Electromagnetic form factors of the nucleon in an improved quark model

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Nucleon electromagnetic form factors are studied in the cloudy bag model (CBM) with center-of-mass and recoil corrections. This is the first presentation of a full set of nucleon form factors using the CBM. The center-of-mass motion is eliminated via several different momentum projection techniques and the results are compared. It is found that the shapes of these form factors are significantly improved with respect to the experimental data if the Lorentz contraction of the internal structure of the baryon is also appropriately taken into account. [S0556-2813(98)03205-1]

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I. INTRODUCTION

Form factors characterize the internal structure of subatomic particles and, in particular, electromagnetic probes of hadrons provide important information on the underlying quark and gluon degrees of freedom. In the nonperturbative regime (i.e., at low momentum transfer), QCD-motivated, effective hadronic models continue to play an important role in analyzing and understanding a wealth of experimental data. The MIT bag model [1] was an early attempt to include the key features of confinement and asymptotic freedom in a quark based model of hadronic structure. The cloudy bag model (CBM) [2] improves on the MIT bag model significantly by introducing an elementary pion field coupled to the quarks inside the bag such that chiral symmetry is restored. The introduction of the pion field not only improves the static nucleon properties, but also provides a convenient connection to the study of conventional intermediate energy physics such as πN and NN scattering.

There are many calculations of the nucleon electromagnetic form factors within different hadronic models. Indeed, the understanding of these form factors is extremely important in any effective theory or model of the strong interaction. However, there is, to our knowledge, no truly satisfactory means of forming fully Lorentz covariant momentum eigenstates from any static model. In this work we suggest an improved treatment (a hybrid method of Galilean momentum projection combined with an appropriate Lorentz contraction) for a model which has been widely used for many years — the CBM — and bring it to larger momentum transfers. The present results not only remind us of the effectiveness of chiral quark models at moderate momentum transfer, but also remind us that they can serve as an essential first step in the investigation of the electromagnetic interaction in quark based nuclear models, in particular, the electromagnetic interaction in the quark-meson coupling (QMC) model [3].

In the CBM, as in the MIT bag model, quarks are independent particles confined in a rigid spherical well. The bag model wave function for a baryon is a direct product of individual quark wave functions, analogous to nuclear shell model wave functions. A static bag cannot carry a definite momentum and so bag-model baryon states are not total momentum eigenstates, in spite of the fact that the Hamiltonian commutes with the total momentum operator. Matrix elements evaluated between such states contain spurious center-of-mass motion. This defect compromises some of the predictive power of the model, such that only observables involved in very low-momentum transfer processes ($q^2/4m_N^2 \ll 1$) are typically assumed to be reliable.

Over the years, a number of prescriptions for the correction of the center-of-mass motion have been developed (for an overview, see for example Refs. [4]). The diversity of approaches may be viewed as an indication of the uncertainty associated with this correction. In contrast to the nonrelativistic case, the internal motion of a composite object cannot be explicitly separated from the collective motion in a covariant description. For the calculation of the form factors, a satisfactory treatment may result from a combination of relativistic boost, momentum projection, and a variational procedure. Betz and Goldflam [5] argued that a static soliton bag can be boosted consistently to a soliton bag moving with a finite velocity. However, this approach is impractical for boosting the MIT bag because of the sharp surface which prevents the construction of a simple boost operator [6]. A number of nonrelativistic methods for the center-of-mass correction exist in the literature [7–9]. Analytic forms of the recoil corrections can be obtained in a relativistic harmonic oscillator quark model [10]. Unfortunately different groups do not always agree with each other and sometimes even result in a correction with the opposite sign.

In this work we compare several intuitively simple momentum-projection procedures for the calculation of the nucleon electromagnetic form factors. The basic idea is to extract the momentum eigenstates from the static solutions by appropriate linear superpositions. The simplest prescription for this approach was proposed by Peierls and Yoccoz (PY) [11]. We will assume that a baryon is composed of three constituents. Hence the wave function for a moving baryon with total momentum $\mathbf{p}$ is constructed as

$$\Psi_{\rm PY}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p}) = N_{\rm PY}(\mathbf{p}) \int d^3\mathbf{x}\, e^{i\mathbf{p}\cdot\mathbf{x}} \Psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{x}), \tag{1}$$

where $N_{\rm PY}(\mathbf{p})$ is a momentum dependent normalization constant. The localized state is simply given by a product of the three individual quark wave functions,

$$\Psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{x})=q(\mathbf{x}_1-\mathbf{x})q(\mathbf{x}_2-\mathbf{x})q(\mathbf{x}_3-\mathbf{x}), \qquad (2)$$

where $\mathbf{x}$ refers to the location of the center of the static bag, and $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ specify the positions of the three constituent quarks. With the PY wave function, the predictions of the static baryon properties are generally improved [4]. It reduces the rms radius, increases g_A, and on the whole produces a better mass spectrum. However, it is unreliable for calculations of dynamic observables which involve large momentum transfers, since the PY wave function does not transform appropriately under Lorentz boosts.

A closely related method for eliminating the center-of-mass motion is called the Peierls-Thouless (PT) projection [12,13]. There the wave function is constructed through one further linear superposition in terms of the PY wave function,

$$\Psi_{\rm PT}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p})=N(\mathbf{p})\int d^3p'\,w(\mathbf{p}') \times e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{x}_{\rm c.m.}}\Psi_{\rm PY}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p}'), \qquad (3)$$

where $\mathbf{x}_{\rm c.m.}=(\mathbf{x}_1+\mathbf{x}_2+\mathbf{x}_3)/3$ is the center of mass of the baryon (we assume equal mass quarks here). Ideally the weight function, $w(\mathbf{p}')$, should be chosen to minimize the total energy, but this is quite complicated to implement in practice. As in Ref. [13], we make the choice $w(\mathbf{p}')=1$ for simplicity and convenience. Then integrations over $\mathbf{x}$ and $\mathbf{p}'$ can be carried out explicitly. This leads to a comparatively simple PT wave function for the baryon,

$$\Psi_{\rm PT}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p})=N_{\rm PT}e^{i\mathbf{p}\cdot\mathbf{x}_{\rm c.m.}}q(\mathbf{x}_1-\mathbf{x}_{\rm c.m.})q(\mathbf{x}_2-\mathbf{x}_{\rm c.m.}) \times q(\mathbf{x}_3-\mathbf{x}_{\rm c.m.}), \qquad (4)$$

where $N_{\rm PT}$ is determined by the requirement that it satisfy the normalization condition

$$\int d^3x_1d^3x_2d^3x_3\Psi^\dagger_{\rm PT}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p}')\Psi_{\rm PT}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3;\mathbf{p}) =(2\pi)^3\delta^{(3)}(\mathbf{p}'-\mathbf{p})\,\eta_{\mathbf{p}}, \qquad (5)$$

with $\eta_p=1$ for the nonrelativistic normalization and $\eta_p=E(p)/m_N$ if we wish to adopt a standard relativistic normalization for the baryon wave function.

