with $C_3^A(0) = 0$, $C_4^A(0) = -0.3$, $C_5^A(0) = 1.2$, $a_4 = a_5 = -1.21$, $b_4 = b_5 = 2$ GeV^2, and $M_A = 1.0$ GeV. Recently, these form factors have been calculated in some quark models and a comparative study of various models has been presented [16]. For comparison with the phenomenological form factors, we also use a nonrelativistic quark model calculation [16]

$$C_5^A(q^2) = \left(\frac{2}{\sqrt{3}} + \frac{1}{3\sqrt{3}}\frac{q_0}{m}\right)e^{-\bar{q}^2/6},$$

$$C_4^A(q^2) = -\frac{1}{3\sqrt{3}}\frac{M^2}{M'm}e^{-\bar{q}^2/6}, \quad C_3^A(q^2) = 0. \quad (19)$$

C. $e^- + p \to N^* + \nu_e$

The Roper resonance N^*, with mass 1440 MeV and decay width of about 350 MeV [23], is the next higher resonance which has appreciable strength into $N\pi$ decay channel. The $N\pi$ events coming from the N^* decay can lie in the invariant mass region of the Δ resonance. Therefore, we also calculate the production cross section of N^* resonance and the possibility to separate it from a Δ resonance signal. It is to be noted that there is no corresponding reaction with an e^+ beam, thus the Δ^{++} production signal is cleaner than the Δ^0 production signal.

The matrix element for the process $e^-(k) + p(p) \to N^*(p') + \nu_e(k')$ is written assuming standard properties of the charged weak current J^α in the $\Delta S = 0$ sector, neglecting second class currents [24]. Using constraints free form factors and manifestly gauge invariant operators for the vector current matrix element [25], J^α is written as

$$J^\alpha = \bar{u}_{N*}(p')[F_1^V(q^2)(\slashed{q}q^\alpha - q^2\gamma^\alpha) + iF_2^V(q^2)\sigma^{\alpha\beta}q_\beta$$
$$+ F_A^V(q^2)\gamma^\alpha\gamma_5 + F_P^V(q^2)q^\alpha\gamma_5]u(p), \quad (20)$$

where $F_{1,2}^V(q^2)$ and $F_{A,P}^V(q^2)$ are the isovector vector and axial vector form factors.

The expression for the differential cross section $d\sigma/d\Omega_{N*}$ is given by Eq. (5) with M' and Γ replaced by the N^* mass and its width, respectively. For the N^* width we have chosen the model described in the Appendix of Ref. [26], where both $\pi + N$ and $\pi + \pi + N$ partial decay channels are taken into account. The $\pi + \pi + N$ decay is assumed to go through a $\pi + \Delta$ intermediate state.

D. N-N^* transition form factors

Using the matrix element in Eq. (20) $F_1^V(q^2)$ and $F_2^V(q^2)$ can, in principle, be determined from the available experimental data on photo and electroproduction of Roper resonance from protons and neutrons. The data on the photoproduction of protons and neutrons fix only $F_2^V(0)$. The electroproduction of Roper resonance has been measured only for the proton, and data are not of very good quality [27]. In the absence of any data on the neutron target, we have to rely on a model to determine the isovector form factors $F_1^V(q^2)$ and $F_2^V(q^2)/F_2^V(0)$. There are many models in the literature for the electroproduction of Roper resonance with very different results [28]. For the purpose of the present estimates, we use the following relations for the isovector transverse and longitudinal helicity amplitudes $A_{1/2}^V(q^2)$ and $S_{1/2}^V(q^2)$, respectively:

$$A_{1/2}^V(q^2) = \frac{5}{3}A_{1/2}^p(q^2) \quad \text{and} \quad S_{1/2}^V(q^2) = S_{1/2}^p(q^2). \quad (21)$$

