# Free surface response in a finite Fermi system

V. I. Abrosimov,[1] O. I. Davidovskaja,[1] V. M. Kolomietz,[1,2] and S. Shlomo[2]
[1]*Institute for Nuclear Research, 252022 Kiev, Ukraine*
[2]*Cyclotron Institute, Texas A&M University, College Station, Texas 77843-3366*
(Received 24 November 1997)

Collective vibrations in a finite Fermi system are studied within a phase space approach which is based on the Landau-Vlasov kinetic equation. The linear response theory is used and the semiclassical internal and collective response functions are evaluated with a continuous single-particle angular momentum $l$. We focus on the strength function and fragmentation width of the vibrations. We determine the contributions to the collective strength function which are associated with different values of relevant single-particle angular momentum. Applications to the nuclear isoscalar vibrations with multipolarities $L=0$, 2, and 3 are presented.
[S0556-2813(98)06805-8]

PACS number(s): 21.60.Ev, 24.30.Cz

## I. INTRODUCTION

A phase space description of collective motion in a finite Fermi system on the basis of the kinetic equation with self-consistent mean field [the Landau-Vlasov kinetic equation (LVKE)] has been the subject of many investigations [1–8]. For studying the excitations of the surface region of a many-body system it is convenient to use explicitly macroscopic collective variables describing the displacement of the effective surface of the system from its equilibrium position. In the present work we consider collective vibrations in a finite Fermi system by using the semiclassical approach proposed in Refs. [9–11]. In contrast to the nuclear fluid dynamics approximation [2,4], where the LVKE is reduced to the equations of motion for the local quantities like particle density, current density, etc., we will solve the collisionless Landau-Vlasov equation directly for the Wigner distribution function. An advantage of such an approach is that this allows us to study the fragmentation width of the collective excitations which is absent in the fluid dynamics approximation. Similarly to the quantum random phase approximation (RPA), the fragmentation width is not related here to the dissipation of the collective energy but caused by the interaction of particles with the time-dependent mean field [9].

This paper is organized as follows. In Sec. II, we find the response function of the finite Fermi system to a periodic external force. In Sec. III, we calculate the strength function for the nuclear isoscalar collective vibrations with multipolarities $L=0$, 2, and 3. We pay attention to the contributions of the different orbits, associated with single-particle angular momentum $l$, to the strength function of the low-lying states and the giant multipole resonance (GMR). In contrast to Refs. [5,9], we do not assume here a discretization of the single-particle angular momentum $l$. We thus provide a consistent consideration of $l$ in the framework of the semiclassical Landau-Vlasov kinetic theory. The conclusions are given in Sec. IV.

## II. RESPONSE FUNCTION FOR A FINITE FERMI SYSTEM

We consider a Fermi system bound by the surface

$$r=R+\delta R(\vartheta,\varphi,t), \tag{1}$$

which is a sphere with radius $R$ in equilibrium. The macroscopic variable $\delta R(\vartheta,\varphi,t)$ describes the local displacement of the surface $R(\vartheta,\varphi,t)$ from its equilibrium position.

A change in $R$ induces motion of the particles inside the sphere. The latter can be represented by a variation of the distribution function $\delta n(\vec{r},\vec{p},t)$ in phase space. The equation of motion for $\delta n(\vec{r},\vec{p},t)$ is given by a linearized Landau-Vlasov equation [12]

$$\frac{\partial}{\partial t}\delta n(\vec{r},\vec{p},t)+\vec{v}\frac{\partial}{\partial\vec{r}}\left[\delta n(\vec{r},\vec{p},t)-\frac{dn_0}{d\epsilon}\int d\vec{p}\,'\mathcal{F}(\hat{p},\hat{p}\,')\,\delta n(\vec{r},\vec{p}\,',t)\right]=0, \tag{2}$$

where $\vec{v}=\vec{p}/m$. The amplitude $\mathcal{F}(\hat{p},\hat{p}\,')$ describes the interactions of the particles. In Landau-Migdal Fermi-liquid theory [13,14], the scattering amplitude $\mathcal{F}(\hat{p},\hat{p}\,')$ depends also on the spin-isospin variables and is given by the expansion

$$\mathcal{F}(\hat{p},\hat{p}\,')=\sum_{l=0}^{\infty}\left[F_l+F_l'(\vec{\tau}\cdot\vec{\tau}\,')+G_l(\vec{\sigma}\cdot\vec{\sigma}\,')+G_l'(\vec{\sigma}\cdot\vec{\sigma}\,')(\vec{\tau}\cdot\vec{\tau}\,')\right]P_l(\hat{p}\cdot\hat{p}\,'). \tag{3}$$