Notice that the above methods of momentum projection act only on the center-of-mass coordinate and the individual quark wave functions are not affected. However, since baryons are composite objects, once they have nonzero momentum, their internal structure should be subsequently modified. For example, the bag surface is no longer spherical in the Breit frame, rather it should be contracted along the direction of motion. We take care of this effect in terms of the prescription by Licht and Pagnamenta [14].

It should be noted that the present work is the first presentation of calculations of the nucleon electromagnetic form factors using the CBM, besides the obvious improvement of the treatment. The outline of the paper is as follows. Firstly, we briefly review the electromagnetic interactions of the CBM in Sec. II. The calculation of electromagnetic form factors for the bare bag with momentum projection is then presented in Sec. III. In Sec. IV, we discuss the necessary scaling of the form factors due to the effects of Lorentz contraction. Pionic corrections are then given in Sec. V. The numerical results are presented and discussed in Sec. VI before the concluding remarks in Sec. VII. Some technical details and explicit proof of gauge invariance of the calculations are provided in the Appendix.

II. ELECTROMAGNETIC CURRENTS IN THE CBM

The linearized CBM Lagrangian with the pseudoscalar pion-quark coupling (up to order $1/f_\pi$) is given by [2]

$$\mathcal{L}=(i\bar{q}\gamma^\mu\partial_\mu q-B)\theta_V-\frac{1}{2}\bar{q}q\,\delta_S +\frac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\frac{1}{2}m_\pi^2\boldsymbol{\pi}^2-\frac{i}{2f_\pi}\bar{q}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}q\,\delta_S, \qquad (6)$$

where B is a bag constant, f_π is the π decay constant, θ_V is a step function (unity inside the bag volume and vanishing outside), and δ_S is a surface delta function. In a lowest order perturbative treatment of the pion field, the quark wave function is not affected by the pion field and is simply given by the MIT bag solution [1]

$$q(\mathbf{r})=\begin{pmatrix} g(r) \\ i\boldsymbol{\sigma}\cdot\hat{r}f(r)\end{pmatrix}\phi\;\theta(R-r), \qquad (7)$$

where ϕ contains the spin-isospin information for the wave function of the quark, $\boldsymbol{\sigma}$ is the usual Pauli spin operator, and R is the spherical bag radius. For the ground state of a massless quark $g(r)=N_sj_0(\omega_sr/R)$, $f(r)=N_sj_1(\omega_sr/R)$, where $\omega_s=2.0428$ and $N_s^2=\omega_s/8\pi R^3j_0^2(\omega_s)(\omega_s-1)$.

From the CBM Lagrangian given in Eq. (6), the conserved local electromagnetic current can be derived using the principle of minimal coupling $\partial_\mu\to\partial_\mu+iqA_\mu$, where q is the charge carried by the field upon which the derivative operator acts. The total electromagnetic current is then

$$J^\mu(x)=j^{\mu(Q)}(x)+j^{\mu(\pi)}(x), \qquad (8)$$

$$j^{\mu(Q)}(x)=\sum_f Q_fe\bar{q}_f(x)\gamma^\mu q_f(x), \qquad (9)$$

$$j^{\mu(\pi)}(x)=-ie[\pi^\dagger(x)\partial^\mu\pi(x)-\pi(x)\partial^\mu\pi^\dagger(x)], \qquad (10)$$

where $q_f(x)$ is the quark field operator for the flavor f, Q_f is its charge in units of e, and $e\equiv|e|$ is the magnitude of the electron charge. The charged pion field operator is defined as

$$\pi(x)=\frac{1}{\sqrt{2}}[\pi_1(x)+i\pi_2(x)], \qquad (11)$$

which either destroys a negatively charged pion or creates a positively charged one.

The physical baryon state is then a dressed bag, consisting of a superposition of a bare bag and a bag with a pion cloud. Algebraically, it has the form

$$|A\rangle = \sqrt{Z_2^A}[1+(m_A - H_0 - \Lambda H_I \Lambda)^{-1} H_I]|A_0\rangle, \quad (12)$$

where Z_2^A is the bare baryon probability in the physical baryon states,

$$Z_2^A = \left[1 + \sum_B \left(\frac{f_0^{AB}}{m_\pi}\right)^2 \frac{1}{12\pi^2} \; \mathrm{P}\int_0^\infty \frac{dk\; k^4 u^2(kR)}{\omega_k (m_A - m_B - \omega_k)^2}\right]^{-1}, \quad (13)$$

where Λ is a projection operator which annihilates all the components of $|A\rangle$ without at least one pion, and H_I is the interaction Hamiltonian which describes the process of emission and absorption of pions. We follow the traditional CBM treatments and consider only states with at most one pion. The matrix elements of H_I between the bare baryon states and their properties are then given by [15]

$$v_{0j}^{AB}(\vec{k}) \equiv \langle A_0 | H_I | \boldsymbol{\pi}_j(\vec{k}) B_0\rangle = \frac{i f_0^{AB}}{m_\pi} \frac{u(kR)}{[2\omega_k (2\pi)^3]^{1/2}} \times \sum_{m,n} C_{S_B 1 S_A}^{s_B m s_A} (\hat{s}_m^* \cdot \vec{k}) C_{T_B 1 T_A}^{t_B n t_A} (\hat{t}_n^* \cdot \vec{e}_j), \quad (14)$$

$$w_{0j}^{AB}(\vec{k}) \equiv \langle A_0 \boldsymbol{\pi}_j(\vec{k}) | H_I | B_0\rangle = [v_{0j}^{BA}(\vec{k})]^* = -v_{0j}^{AB}(\vec{k}) = v_{0j}^{AB}(-\vec{k}), \quad (15)$$

where the pion has momentum $\vec{k}$ and isospin projection j. Note also that f_0^{AB} is the reduced matrix element for the $\pi B_0 \to A_0$ transition vertex, $u(kR) \equiv 3 j_1(kR)/kR$, $\omega_k = \sqrt{k^2 + m_\pi^2}$, and $\hat{s}_m$ and $\hat{t}_n$ are spherical unit vectors for spin and isospin, respectively.