These relations are valid in the nonrelativistic constituent quark model without configuration mixing and are deduced from the results given in Ref. [29]. The helicity amplitudes $A_{1/2}^p$ and $S_{1/2}^p$, the superscript p referring to the proton are defined in the standard way [30] i.e.,

$$A_{1/2}^p = \sqrt{\frac{2\pi\alpha}{k_R}} \langle N^* \uparrow | \sum_{\text{pol}} \epsilon \cdot J_{em}^p | N \downarrow \rangle,$$

$$S_{1/2}^p = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\boldsymbol{q}|}{\sqrt{-q^2}} \langle N^* \uparrow | \sum_{\text{pol}} \epsilon \cdot J_{em}^p | N \uparrow \rangle, \tag{22}$$

where the electromagnetic current J_α^p is given by [25]

$$J_\alpha^p = \bar{u}_{N^*}(p')[F_1^p(q^2)(\not{q}q_\alpha - q^2\gamma_\alpha) + F_2^p i\sigma_{\alpha\beta}q^\beta]u(p) \tag{23}$$

and $F_{1,2}^p(q^2)$ are the electromagnetic transition form factors for the N-N^* transitions, $\boldsymbol{q}$ is the three momentum of the virtual photon and k_R is the energy of an equivalent real photon both in the rest frame of N^*. They are given by

$$k_R = \frac{W^2 - M^2}{2W}, \quad \boldsymbol{q}^2 = \frac{(W^2 - M^2 + q^2)^2}{4W^2} - q^2,$$

$$W^2 = (k+p)^2. \tag{24}$$

Using Eqs. (21) and (22), isovector helicity amplitudes are derived to be

$$A_{1/2}^V(q^2) = |\boldsymbol{q}|g(q^2)\left[F_2^V(q^2) - \frac{q^2}{W+M}F_1^V(q^2)\right], \tag{25}$$

$$S_{1/2}^V(q^2) = \frac{1}{\sqrt{2}}\boldsymbol{q}^2 g(q^2)\left[F_1^V(q^2) - \frac{F_2^V(q^2)}{W+M}\right], \tag{26}$$

with

$$g(q^2) = \sqrt{\frac{8\pi\alpha(W+M)W^2}{M(W-M)[(W+M)^2 - q^2]}}. \tag{27}$$

Inverting Eqs. (25) and (26) we calculate the isovector form factors $F_1^V(q^2)$ and $F_2^V(q^2)$ in terms of the helicity amplitudes and use Eq. (21) to obtain them from the presently available data on $A_{1/2}^p(q^2)$ and $S_{1/2}^p(q^2)$, quoted by Li et al. [28].

In the case of axial vector form factors $F_A^V(q^2)$ and $F_P^V(q^2)$, there is no experimental information available. We use the pion pole dominance of the divergence of axial current (PDDAC) hypothesis, as done in Sec. II B, to relate $F_A^V(q^2)$ and $F_P^V(q^2)$ to each other and also to obtain a corresponding Goldberger Treiman relation (GTR) relating $F_A^V(0)$ to the $N^* \to N\pi$ coupling $\widetilde{f}$ and f_π. A straightforward calculation gives [24]

$$F_A^V(0) = \sqrt{2}f_\pi \frac{\widetilde{f}}{m_\pi}, \quad F_P^V = \frac{M+M^*}{m_\pi^2 - q^2}F_A^V, \tag{28}$$

where $\widetilde{f}$ is the $N^* \to N\pi$ coupling determined from the experimental decay rate for this channel and defined through the $N^*N\pi$ Lagrangian using pseudovector coupling, i.e.,

$$\mathcal{L}_{\text{int}} = i\frac{\widetilde{f}}{m_\pi}\bar{\psi}_{N^*}\gamma^\mu\gamma_5\boldsymbol{\tau}(\partial_\mu\boldsymbol{\phi})\psi + \text{H.c.} \tag{29}$$

A dipole form for the q^2 dependence of $F_A^V(q^2)$ is used

$$F_A^V(q^2) = \frac{F_A^V(0)}{(1 - q^2/M_A^2)^2}, \tag{30}$$

where $M_A = 1.0$ GeV as taken in the case of N-Δ form factors. Using Eqs. (28) and (30) for the axial vector form factor $F_A^V(q^2)$ [$F_P^V(q^2)$ contribution is negligible] and Eqs. (25)–(27) for the vector form factor, numerical results are presented in the next section.