Moreover, in a finite Fermi system, the Landau parameters $F_l$, $F_l'$, $G_l$, and $G_l'$ are functions of $r$ [14]. This last aspect and the relation of Landau parameters $F_l$, $F_l'$, $G_l$, and $G_l'$ to the saturated Skyrme forces were studied in Ref. [15]. An explicit form of the amplitude (3) is important to provide a self-consistent and detailed description of the ground state and excitations in the nucleus; see Refs. [15,16]. In our approach to the nuclear collective excitations, the ground state is assumed to be fixed by the external mean field. The approach is based on the classical Landau-Vlasov equation (2) applied to the finite Fermi system and we do not consider here the spin degrees of freedom, related to the $G_l$ and $G_l'$ components in Eq. (3). We point out also that the term with $F_l'$ in Eq. (3) is responsible for the isovector excitations. Below we will restrict ourselves to the isoscalar excitations only. It was noted earlier [17] that the different choices of $F_l$ in Eq. (3) do not affect much the description of the isoscalar giant multipole resonances as long as one does not include Landau parameters $F_l$ with $l \neq 0$. An inclusion of the Landau parameter $F_1$ reduces the effective mass $m^*$ of the nucleon and can be taken into consideration phenomenologically through a corresponding change of $m^*$ in the final result. Finally, the highest components with $l \geqslant 2$ in Eq. (3) cannot be derived from the Skyrme forces and they are usually omitted [15,16].

The main aim of this work is to study the contributions of different orbits (different single-particle angular momentum) to the low-lying excited states and the giant multipole resonances in the isoscalar channel; see also the WKB analysis of the RPA response function given in Refs. [19–21]. Conceptually, this problem is analogous to that of a semiclassical study of the contributions of the different classical single-particle orbits to the single-particle level density in the static case [22]. We point out that both considerations are not influenced significantly by the residual interaction $\mathcal{F}(\hat{p},\hat{p}')$ because the classical orbits represent here the gross-shell effects. Thus, we put below $\mathcal{F}(\hat{p},\hat{p}')=0$ and reduce the dynamical problem to a solution of the following simple kinetic equation (see also Refs. [5,8,9]):

$$\frac{\partial}{\partial t}\delta n(\vec{r},\vec{p},t)+\vec{v}\frac{\partial}{\partial \vec{r}}\delta n(\vec{r},\vec{p},t)=0. \tag{4}$$

An additional reason for the applicability of Eq. (4) to the nuclear problems is the fact that the Landau parameter $F_0$ is small enough in the nuclear interior [15,18]. It was, however, shown in [15] that there is a significant enhancement of $F_0$ in the surface region of the nucleus. We will take into account this feature of $F_0$ phenomenologically through the appropriate choice of the boundary condition for the pressure on the nuclear surface; see below Eq. (6). The Landau parameter $F_1$ is approximately constant inside the nucleus and goes sharply to zero in the surface region [15]. This one can be taken into consideration in our approach phenomenologically by introducing a constant effective mass $m^*$ of the nucleon.

In spite of the absence of the interparticle interaction in Eq. (4), the collective eigenmodes in a finite system can be described with Eq. (4) by an appropriate choice of the boundary conditions. Following Refs. [10,23,24], Eq. (4), valid inside a system at $r<R$, is augmented by mirror reflection boundary conditions at the moving surfaces, Eq. (1),

$$[\delta n(\vec{r},\vec{p}_\perp,p_r,t)-\delta n(\vec{r},\vec{p}_\perp,-p_r,t)]|_{r=R} = -2p_r\frac{dn_0}{d\epsilon}\frac{\partial}{\partial t}\delta R(\vartheta,\varphi,t), \tag{5}$$

where $p_r$ is the radial momentum, $\vec{p}_\perp=(0,p_\vartheta,p_\varphi)$, $n_0$ is the equilibrium distribution function, and $\epsilon=p^2/2m$.