III. MOMENTUM PROJECTION CALCULATIONS FOR A BARE BAG

It is customary to define the nucleon electric (G_E) and magnetic (G_M) form factors in the Breit frame by

$$\left\langle N_{s'}\left(\frac{\vec{q}}{2}\right) \middle| J^0(0) \middle| N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi_{s'}^\dagger \chi_s G_E(q^2), \quad (16)$$

$$\left\langle N_{s'}\left(\frac{\vec{q}}{2}\right) \middle| \vec{J}(0) \middle| N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi_{s'}^\dagger \frac{i\boldsymbol{\sigma}\times\vec{q}}{2m_N} \chi_s G_M(q^2), \quad (17)$$

where χ_s and $\chi_{s'}^\dagger$ are Pauli spinors for the initial and final nucleons, $\vec{q}$ is the Breit-frame three momentum transfer, i.e., $q^2 = q_0^2 - \vec{q}^2 = -\vec{q}^2 = -Q^2$. We choose $\vec{q}$ to define the z axis. The major advantage of the Breit frame is that G_E and G_M are explicitly decoupled, and can be determined respectively by the time and space components of the electromagnetic current operator J^μ.

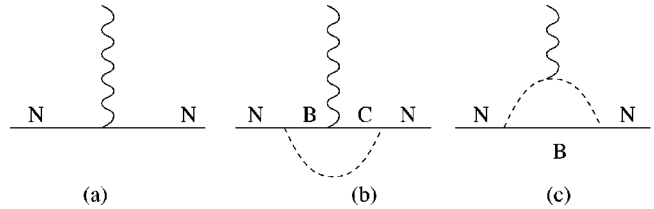

FIG. 1. Diagrams illustrating the various contributions included in this calculation (up to one pion loop). The intermediate baryons B and C are restricted to the N and Δ.

In the definition above [i.e., Eqs. (16) and (17)], both initial and final states are physical states. Using Eqs. (8) and (12), the total electromagnetic form factors can be expressed in terms of the three processes shown in Fig. 1. In this section we calculate the contribution from the bare bag only, and leave the pion loop effects to be included in a later section. For the three-momentum eigenstates, Eqs. (1) and (4), we can proceed to calculate the electromagnetic form factors in a relatively straightforward way. For the PY projection, we obtain,

$$G_E^{\mathrm{PY}}(q^2) = I_E(q^2)/D_{\mathrm{PY}}(q^2), \quad (18)$$

$$G_M^{\mathrm{PY}}(q^2) = I_M(q^2)/D_{\mathrm{PY}}(q^2), \quad (19)$$

where

$$I_E(q^2) = \int_0^\infty dz z^2\; N_Q^2(z) \int d^3r\; j_0(qr) \times \left[g_+ g_- + \frac{f_+ f_-}{r_+ r_-}\left(r^2 - \frac{z^2}{4}\right)\right]\Theta, \quad (20)$$

$$I_M(q^2) = 2m_N \int_0^\infty dz z^2\; N_Q^2(z) \int d^3r\; \frac{j_1(qr)}{qr} \times \left[r^2\left(\frac{g_- f_+}{r_+} + \frac{g_+ f_-}{r_-}\right) + \frac{\vec{r}\cdot\vec{z}}{2}\left(\frac{g_- f_+}{r_+} - \frac{g_+ f_-}{r_-}\right)\right]\Theta, \quad (21)$$

$$D_{\mathrm{PY}}(q^2) = \int_0^\infty dz z^2\; N_Q^3(z)\; j_0(qz/2)\Theta, \quad (22)$$

$$N_Q(z) = \int d^3r \left[g_+ g_- + \frac{f_+ f_-}{r_+ r_-}\left(r^2 - \frac{z^2}{4}\right)\right]\Theta. \quad (23)$$

Here $D_{\mathrm{PY}}(q^2)$ is the momentum dependent normalization factor and $N_Q(z)$ is the overlap integral associated with each quark spectator. The following shorthand notation has been used in the above equations:

$$r_\pm \equiv \left|\vec{r} \pm \frac{\vec{z}}{2}\right|, \quad (24)$$

$$g_\pm \equiv g(r_\pm), \quad f_\pm \equiv f(r_\pm), \quad (25)$$

$$\Theta \equiv \theta(R - r_+)\,\theta(R - r_-). \quad (26)$$

Similarly, for the PT projection, we obtain

$$G_E^{\rm PT}(q^2)=\int d^3r j_0(qr)\rho(r)K(r)/D_{\rm PT}, \qquad (27)$$

$$G_M^{\rm PT}(q^2)=\int d^3r j_1(qr)g(r)f(r)K(r)/D_{\rm PT}, \qquad (28)$$

where

$$D_{\rm PT}=\int d^3r\rho(r)K(r), \qquad (29)$$

with $\rho(r)\equiv g^2(r)+f^2(r)$ and $K(r)\equiv\int d^3x\rho(\vec{x})\rho(-\vec{x}-\vec{r})$ is the recoil function to account for the correlation of the two spectator quarks.

As expected, without the momentum projection, Eqs. (18, 19) and Eqs. (27, 28) would reduce to the familiar results for the static, spherical MIT bag, i.e.,

$$G_E^{\rm (static)}(q^2)=\int d^3r\ j_0(qr)\ [g^2(r)+f^2(r)], \qquad (30)$$

$$G_M^{\rm (static)}(q^2)=2m_N\int d^3r\ \frac{j_1(qr)}{q}\,[2g(r)f(r)]. \qquad (31)$$

Note that nonrelativistic normalization of the nucleon wave functions has been used here [see Eq. (5)], as is appropriate for the simple (Galilean) three-momentum projections being used here.

IV. CORRECTIONS FROM THE LORENTZ CONTRACTION

A complete solution of a covariant many-body problem is extremely difficult. There is a substantial body of literature which uses light-cone dynamics [16] for the constituent quarks. With a few parameters this approach can reproduce experimental data over quite a large momentum transfer range. For the bag model, there is no Lorentz covariant solution for an extended quantum object in more than two dimensions [17]. Thus we use a semiclassical prescription here.