III. NUMERICAL RESULTS

In this section we present numerical results for the differential cross section for the processes $e^- + p \to \Delta^0 + \nu_e$, $e^+ + p \to \Delta^{++} + \bar{\nu}_e$, $e^- + p \to N^* + \nu_e$ and study them using various form factors. We stress here, in particular, the importance of the decay width in the angular dependence of the cross sections and the effect of changing various form factors in the vector and axial vector sectors.

A. $e^- + p \to \Delta^0 + \nu_e$

We present in Fig. 1 the Δ angular distribution for energies $E_e = 0.5$, 0.855, and 4.0 GeV using the expressions for the form factors of N-Δ transition, given in Sec. II B. The electron energies are chosen to correspond to the Mainz and TJNAF accelerators. The invariant mass has been restricted to $W < 1.4$ GeV to include the Δ dominated events only. The differential cross section is found to be forward peaked at lower electron energies, for example at $E_e = 500$ MeV, but the peak shifts to higher angles as we increase the energy. There is a gain of 50% in the total cross section as we go from the maximum Mainz energy (0.855 GeV) to 4 GeV. We also study the cross section sensitivity to the transition form factors. We do this by calculating the cross section for three sets of vector and axial vector form factors. In the first set, we use Eqs. (12) and (13) for the vector form factors and Eq. (18) for the axial vector form factors, and the results are shown in Fig. 1 (solid line). In the second set, we take the form factors recently discussed by Hemmert et al. [17], which use Eqs. (12) and (14) for the vector form factors and Eq. (18) with $C_5^A(0) = 0.87$, $C_4^A(0) = -0.29$ MeV for the axial vector form factors. The results are shown by the short-dashed line. In the third set, we use the nonrelativistic quark model form factors given by Eqs. (15) and (19) taken from Liu et al. [16] and the results are shown by the long-dashed line.