An essential property of a finite Fermi system having a free surface is that the motion of the surface should be consistent with the motion of the particles inside the system. This can be achieved by imposing the following ''subsidiary condition'':

$$P_{rr}(\vec{r},t)|_{r=R+\delta R(\vartheta,\varphi,t)}=P_\sigma(\theta,\varphi,t)+F(\vartheta,\varphi,t). \tag{6}$$

Here $P_{rr}(\vec{r},t)$ is the normal component of the momentum flux tensor $P_{ik}(\vec{r},t)$ (see, e.g., Chap. 1 of Ref. [12]),

$$P_{ik}(\vec{r},t)=\int\frac{d\vec{p}}{h^3}p_i v_k\delta n(\vec{r},\vec{p},t), \tag{7}$$

and $P_\sigma(\vartheta,\varphi,t)$ is the additional pressure resulting from the surface tension. We would like to point out that the boundary condition (6) plays a central role in our description of the eigenvibrations. An introduction of the phenomenological surface pressure $P_\sigma(\vartheta,\varphi,t)$ provides the self-consistency condition in lieu of the self-consistent mean field in Eq. (4). In part, the pressure $P_\sigma(\vartheta,\varphi,t)$ is created by the interparticle interaction in the surface region. By adopting a phenomenological $P_\sigma(\vartheta,\varphi,t)$ [see Eq. (24)], one can take into account the above-mentioned enhancement of the Landau parameter $F_0$ in the surface region.

In Eq. (6) we have also introduced an external pressure $F(\vartheta,\varphi,t)$. This has been done simply to enable us to identify later the response of the system to such an external probe and thus to benefit from the tools of the linear response theory. In the present study, the $F(\vartheta,\varphi,t)$ is chosen to have the following form:

$$F(\vartheta,\varphi,t)=\sum_M F_{LM}(\omega)Y_{LM}(\vartheta,\varphi)\cos(\omega t)\exp(\eta t), \tag{8}$$

with $\eta=+0$ representing an infinitesimally small quantity to guarantee that the external field is turned on adiabatically at $t=-\infty$. Similar expressions can also be written for $P_{rr}$ and $P_\sigma$.

To find solutions of Eqs. (4) and (5) it is convenient to change the variables $(\vec{r},\vec{p})$ to a new set of variables $(r,\epsilon,l,\alpha,\beta,\gamma)$ as proposed in Ref. [9],

$$(\vec{r},\vec{p})\to(r,\epsilon,l,\alpha,\beta,\gamma). \tag{9}$$

The new variables are the single-particle energy $\epsilon$, single-particle angular momentum $l=|\vec{r}\times\vec{p}|$, radius $r$, and Euler angles $(\alpha,\beta,\gamma)$. The Euler angles are defined by the rotation of the laboratory frame $(x,y,z)$ to $(x',y',z')$ with $\hat{\vec{z}'}$ along $\vec{l}$ and $\hat{\vec{y}'}$ along $\vec{r}$. Let us introduce the single-particle radial velocity

$$v(r,\epsilon,l)=\pm\{(2/m)[\epsilon-(\hbar l)^2/2mr^2]\}^{1/2}. \tag{10}$$

Here and below the single-particle angular momentum $l$ is given in units of $\hbar$. Now the distribution functions with positive radial velocities, $\delta n^+(r,\epsilon,l,\alpha,\beta,\gamma,t)$, and with negative ones, $\delta n^-(r,\epsilon,l,\alpha,\beta,\gamma,t)$, are considered separately.

Of interest are those solutions of Eqs. (4) and (5) for $\delta R(\vartheta,\varphi,t)$ which are consistent with the special form of the external pressure (8). Thus we may write

$$\delta R_L(\vartheta,\varphi,t)=\mathrm{Re}\left[\sum_M \delta R_{LM}(\omega)Y_{LM}(\vartheta,\varphi)\exp(-i\omega t)\right]. \tag{11}$$

In terms of the new variables, Eq. (9), these solutions can be written as

$$\delta R_L(\vartheta,\varphi,t)=\mathrm{Re}\left[\sum_{M,N}\delta R_{LM}(\omega)Y_{LN}\left(\frac{\pi}{2},\frac{\pi}{2}\right)\times[\mathcal{D}^L_{MN}(\alpha,\beta,\gamma)]^*\exp(-i\omega t)\right], \quad (12)$$

where we used the following expansion:

$$Y_{LM}(\vartheta,\varphi)=\sum_{N=-L}^{L}[\mathcal{D}^L_{MN}(\alpha,\beta,\gamma)]^*Y_{LN}\left(\frac{\pi}{2},\frac{\pi}{2}\right). \quad (13)$$