As mentioned in the introduction, the spherical bag is expected to undergo a Lorentz contraction along the direction of motion once it acquires a momentum. An intuitive prescription by Licht and Pagnamenta [14] suggested that, in the preferred Breit frame, the interaction of the individual constituents of a cluster with the projectile may be regarded as instantaneous to a good approximation. Relativistic form factors can be simply derived from the corresponding nonrelativistic ones by a simple substitution rule. In the case of the bag model, once the spurious center-of-mass motion is subtracted using the PY or PT procedure, it is natural to rescale the quark internal coordinates as well, i.e.,

$$\Psi(\vec{x}_1,\vec{x}_2,\vec{x}_3;\vec{0})\xrightarrow{\text{projection}}\Psi(\vec{x}_1,\vec{x}_2,\vec{x}_3;\vec{p})$$
$$\xrightarrow{\text{contraction}}\Psi(\vec{x}'_1,\vec{x}'_2,\vec{x}'_3;\vec{p}) \qquad (32)$$

where the quark coordinates $\vec{x}'_i$ for the moving bag are related to the $\vec{x}_i$ by a Lorentz transformation. Without loss of generality, we again choose the photon momentum $\vec{q}$ along the z direction. Then in the Breit frame, the quark displacements must contract in the z direction while they remain unchanged in the x and y directions. Thus we have

$$z'_i=\frac{m_N}{E}z_i, \qquad (33)$$

$$d^3x'_1d^3x'_2=\left(\frac{m_N}{E}\right)^2d^3x_1d^3x_2, \qquad (34)$$

where we have assumed $t=0$ for all constituents, i.e., the instantaneous approximation. Note that m_N is the nucleon mass and E is the on-shell nucleon energy in the Breit frame. The $(m_N/E)^2$ factor is due to the Lorentz contraction of the coordinates of the two spectator quarks along the direction of motion. As an example, the proton charge form factor in the PT scheme is thus

$$\begin{aligned}G_E(q^2)&=\int\prod_i d^3x'_ie^{-iqz'_3}[q_{\vec{p}}^{\dagger}(\vec{x}'_3-\vec{x}'_{\rm c.m.})\,\delta(\vec{x}'_3)\\&\quad\times q_{\vec{p}}(\vec{x}'_3-\vec{x}'_{\rm c.m.})]\ \rho_{\vec{p}}(\vec{x}'_1-\vec{x}'_{\rm c.m.})\\&\quad\times\rho_{\vec{p}}(\vec{x}'_2-\vec{x}'_{\rm c.m.})\\&=\left(\frac{m_N}{E}\right)^2\int d^3x_1d^3x_2e^{-iqz(m_N/E)}\rho(\vec{x}_1-\vec{x}_{\rm c.m.})\\&\quad\times\rho(\vec{x}_2-\vec{x}_{\rm c.m.})\rho(-\vec{x}_{\rm c.m.})\\&=\left(\frac{m_N}{E}\right)^2G_E^{\rm sph}(q^2m_N^2/E^2),\end{aligned} \qquad (35)$$

where $q_{\vec{p}}$ is the quark wave function in the Breit frame (in a deformed bag), $\rho_{\vec{p}}$ is the probability density of the quark, and $G_E^{\rm sph}$ is the charge form factor calculated with the spherical static bag wave function [such as Eqs. (18) and (27)]. In the second step of the derivation we have used the fact that a probability amplitude is a constant in different Lorentz frames, hence, the identity $q_{\vec{p}}(\vec{x})'\equiv q(\vec{x})$ has been used as in Ref. [14]. For the magnetic form factor, a similar expression can be derived, $G_M(q^2)=(m_N/E)^2G_E^{\rm sph}(q^2m_N^2/E^2)$. Note that we used the fact that all three quarks have the same spatial wave function in obtaining Eq. (35). The scaling factor in the argument is due to the coordinate change of the struck quark and the factor in the front, $(m_N/E)^2$, comes from the reduction of the integral measure of two spectator quarks in the Breit frame. Note that this prescription is similar, but not identical, to the Lorentz contraction arguments used in the Skyrme model [18]. The difference is the $(m_N/E)^2$ factor in front of $G_E^{\rm sph}$ which is absent in Ref. [18]. It might be argued that, since we use only a nonrelativistic momentum projection [i.e., η_p in Eq. (5) cannot be fixed unambiguously], this factor is not well determined.

V. CORRECTIONS FROM THE PERTURBATIVE PION CLOUD

In the CBM with a bag radius above 0.7 fm the pion field is relatively weak and the pionic effects can be included perturbatively [19]. As usual we assume that there is *no more than one pion in the air*. There are two processes contributing to the nucleon electromagnetic form factors due to the pion cloud. One involves the direct $\gamma\pi\pi$ coupling shown in Fig. 1(c), and the other is the γqq coupling inside a pion loop, as in Fig. 1(b).

Figure 1(c) actually contains three time-ordered subdiagrams, and has been evaluated in the CBM by Théberge and Thomas [15]. They gave

$$G^{(\pi)}_{(E,M)}(q^2)=G^{(\pi)}_{(E,M)}(q^2;N)+G^{(\pi)}_{(E,M)}(q^2;\Delta), \tag{36}$$

where the two terms correspond to two cases with different intermediate baryons (N and Δ). For completeness, we quote the individual contributions explicitly,

$$G^{(\pi)}_E(q^2;N)=\frac{1}{36\pi^3}\left(\frac{f^{NN}}{m_\pi}\right)^2 \times\int d^3k\,\frac{u(kR)\,u(k'R)\,\vec{k}\cdot\vec{k}'}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})}\langle N|\tau_3|N\rangle, \tag{37}$$

$$G^{(\pi)}_E(q^2;\Delta)=-\frac{1}{72\pi^3}\left(\frac{f^{N\Delta}}{m_\pi}\right)^2 \times\int d^3k\,\frac{u(kR)\,u(k'R)\,\vec{k}\cdot\vec{k}'}{(\omega_{\Delta N}+\omega_k)(\omega_{\Delta N}+\omega_{k'})(\omega_k+\omega_{k'})} \times\langle N|\tau_3|N\rangle, \tag{38}$$

$$G^{(\pi)}_M(q^2;N)=\frac{2m_N}{72\pi^3}\left(\frac{f^{NN}}{m_\pi}\right)^2 \times\int d^3k\,\frac{u(kR)\,u(k'R)\,(\hat{q}\times\vec{k})^2}{(\omega_k\omega_{k'})^2}\langle N|\tau_3|N\rangle, \tag{39}$$

$$G^{(\pi)}_M(q^2;\Delta)=\frac{2m_N}{288\pi^3}\left(\frac{f^{N\Delta}}{m_\pi}\right)^2\int d^3k \times\frac{(\omega_{\Delta N}+\omega_k+\omega_{k'})\,u(kR)\,u(k'R)\,(\hat{q}\times\vec{k})^2}{(\omega_{\Delta N}+\omega_k)(\omega_{\Delta N}+\omega_{k'})(\omega_k\omega_{k'})(\omega_k+\omega_{k'})} \times\langle N|\tau_3|N\rangle, \tag{40}$$

where $\vec{k}'=\vec{k}+\vec{q}$, $\omega_{BN}\simeq m_B-m_N$, f^{NB} is the renormalized πNB coupling constant, and τ_3 is the third nucleon isospin Pauli matrix.