In Fig. 2, using the first set of form factors, we show the effect of the decay width Γ on the differential cross sections $d\sigma/d\Omega_\Delta$ for $E_e = 500$ and 4000 MeV. It is clear from Fig. 2 that the Δ width plays an important role in the angular cross section. In the limit of $\Gamma \to 0$, our results qualitatively agree with those of Hwang et al. [15]. The narrow angular range in
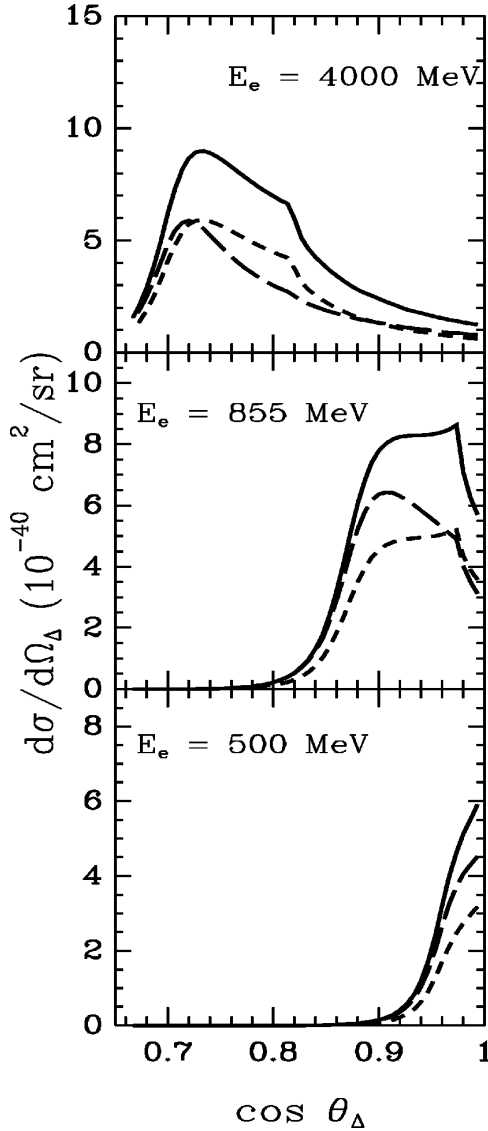
FIG. 1. Δ^0 angular distribution for the reaction $e^- + p \rightarrow \Delta^0 + \nu_e$ with three different sets of form factors, as explained in the text.
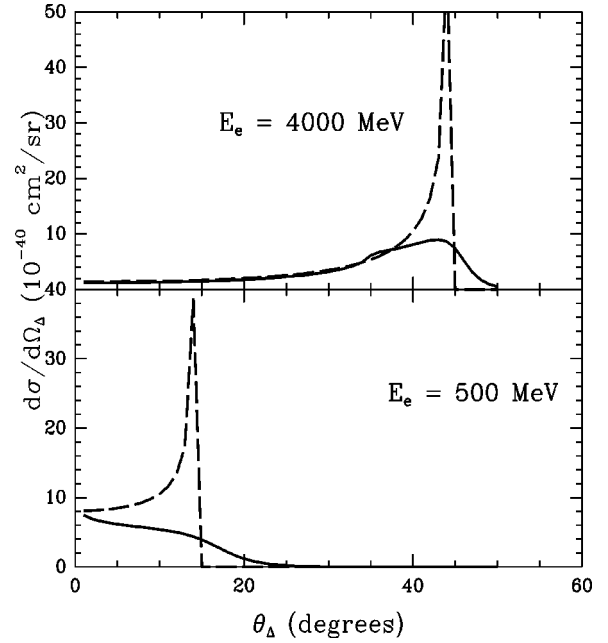

FIG. 2. Δ^0 angular distribution for the reaction $e^- + p \rightarrow \Delta^0 + \nu_e$ with finite (solid line) and zero (dashed line) widths. The form factors are taken to be the same as for the solid line of Fig. 1.

which the cross sections were earlier predicted to dominate is not there when the effect of decay width is taken into account. On the other hand there is a considerable cross section over a wide angular region, which increases as energy raises and corresponds to $0 < \theta < 45°$ for $E_e = 4.0$ GeV. Therefore, a high angular resolution is not really needed in the experiments and large acceptance detectors can be used to study this reaction. This feature of angular dependence of the cross section is maintained with all the form factors used in this study.

B. $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$

In Fig. 3, we present the results for $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$. For this process, the cross section is overall enhanced by an isospin factor of 3 and reduced due to the different sign of the interference term, which depends on energy and momentum transfer. The angular dependence of the cross section and its increase with the energy are, otherwise, quite similar to the $e^- + p \rightarrow \Delta^0 + \nu_e$.

The role of interference terms is very interesting in the case of N-Δ transition. As a comparison of Figs. 1 and 3 shows, the suppression due to the opposite sign of interference terms is quite large at lower energies to overtake the overall increase by a factor of 3 due to isospin. As the energy increases, the relative importance of the interference terms becomes small and the cross section $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$ dominates. At around $E_\nu \sim 1.5$ GeV, the cross sections for $e^- + p \rightarrow \Delta^0 + \nu_e$ and $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$ are comparable. The effect of the decay width of Δ is same as discussed in Sec. III A, and our results with $\Gamma = 0$ are in qualitative agreement with the results of Hwang *et al.* [15], except that we obtain a larger cross section compared to the cross sections obtained by them in the region away from the peak.