For the $\delta n(\vec{r},\vec{p},t)$, we seek a $\mathcal{D}$-function expansion of the form

$$\delta n_L(\vec{r},\vec{p},t)=\frac{dn_0}{d\epsilon}\mathrm{Re}\left[\sum_{M,N}[f^+_{N,LM}(r,\epsilon,l,\omega)+f^-_{N,LM}(r,\epsilon,l,\omega)][\mathcal{D}^L_{MN}(\alpha,\beta,\gamma)]^*\times\exp(-i\omega t)\right]. \quad (14)$$

The functions $f^\pm(r,\epsilon,l,\omega)$ correspond to the distributions $\delta n^\pm(r,\epsilon,l,\alpha,\beta,\gamma,t)$ and represent a change of the phase space distribution for particles with energy $\epsilon$ and angular momentum $l$ at the distance $r$ from the center of the system. They describe the Fermi-surface distortions.

Using in Eq. (4) the new variables (9) and taking into account the expansion (14) for $\delta n_L(\vec{r},\vec{p},t)$ together with the boundary condition (5), we obtain a system of differential equations over $r$ for the functions $f^\pm$. Their solution is found to be (see also Ref. [10])

$$f^\pm_{N,LM}(r,\epsilon,l,\omega)=\frac{\exp\{\pm i[\omega\tau(r,\epsilon,l)-N\gamma(r,\epsilon,l)]\}}{\sin\{(1/2)[\omega T(R,\epsilon,l)-N\Gamma(R,\epsilon,l)]\}}\times Y_{LN}\left(\frac{\pi}{2},\frac{\pi}{2}\right)\omega p(R,\epsilon,l)\delta R_{LM}(\omega). \quad (15)$$

Here $p(r,\epsilon,l)=m|v(r,\epsilon,l)|$. The quantities $\tau(r,\epsilon,l)$ and $\gamma(r,\epsilon,l)$ are introduced as in Ref. [9],

$$\tau(r,\epsilon,l)=\int_{r_1}^{r}dr'\frac{1}{|v(r',\epsilon,l)|}, \quad (16)$$

$$\gamma(r,\epsilon,l)=\int_{r_1}^{r}dr'\frac{\hbar l}{mr'^2}\frac{1}{|v(r',\epsilon,l)|}. \quad (17)$$

The radial turning point $r_1$ is a solution of $v(r_1,\epsilon,l)=0$. The quantities $T(R,\epsilon,l)$ and $\Gamma(R,\epsilon,l)$ are, respectively, the radial and angular periods for a particle orbit with given $\epsilon$ and $l$. Namely,

$$T(R,\epsilon,l)=2\tau(R,\epsilon,l) \quad (18)$$

and

$$\Gamma(R,\epsilon,l)=2\gamma(R,\epsilon,l). \quad (19)$$

We may now present the solutions (12) in terms of the response function $\chi^L_{PR}(\omega)$. The latter may be defined as [25]

$$\delta R_{LM}(\omega)/R=-\chi^L_{PR}(\omega)F_{LM}(\omega)/P_0, \quad (20)$$

where the equilibrium pressure of the Fermi gas, $P_0=(2/5)\epsilon_F\rho_0$, is introduced for convenience, $\rho_0$ is the bulk particle density, and $\epsilon_F$ is the Fermi energy. The quantity $\chi^L_{PR}(\omega)$, defined by Eq. (20), represents a nondiagonal surface-pressure response function. It describes the response of the nuclear surface to the external pressure. At the end of this section we will also introduce the diagonal pressure-pressure collective response function. The response function $\chi^L_{PR}(\omega)$ can be expressed through the so-called internal response function $\chi^L_{\mathrm{int}}(\omega)$ defined by the following relation:

$$P_{rr,LM}(R,\omega)/P_0=\chi^L_{\mathrm{int}}(\omega)\delta R_{LM}(\omega)/R. \quad (21)$$

Using Eqs. (7), (14), and (15) the internal response function $\chi^L_{\mathrm{int}}(\omega)$ is found to be

$$\chi^L_{\mathrm{int}}(\omega)=-\frac{60\pi}{2L+1}\frac{\omega R}{v_F}\sum_{N=-L}^{L}\left|Y_{LN}\left(\frac{\pi}{2},\frac{\pi}{2}\right)\right|^2\int_0^1 d\lambda\lambda\times(1-\lambda^2)\mathrm{ctg}\left(\frac{\omega R}{v_F}(1-\lambda^2)^{1/2}-N\arccos(\lambda)\right), \quad (22)$$

where the dimensionless angular momentum $\lambda=l/(k_F R)$ is introduced, $v_F$ is the Fermi velocity, and $k_F=mv_F/\hbar$.