Corresponding to Fig. 1(b), the transition matrix element can be written as

$$\langle N(\vec{q}/2)|j^{\mu(Q)}(0)|N(-\vec{q}/2)\rangle=\sum_{BC,j}\int d^3k \times\frac{\langle N(\vec{q}/2)|H_I|N(\vec{p}\,'),\pi_j(\vec{k})\rangle\langle B(\vec{p}\,')|j^{\mu(Q)}(0)|C(\vec{p})\rangle\langle C(\vec{p}),\pi_j(\vec{k})|H_I|N(-\vec{q}/2)\rangle}{(\omega_{BN}+\omega_k)(\omega_{CN}+\omega_k)}, \tag{41}$$

where $\vec{p}\,'=(\vec{q}/2)+\vec{k}$ and $\vec{p}=-(\vec{q}/2)+\vec{k}$ are the momenta for the intermediate baryons B and C. With the dynamical baryons and pion here, we have to evaluate the electromagnetic matrix elements for the intermediate processes in an arbitrary frame. Thus the matrix elements of J^0 might contain both $G_E(q^2)$ and $G_M(q^2)$, as do those of $\vec{J}$. It is convenient to use the identity

$$\langle p'|j^{\mu(Q)}(0)|p\rangle=\overline{u}(p')\left[\gamma^\mu F_1(q^2)+\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2(q^2)\right]u(p), \tag{42}$$

where $F_1(q^2)=[G_E(q^2)+\eta G_M(q^2)]/(1+\eta)$ and $F_2(q^2)=[G_M(q^2)-G_E(q^2)]/(1+\eta)$ with $\eta=-q^2/4m_N^2$. Both $F_1(q^2)$ and $F_2(q^2)$ are Lorentz scalar functions and hence can be evaluated in any frame. However, it can be shown that after integrating over the loop momentum, $\vec{k}$, the time (G_E) and space (G_M) components of Eq. (41) decouple again as long as the overall matrix element is evaluated in the Breit frame. The detailed expressions are messy and are therefore given in the Appendix.

VI. RESULTS AND DISCUSSION

In this work, we have adopted the usual philosophy for the renormalization in the CBM, using the approximate relation, $f^{AB}\simeq(f_0^{AB}/f_0^{NN})f^{NN}$. There are uncertain corrections on the bare coupling constant f_0^{NN}, such as the nonzero quark mass and the correction for spurious center-of-mass motion. Therefore, we use the renormalized coupling constant in our calculation, $f^{NN}\simeq 3.03$, which corresponds to the usual πNN coupling constant, $f^2_{\pi NN}\simeq 0.081$.

It should be pointed out that there is no unambiguous way to implement strict gauge invariance (the Ward-Takahaski identities) for a composite particle [20]. In this work, we ensure a somewhat weak requirement — electromagnetic

TABLE I. Magnetic moments of the nucleon. The static case refers to the original CBM results without center-of-mass correction, and PY and PT are for two calculations with momentum projected wave functions. The experimental values are $2.79\mu_B$ and $-1.91\mu_B$, respectively.

	Proton			Neutron		
R(fm)	static	PY	PT	static	PY	PT
0.8	2.49	2.25	2.36	−2.06	−1.89	−1.97
0.9	2.44	2.18	2.30	−1.96	−1.78	−1.86
1.0	2.46	2.18	2.31	−1.92	−1.73	−1.81
1.1	2.53	2.23	2.36	−1.93	−1.71	−1.81

current conservation [21]. The explicit proof is given in the Appendix. Recall that Eqs. (37), (38), (39), and (40) are results evaluated under a heavy baryon approximation. Ideally, Fig. 1(c) should be evaluated on the same footing as Fig. 1(b). Numerical calculations show that the recoil effects for intermediate baryon in the pion loop are negligible, and we may therefore ignore this recoil and use the standard static CBM results for the pionic correction. Consequently, the charge form factors at zero momentum transfer automatically satisfy the requirement of charge conservation, i.e., $G_E(0) = G_E^{(Q)}(0) + G_E^{(\pi)}(0) = e_N$, where e_N is 1 for the proton and 0 for the neutron.

The magnetic moments are simply the values of the magnetic form factors at zero momentum transfer, $\mu \equiv G_M^{(Q)}(0) + G_M^{(\pi)}(0)$. Note that the expression for the contribution from Fig. 1(b) is somewhat different from Ref. [15]. Here, there is no Z_2 factor for Fig. 1(b) consistent with the charge conservation. As a result of this choice the numerical contribution from Fig. 1(b) increases by roughly 30% ($Z_2^N \simeq 0.73$ for $R=1$ fm), bringing the total nucleon magnetic moments a few percent closer to the experimental data.

Table I gives the nucleon magnetic moments in this calculation. The center-of-mass correction reduces the static values by $5-10$ %. This is in contradiction with Ref. [7] but is consistent with Refs. [9,22]. The bag radius dependence is significantly reduced by the pion cloud in the CBM, with little variation over the range $R=0.8-1.0$ fm. The deficiency of the nucleon magnetic moments may be attributed to the higher order pionic corrections and explicit vector meson contributions [23].

The difference between the two choices of normalization of the wave function [i.e., the factor $\eta_p = 1$ or E/m_N in Eq. (5)] is not significant with respect to the shape of the form factors. However it will smoothly scale these form factors. For the proton charge form factor, for example, the relativistic normalization raises the form factor roughly 5% at $Q^2 = 0.5\ \mathrm{GeV}^2$ and 10% at $Q^2 = 1.0\ \mathrm{GeV}^2$. For clarity, we have always used $\eta_p = 1$ in the following figures as previously stated.

The characteristic effect of the center-of-mass correction on the charge form factor of the bare proton bag is illustrated in Fig. 2 with the bag radius $R=1$ fm. The ''dipole'' refers to the standard dipole fit, $F(Q^2) = 1/(1+Q^2/0.71\ \mathrm{GeV}^2)^2$. The bare charge form factors calculated with the static bag usually drop too quickly. With the PY projection procedure, the form factor at moderate momentum transfer ($Q^2 \sim 0.5\ \mathrm{GeV}^2$) increases nearly 100%. However the shape of the form factor does not change very much. It is generally too stiff and drops too fast, which is mainly due to the sharp surface of the cavity approximation and lack of translational invariance of the wave function. Using the translational invariant PT projection procedure leads to improved behavior of the form factors. In particular, after including the correction arising from Lorentz contraction, the shape of the form factors is significantly improved. It is reassuring to see that the combination of Lorentz contraction and Galilean (nonrelativistic) momentum projection is less scheme dependent than the momentum projection alone, e.g., compare the pairs of curves PY and PT with PYL and PTL in Fig. 2.