C. $e^- + p \rightarrow N^* + \nu_e$

In Fig. 4, we present the results for the $d\sigma/d\Omega_{N^*}$ at $E_e = 4$ GeV with the same invariant mass cut for N^* production as for Δ production ($W < 1.4$ GeV). We use the form factors obtained from Eqs. (25)–(27). In order to calculate them we take the amplitudes $S^p_{1/2}$ and $A^p_{1/2}$ of Gerhardt [27] as quoted by Li *et al.* [28]. The solid and dashed curves shown in Fig. 4 correspond to the two parametrizations of $S^p_{1/2}$ and $A^p_{1/2}$ of Ref. [28]. We find that the cross sections for N^* production are smaller than the Δ production cross sections by an order of magnitude. Furthermore, the N^* angular cross section peaks around $\cos\theta = 0.82$ as compared to the Δ production that peaks around $\cos\theta = 0.73$. The present uncertainty in the determination of the form factors $F_1^V(q^2)$ and $F_2^V(q^2)$ leads to an uncertainty of 20% in the cross section in the peak region, as shown in Fig. 4. This uncertainty does not affect the main conclusion of this study as the contribution of N^* production in the kinematic region of pions coming from Δ decay is still too small and peaks at a different angle than the
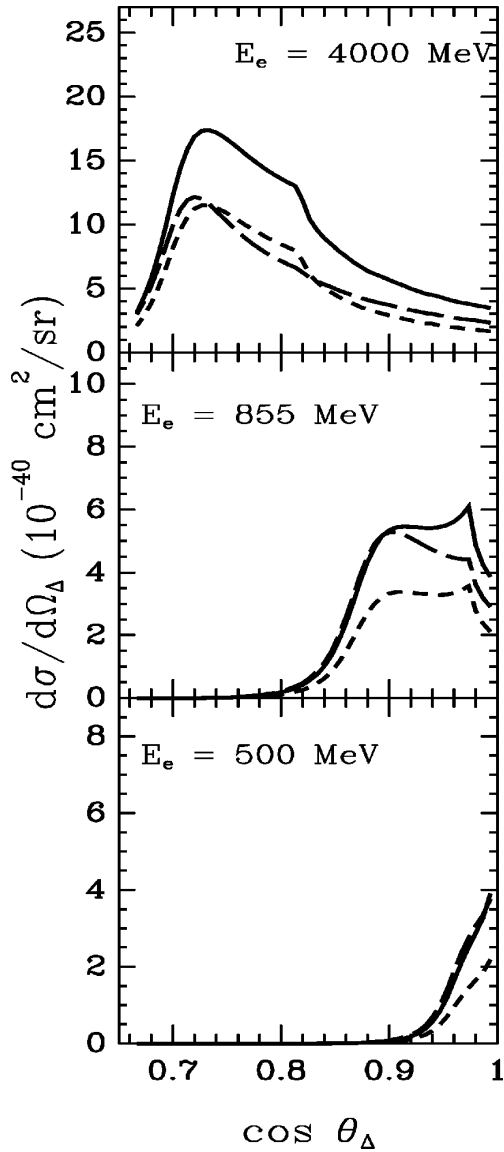
FIG. 3. Same as Fig. 1 for the reaction $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$.

Δ's. Increasing the invariant mass cut from 1.4 GeV to $(M^* + m_\pi)$, shifts the peak of N^* to still lower angles. Therefore, the larger width of the N^* affects pion production rates in an angular region well separated from the Δ produced pions.

IV. DISCUSSION AND CONCLUSIONS

We now address ourselves to the present experimental situation and the possibility of observing these reactions at Mainz and/or TJNAF accelerators. At these accelerators luminosities of the order of 10^{38} cm^{-2} sec^{-1} or more are expected. The estimated count rate is given by