The collective response function $\chi^L_{PR}(\omega)$ can be found now from Eqs. (6), (20), and (21) as

$$\chi^L_{PR}(\omega)=\left[-\chi^L_{\mathrm{int}}(\omega)+\frac{\sigma(L-1)(L+2)}{P_0R}\right]^{-1}=\frac{\kappa}{1-\kappa\chi^L_{\mathrm{int}}(\omega)}, \quad (23)$$

where

$$\kappa=P_0R/\sigma(L-1)(L+2).$$

To obtain Eq. (23), it was assumed that the additional pressure $P_{\sigma,LM}(\omega)$ [see Eq. (6)] has the same form as for surface deformation of the liquid drop of the multipolarity $L$ (see, e.g., Ref. [26]),

$$P_{\sigma,LM}(\omega)=\sigma(L-1)(L+2)R^{-2}\delta R_{LM}(\omega), \quad (24)$$

where $\sigma$ is the surface tension.

The diagonal collective pressure-pressure response function $\chi^L_{PP}(\omega)\equiv\chi^L(\omega)$ can be derived through the following relation:

$$P_{rr,LM}(R,\omega)/P_0=-\chi^L(\omega)F_{LM}(\omega)/P_0. \quad (25)$$

From this definition and Eqs. (6) and (21) we obtain the relation between the collective and internal response functions. Namely,

$$\chi^L(\omega)=\frac{\kappa\chi^L_{\mathrm{int}}(\omega)}{1-\kappa\chi^L_{\mathrm{int}}(\omega)}. \quad (26)$$

This relation is similar to the one derived for the quantum collective response function in the case of separable particle-particle interaction; see Eq. (6-243) of Ref. [27]. We point out that the imaginary parts of both response functions $\chi^L_{PR}(\omega)$ and $\chi^L(\omega)$, i.e., corresponding strength functions, are proportional to each other,

$$\mathrm{Im}\ \chi^L_{PR}(\omega)=\kappa^2\,\frac{\mathrm{Im}\ \chi^L_{\mathrm{int}}(\omega)}{[1-\mathrm{Re}\ \chi^L_{\mathrm{int}}(\omega)]^2+[\kappa\ \mathrm{Im}\ \chi^L_{\mathrm{int}}(\omega)]^2}=\kappa\ \mathrm{Im}\ \chi^L(\omega). \tag{27}$$

The poles of the collective response functions (23) or (26) determine the eigenfrequencies for the oscillations in a finite Fermi system. In the next section we study some properties of the imaginary part of the response functions (22) and (26) for the vibrations of multipolarities $L=0$, 2, and 3.

In order to clarify the physical origin of both quantities $l$ and $N$ in the internal response function $\chi^L_{\mathrm{int}}(\omega)$ of Eq. (22), we point out that there is a close connection between the LVKE approach and the semiclassical WKB approximation to the quantum RPA; see Refs. [9,19–21]. It can be shown [20] that in the general case of the response of a finite Fermi system to an external field,

$$V_{\mathrm{ext}}(\vec{r},t)=\hat{Q}(\vec{r})e^{-i\omega t}+\mathrm{c.c.},$$

one has

$$\chi^L_{\mathrm{int}}(\omega,\mathrm{LVKE})\approx\chi^L_{\mathrm{int}}(\omega,\mathrm{WKB}), \tag{28}$$

where $\chi^L_{\mathrm{int}}(\omega,\mathrm{LVKE})\equiv\chi^L_{\mathrm{int}}(\omega)$ is given by Eq. (22) and $\chi^L_{\mathrm{int}}(\omega,\mathrm{WKB})$ is the internal (''shell model'') response function evaluated in the semiclassical WKB approximation. One can expect that the relationship (28) is preserved in the particular case of the response to the external pressure in the form of Eq. (8).