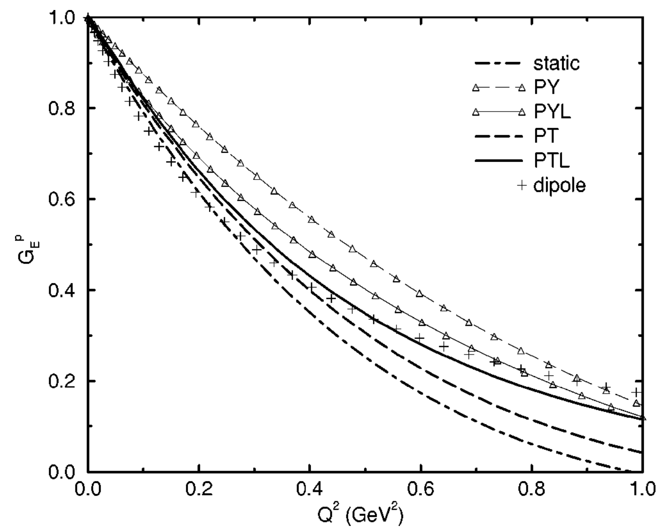

FIG. 2. The effect of the center-of-mass correction and Lorentz contraction for the charge form factor of the bare proton. The bag radius is taken to be 1.0 fm. The ''static'' curve refers to the naive MIT cavity approximation, PY and PT stand for Peierls-Yoccoz and Peierls-Thouless projection, respectively, and PYL and PTL for the corresponding versions with the Lorentz contraction.

Figures 3 and 4 show the individual contributions to the

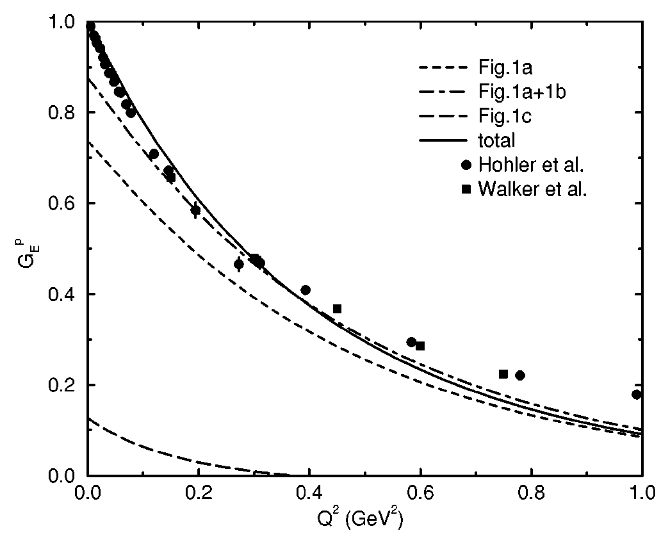

FIG. 3. The individual contributions to the proton charge form factor with the bag radius $R=1.0$ fm. The quark part is calculated using the Lorentz contracted PT wave functions. Experimental data are taken from Ref. [25].

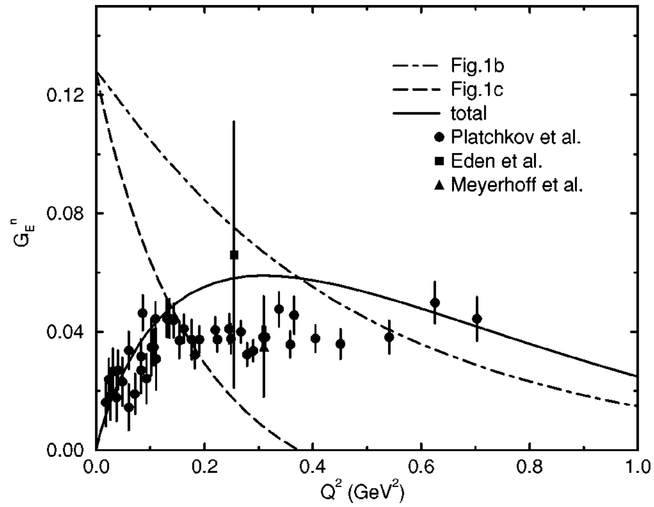

FIG. 4. The individual contributions to the neutron charge form factor. The key is as in Fig. 3, except that the experimental data is from Ref. [26]. As the contribution from Fig. 1(c) is negative, we show its magnitude for convenience.

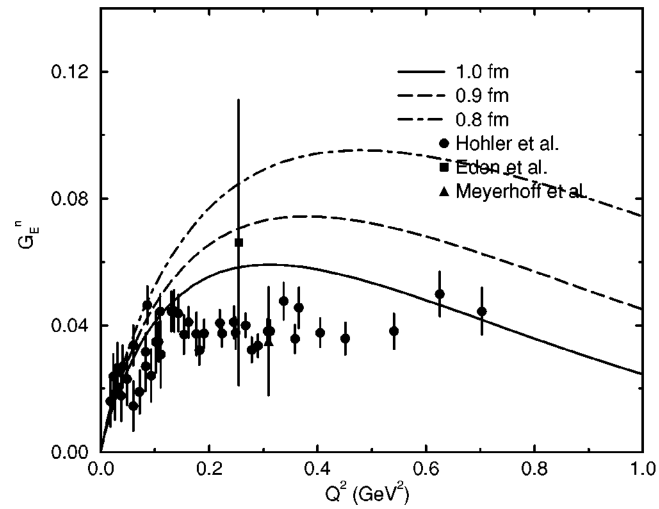

FIG. 6. The neutron charge form factor using Lorentz contracted PT wave functions. Data are the same as in Fig. 4.

charge form factors from the quarks and the pion cloud, for a typical bag radius of $R=1$ fm. For the proton charge form factor, it is clear that the γqq coupling terms dominate the form factor where the bare photon-bag coupling contributes nearly 75%. The correction from the $\gamma\pi\pi$ coupling decreases very quickly as the momentum transfer increases. A smaller bag radius will lead to a larger pionic contribution. For the neutron charge form factors, the contribution from the photon-bare bag coupling [Fig. 1(a)] vanishes due to the SU(6) structure. The severe cancellation between Fig. 1(b) and Fig. 1(c) results in a small but nonvanishing neutron charge form factor, with a negative mean square radius as one would expected simply from the Heisenberg Uncertainty Principle. With the bag radius $R=1$ fm, we obtain the neutron charge rms radius of $\langle r^2\rangle_{En}=-0.14\ \mathrm{fm}^2$, to be compared with the experimental value of $-0.12\ \mathrm{fm}^2$ [24].