$$\text{counts/hour} = \Delta\Omega \times (d\sigma/d\Omega)$$

$$\times \text{luminosity} \times 3600 \text{ sec/hour}$$

$$\times \text{detector efficiency}. \quad (31)$$

Using this formula, for example at 4.0 GeV, in the peak region of around 40° where the cross sections are of the order of 10^{-39} cm^2, we find the count rate to be
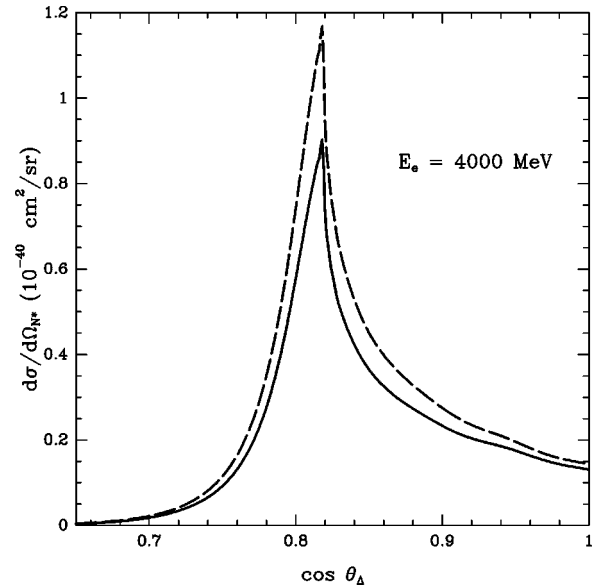

FIG. 4. N^* angular distribution for the reaction $e^- + p \rightarrow N^* + \nu_e$ with two different parametrizations of the vector form factors extracted from Gerhardt's analysis [27], as explained in the text.

$$\text{counts/hour} \sim 360 \times \Delta\Omega \text{ (sr)} \times \text{detector efficiency/hour.} \quad (32)$$

A similar count rate is expected at $E_e = 855$ MeV in the vicinity of 20°, where the cross sections are of the same order. Keeping in mind the finite angular range over which the cross sections are appreciably larger than 10^{-40} cm^2, the estimates made above suggest that the number of counts could be high enough for considering the feasibility of doing such an experiment.

Finally, to summarize this paper, we have made a theoretical study of the weak production of Δ and $N^*(1440)$ through the charge changing reactions induced by electron beams of energies corresponding to Mainz and TJNAF accelerators. We find that:

(1) The differential cross section for the weak production of Δ resonance with electron beams of the order of 10^{-39} cm^2/sr, which is quite sizable. At $E_e = 855$ MeV, the cross section for the $e^- + p \rightarrow \Delta^0 + \nu_e$ is larger than the cross section for the $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$ while at $E_e = 4.0$ GeV, the cross section for the $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$ process is about a factor of 2 larger than the $e^- + p \rightarrow \Delta^0 + \nu_e$. As we increase the energy from 855 to 4000 MeV the peak in the cross section shifts to a higher angle from 20° to 40°.

(2) There is a large angular region in which the differential cross sections are appreciable. This feature of the differential cross sections facilitates the observation of this reaction at current electron accelerators where large angle acceptance detectors are planned to be used in electron scattering experiments. There is no need for a sharp angular resolution in the vicinity of 0.1° as found earlier, based on a calculation neglecting the decay width of Δ resonance.

(3) The production cross section for N^* is an order of magnitude smaller than the production cross section of Δ and peaks at an angle well separated from the Δ production peak region. This makes the identification of Δ^0 through the measurement of pions and protons quite clean for the invariant

mass cut of $W < 1.4$ GeV. There is no such contamination from N^* resonances in the identification of Δ^{++}.

(4) The production cross section is dominated by the three form factors C_5^A, C_3^V, and C_4^V and an experimental measurement could discriminate between the various models used for these form factors. If the electromagnetic production cross sections for the Δ resonance are precise enough to fix the C_3^V and C_4^V form factors, this will make the determination of C_5^A quite model independent.

(5) There is a very strong energy dependence of the V-A interference terms in the Δ production process which can be used for determining the other form factors like $C_3^A(q^2)$ and $C_4^A(q^2)$. Experimental information on these form factors is extremely important for the theoretical models currently used for nucleon structure as well as for some earlier analyses which use quite different values of C_3^A and C_4^A for explaining the experimental data on neutrino scattering in the intermediate energy region.

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