The structure of the WKB response function $\chi^L_{\mathrm{int}}(\omega,\mathrm{WKB})$ is well known; see Refs. [9,20]. In particular, the factor $|Y_{LN}(\pi/2,\pi/2)|^2$ in Eq. (22) emerges as that of the classical limit of the Clebsh-Gordon coefficients in the matrix element

$$\langle n_p l_p m_p|\hat{Q}(\vec{r})|n_h l_h m_h\rangle \tag{29}$$

appearing in the quantum shell-model response function. The particle and hole angular momenta [correspondingly, $l_p$ and $l_h$ in Eq. (29)] are related to the values $l$ and $N$ in Eq. (22). Namely, for $l_p,l_h\gg L$ one has

$$l_h\approx l,\quad N\approx l_p-l_h. \tag{30}$$

We point out that, in contrast to the quantum integer number $l_h$, the classical angular momentum $l$ in the LVKE approach is a continuous variable.

Taking into account Eq. (30), it is convenient to rewrite the internal response function (22) as a sum (integral) of contributions from the excited particles with angular momentum $l'\equiv l_p=N+l_h\equiv N+l$ for $l,\ l'\geq 0$. One obtains from Eq. (22)

$$\chi^L_{\mathrm{int}}(\omega)=\sum_{l'=0}^{l_{\max}+L}\chi^L_{\mathrm{int}}(\omega,l'), \tag{31}$$

where $l_{\max}=k_FR$. Assuming below that $l_{\max}>L$, we have

$$\chi^L_{\mathrm{int}}(\omega,l')=-\frac{60\pi}{2L+1}\frac{\omega R}{v_F}\frac{\hbar^2}{l^2_{\max}}\left|Y_{LN}\left(\frac{\pi}{2},\frac{\pi}{2}\right)\right|^2\times[g(l'-N,N)\,\theta(l'-N)\,\theta(l_{\max}+N-l')+g(l'+N,-N)(1-\delta_{N0})\,\theta(l')\times\theta(l_{\max}-N-l')], \tag{32}$$

where

$$g(x,y)=x(1-x^2)\mathrm{ctg}[(\omega R/v_F)\sqrt{1-x^2}-y\ \arccos(x)]. \tag{33}$$

The collective response function of Eq. (27) can also be represented in a form similar to Eq. (31). Namely,

$$\mathrm{Im}\ \chi^L(\omega)=\sum_{l'=0}^{l_{\max}+L}\mathrm{Im}\ \chi^L(\omega,l'), \tag{34}$$

where

$$\mathrm{Im}\ \chi^L(\omega,l')=\mathrm{Im}\ \chi^L_{\mathrm{int}}(\omega,l')\ G^L(\omega) \tag{35}$$

and

$$G^L(\omega)=\frac{\kappa}{[1-\mathrm{Re}\ \chi^L_{\mathrm{int}}(\omega)]^2+[\kappa\ \mathrm{Im}\ \chi^L_{\mathrm{int}}(\omega)]^2}. \tag{36}$$

## III. STRENGTH DISTRIBUTION OF COLLECTIVE VIBRATIONS

The strength distribution of the collective vibrations in a finite Fermi system as a function of the external pressure frequency [see Eq. (8)] is described by the imaginary part of the collective response function (26). To obtain this function we have used a phase space approach based on the Landau-Vlasov equation. So in this theory the particle angular momentum λ is a continuous variable. One usually replaces the integration over angular momentum λ in Eq. (22) with a sum over discrete values [9]:

$$\lambda\to\frac{\hbar}{p_FR}\sqrt{l(l+1)}.$$

However, such a kind of replacement is not consistent with the semiclassical character of the initial Landau-Vlasov equation and we will preserve below λ as a continuous variable.

The approach presented above is general enough that it can be applied to any Fermi system. As an example we will consider below the isoscalar vibrations in finite nuclei. The imaginary part of the collective response function (26) has been evaluated for collective vibrations in a nucleus with $A=208$ nucleons. The following values for the nuclear parameters were used: $\sigma=1$ MeV/fm$^2$, $\rho_0=0.17$ fm$^{-3}$, $\epsilon_F=40$ MeV, and $r_0=1.12$ fm. In the calculations we smear