Figures 5–8 show the bag radius dependence of the nucleon electromagnetic form factors. A large bag radius always leads to a softer form factor. We have used PT wave functions with Lorentz contraction in these calculations. The predictions show quite a reasonable agreement with the experimental data to much larger values of the momentum transfer than one has tended to expect.

VII. SUMMARY

We have calculated the electromagnetic form factors of the nucleon within the CBM, including relativistic corrections in the form of momentum projection and the Lorentz contraction of the internal structure. Electromagnetic current conservation is ensured in this calculation which is performed in the Breit frame. This is the first time that a presentation of all the nucleon electromagnetic form factors has been made for the CBM. The two different procedures of

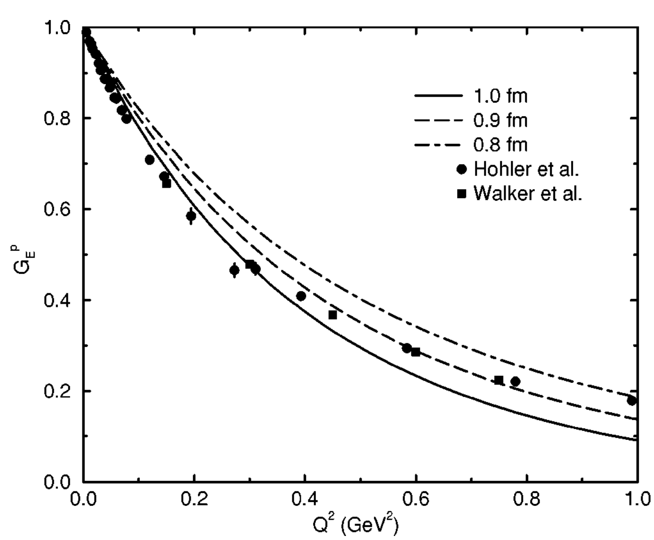

FIG. 5. The proton charge form factor for three different bag radii. Lorentz contracted PT wave functions (with $\eta_p=1$) are used in the calculations. Data are the same as in Fig. 3.

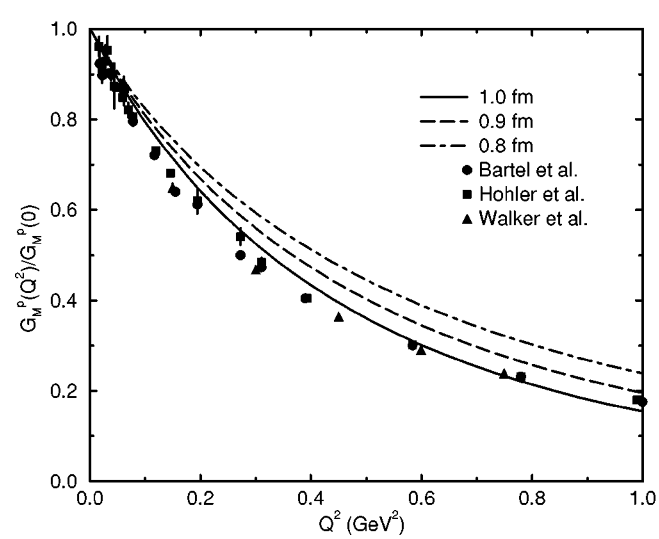

FIG. 7. The proton magnetic form factor using Lorentz contracted PT wave functions. Experimental data are from Refs. [25,27].

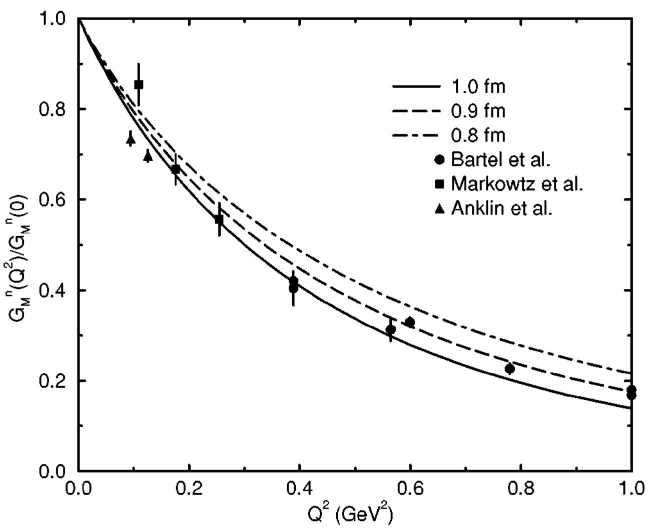

FIG. 8. The neutron magnetic form factor using Lorentz contracted PT wave functions. Experimental data are from Refs. [27,28].

momentum projection for the spurious center-of-mass motion give results which are relatively close to each other when Lorentz contraction effects are included. The Galilean invariant PT projection is generally a little better than the PY method in that it leads to a shape more closely resembling the dipole form. Including the corrections for center-of-mass motion and Lorentz contraction, the numerical predictions are in rather good agreement with data in the region $Q^2 < 1$ GeV^2. This is quite a remarkable result when one realizes the simplicity of the model. In particular, there are no explicit vector meson contributions and one possible future development would be to include $\pi\pi$ interactions.

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APPENDIX

In Fig. 1(b), the intermediate γBC vertex can no longer be in the Breit frame. A straightforward evaluation gives the matrix elements

$$\overline{u}(p')\gamma^0 u(p)=N'N\left[1+\frac{k^2-q^2/4}{(E'+m_N)(E+m_N)}\right]+\mathcal{O}(\vec{k}), \tag{A1}$$

$$\overline{u}(p')\frac{i\sigma^{0\nu}q_\nu}{2m_N}u(p)=-\frac{N'N}{2m_N}\left(\frac{q^2/2+\vec{k}\cdot\vec{q}}{E'+m_N}-\frac{-q^2/2+\vec{k}\cdot\vec{q}}{E+m_N}\right)+\mathcal{O}(\vec{k}), \tag{A2}$$

$$\overline{u}(p')\gamma^a u(p)=\frac{i}{2}(\boldsymbol{\sigma}\times\vec{q})_a N'N\left(\frac{1}{E'+m_N}+\frac{1}{E+m_N}\right)+\mathcal{O}(\vec{k}), \tag{A3}$$

$$\overline{u}(p')\frac{i\sigma^{a\nu}q_\nu}{2m_N}u(p)=\frac{i}{2m_N}(\boldsymbol{\sigma}\times\vec{q})_a N'N\times\left[1-\frac{q^2/4+k^2}{(E'+m_N)(E+m_N)}\right]+\mathcal{O}(\vec{k}), \tag{A4}$$

where $\vec{p}\,'=(\vec{q}/2)+\vec{k}$, $\vec{p}=-(\vec{q}/2)+\vec{k}$, $E=(\vec{p}^{\,2}+m_N^2)^{1/2}$, $E'=(\vec{p}\,'^2+m_N^2)^{1/2}$, and the normalization constants are $N=[(E+m_N)/2m_N]^{1/2}$, $N'=[(E'+m_N)/2m_N]^{1/2}$. The index a in Eqs. (A3) and (A4) denotes a space component, the $\mathcal{O}(\vec{k})$ terms in all equations refer to other pieces which are odd in $\vec{k}$ and will vanish after integration over the loop momentum $\vec{k}$. Substituting the above matrix elements into Eq. (41) and performing some spin and isospin algebra, we obtain the nucleon electric and magnetic form factors originating from the γqq coupling [i.e., the combination of Fig. 1(a) and 1(b) with a proper normalization],

$$G_E^{(Q)}(q^2)=Z_2G_E^{(b)}(q^2)\begin{pmatrix}1\\0\end{pmatrix}+E_{NN}(q^2)\begin{pmatrix}1/3\\2/3\end{pmatrix}+E_{\Delta\Delta}(q^2)\begin{pmatrix}4/3\\-1/3\end{pmatrix}, \tag{A5}$$

$$G_M^{(Q)}(q^2)=Z_2G_M^{(b)}(q^2)\begin{pmatrix}1\\-2/3\end{pmatrix}+M_{NN}(q^2)\begin{pmatrix}1/27\\-4/27\end{pmatrix}+M_{\Delta\Delta}(q^2)\begin{pmatrix}20/27\\-5/27\end{pmatrix}+M_{\Delta N}(q^2)\begin{pmatrix}16\sqrt{2}/27\\-16\sqrt{2}/27\end{pmatrix}, \tag{A6}$$

where the upper and lower coefficients refer to the proton and neutron respectively. Here $G_E^{(b)}$ and $G_M^{(b)}$ are the bare form factors calculated in Sec. III and IV, and E_{BC} and M_{BC} are given by

$$E_{BC}(q^2)=\frac{f^{NB}f^{NC}}{12\pi^2m_\pi^2}\int_0^\infty\frac{dk\;k^4u^2(kR)}{(\omega_{BN}+\omega_k)(\omega_{CN}+\omega_k)\omega_k}\Omega_E(q,k), \tag{A7}$$

$$M_{BC}(q^2)=\frac{f^{NB}f^{NC}}{12\pi^2m_\pi^2}\int_0^\infty\frac{dk\;k^4u^2(kR)}{(\omega_{BN}+\omega_k)(\omega_{CN}+\omega_k)\omega_k}\Omega_M(q,k), \tag{A8}$$

where $\Omega_E(q,k)$ and $\Omega_M(q,k)$ contain the recoil corrections for the intermediate baryons in Fig. 1(b) and are given by

$$\Omega_E(q,k)=\frac{1}{4m_N}\int_{-1}^{1}dx[(E'+m_N)(E+m_N)]^{1/2}\times\left\{\left[1+\frac{k^2-q^2/4}{(E'+m_N)(E+m_N)}\right]F_1(q^2)-\frac{1}{2m_N}\left(\frac{q^2/2+\vec{k}\cdot\vec{q}}{E'+m_N}-\frac{-q^2/2+\vec{k}\cdot\vec{q}}{E+m_N}\right)F_2(q^2)\right\}, \quad (A9)$$

$$\Omega_M(q,k)=\frac{1}{4m_N}\int_{-1}^{1}dx[(E'+m_N)(E+m_N)]^{1/2}\times\left\{m_N\left(\frac{1}{E'+m_N}+\frac{1}{E+m_N}\right)F_1(q^2)+\left[1+\frac{k^2-q^2/4}{(E'+m_N)(E+m_N)}\right]F_2(q^2)\right\}. \quad (A10)$$

Without the recoil of the intermediate baryons (i.e., with $\vec{k}$ set to zero) the usual CBM results are thus recovered,

$$\Omega_E(q,k)=F_1(q^2)-\eta F_2(q^2)=G_E^{(b)}(q^2), \quad (A11)$$

$$\Omega_M(q,k)=F_1(q^2)+F_2(q^2)=G_M^{(b)}(q^2). \quad (A12)$$

Now let us discuss the issue of gauge invariance. In the CBM the baryons are assumed to be on mass shell, thus it only makes sense to discuss current conservation as a weak condition for electromagnetic gauge invariance. In a static treatment, current conservation holds trivially. With center-of-mass corrections, the electromagnetic form factors are most conveniently calculated in the Breit frame. Since $q^0=0$ in this frame, current conservation is ensured provided that

$$\vec{q}\cdot\left\langle N\left(\frac{\vec{q}}{2}\right)|\vec{J}(0)|N\left(-\frac{\vec{q}}{2}\right)\right\rangle=0. \quad (A13)$$

For the quark core [Fig. 1(a)], explicit evaluations in both PY and PT projection methods guarantee that the matrix element of the spatial component of the current is proportional to $\boldsymbol{\sigma}\times\vec{q}$, and thus satisfies Eq. (A13). For Fig. 1(b), as shown in the previous paragraph, all terms which are odd in $\vec{k}$ simply vanish after the angular integration over the loop momentum, and the only surviving term is proportional to $\boldsymbol{\sigma}\times\vec{q}$, therefore this diagram is separately gauge invariant. The proof of gauge invariance for Fig. 1(c) is slightly different since it is evaluated in the heavy baryon approximation. By expanding the pion field in a plane wave and connecting the pion creation/annihilation operators with the CBM Hamiltonian and the physical baryon states, it is easy to show that [15]

$$\vec{j}^{(\pi)}(0)\propto\int d\hat{k}\,\vec{k}\vec{k}\cdot\boldsymbol{\sigma}\times\vec{q}=k^2\boldsymbol{\sigma}\times\vec{q}. \quad (A14)$$

Thus this current is also transverse with respect to $\vec{q}$. Since the total electromagnetic current is just the sum of the three contributions [Figs. 1(a), 1(b) and 1(c)] in the CBM, and hence current conservation, Eq. (A13), is satisfied in this calculation.

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