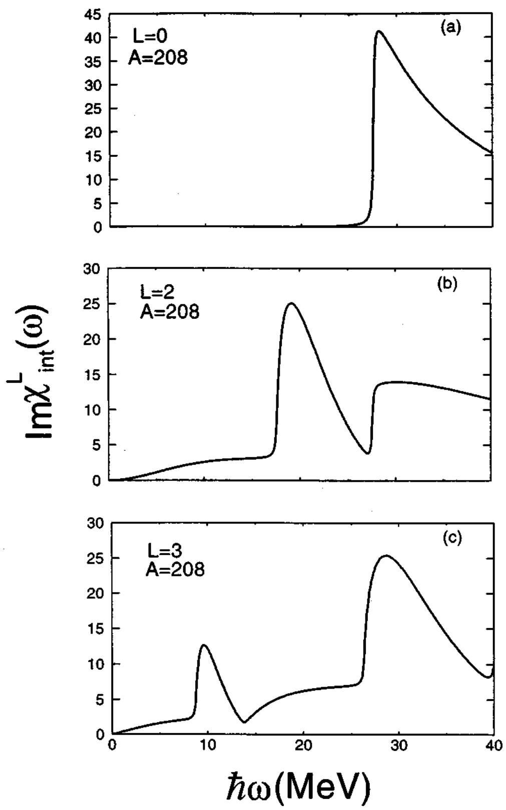


FIG. 1. The imaginary part of the intrinsic response function of $^{208}$Pb [see Eq. (22)] for the isoscalar excitations with $L=0$, 2, and 3 in arbitrary units. The smearing parameter $\eta=0.1$ MeV. The solid and dashed lines correspond to the values of $F_0=0$ and $-0.4$, respectively.

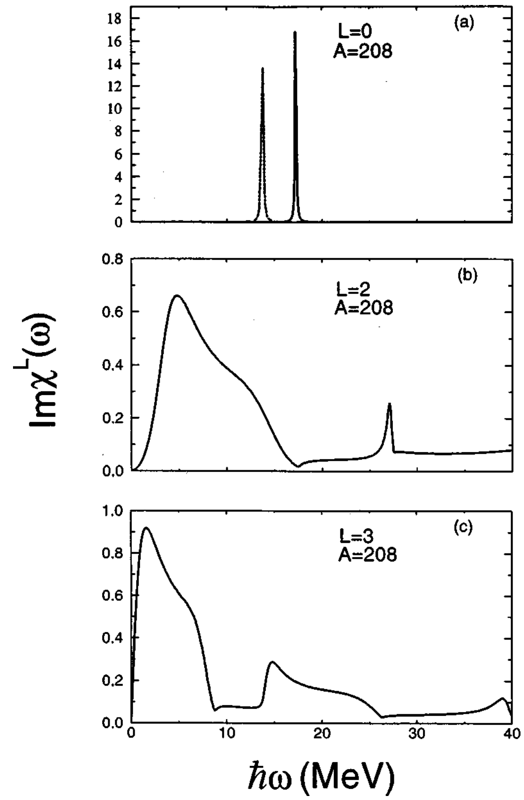


FIG. 2. The same as Fig. 1, for the collective response function; see Eq. (26).

out the $\delta$ functions in the imaginary part of Eq. (22) by giving a finite value to the infinitesimal parameter $\eta=0.2$ MeV at $L\neq 0$ and $\eta=0.1$ MeV at $L=0$.

In Figs. 1(a), 1(b), and 1(c) the imaginary part, $\chi''_{\text{int}}(\omega)=\text{Im}\,\chi^L_{\text{int}}(\omega)$, of the intrinsic response function, Eq. (22), are shown for isoscalar resonances with $L=0$, 2, and 3, respectively. Similarly, in Figs. 2(a), 2(b), and 2(c) we show the corresponding isoscalar collective response function, Eq. (26). A comparison of both sets of figures shows [see also Eq. (27)] that due to the consistency condition (6) one observes a considerable strength redistribution. The strength functions shown in Figs. 2(a), 2(b), and 2(c) have a resonance structure with significant resonance widths in the case of $L>0$. We would like to stress that these widths are closely associated with the so-called fragmentation widths in quantum RPA calculations. In the case of the internal response function $\chi^L_{\text{int}}(\omega)$, our Figs. 1(a), 1(b), and 1(c) produce smooth curves for the corresponding partial particle-hole strength distributions in the quantum shell-model calculations; see also Refs. [5–7,9]. From the point of view of Landau's Fermi-liquid theory, the widths of the excited states in the collective strength function Im $\chi^L(\omega)$ in Figs. 2(a), 2(b), and 2(c) appear due to the Landau (noncollisional) damping. The full description of the widths must also include the contribution from the two-body collisions. We have neglected this collisional damping in our kinetic equation (4). The collisional damping can be easily taken into consideration by adding the collisional integral in the $\tau$ approximation to the right-hand side of Eq. (4). We point out that the inclusion of the smearing parameter $\eta$ in Eq. (22) simulates the collisional damping with $\eta=\hbar/\tau$, where $\tau$ is the relaxation time; see Ref. [29].

We note also that the use of the small smearing parameter $\eta$ leads only to a slight spreading of the resonance structure in Figs. 1 and 2. This can be seen directly in Fig. 2(a) for $L=0$, where the fragmentation width is absent and the width of the monopole resonance is due only to the smearing parameter $\eta$.

It is seen from Fig. 2(a) that there is one collective isoscalar monopole mode at excitation energy $\hbar\omega_0\approx 17.2$ MeV which exhausts all the strength; see also Ref. [8]. The calculated value is higher than the experimental excitation energy of 13.7 MeV [28]. We note that the bulk nucleon-nucleon interaction $\mathcal{F}(\hat{p},\hat{p}')$ was omitted in our consideration; see Eqs. (2) and (4). Inclusion of the interaction $\mathcal{F}(\hat{p},\hat{p}')$ can be easily done in the case of the simplest momentum-isotropic interaction $\mathcal{F}(\hat{p},\hat{p}')=F_0$, where $F_0$ is the Landau parameter [12,13]. A better description of the energy of the GMR is then obtained with an appropriate choice of the parameter $F_0=-0.4$ [the dashed line in Fig. 2(a)]; see also Ref. [29]. Note that the value of the parameter $F_0$ can be different if the Landau parameter $F_1$, i.e., the effective mass of the nucleon, is taken into consideration.

Taking into account the bulk interaction $\mathcal{F}(\hat{p},\hat{p}')$ is more important in the case of monopole excitations where the nuclear interior is involved. In Fig. 2(b) the strength distribution is presented for the collective isoscalar quadrupole vibrations $(L=2)$. Most of the strength is distributed in the energy range between the giant resonance ($\omega$

$\approx 11$ MeV/$\hbar$) and the low-lying collective state ($\omega \approx 4$ MeV/$\hbar$) for $^{208}$Pb. A small part of the strength is observed at energy which is about twice the giant resonance energy. As can be seen from Fig. 2(c), the bulk part of the strength of the collective isoscalar octupole excitations ($L=3$) is distributed in the energy region of the low-lying collective state ($\omega \approx 2.6$ MeV/$\hbar$), the low-frequency octupole resonance ($\omega \approx 5$–8 MeV/$\hbar$), and the high-frequency octupole resonance ($\omega \approx 17$–20 MeV/$\hbar$) for $^{208}$Pb. This strength distribution is in agreement with the observed isoscalar octupole response in $^{208}$Pb [30,31].

As can be seen from Figs. 2(b) and 2(c), the strength functions in the giant quadrupole and octupole resonance regions are quite small. This occurs due to the particular choice of the external field in the form of the external pressure, Eq. (8), instead of the commonly used electromagnetic field. We will evaluate here the contribution of the giant resonance region to the electromagnetic energy-weighted sum rule (EWSR) for the monopole case. The classical transition density, i.e., the time-dependent variation of the particle density $\delta\rho_{L=0}(\vec{r},t)$, can be found from Eqs. (14) and (15) and is given by the following expression (see also Refs. [29,32]):

$$\delta\rho_{L=0}(\vec{r},t) = \int \frac{d\vec{p}}{(2\pi\hbar)^3}\,\delta n_{L=0}(\vec{r},\vec{p},t) = \eta(\vec{r},t)\rho_0\theta(R-r) + q(t)\rho_0 R\,\delta(R-r), \tag{37}$$

where

$$\eta(\vec{r},t) \approx -\frac{3}{R}\left[1+\frac{\omega^2R^2}{6v_F^2}\left(1-\frac{5}{3}\frac{r^2}{R^2}\right)\right]\delta R(t) \tag{38}$$

and

$$q(t) = \frac{1}{R}\,\delta R(t). \tag{39}$$

We assume also that the condition $s=\omega R/\pi v_F < 1$ is fulfilled. Taking into account the continuity equation and Eq. (37), one can find the velocity field $\vec{u}(\vec{r},t)$ in the following form:

$$\vec{u}(\vec{r},t) = -\frac{\vec{r}}{r^3\rho_0}\int_0^r dr_1 r_1^2 \frac{d}{dt}\,\delta\rho_{L=0}(\vec{r},t) = -\frac{\vec{r}}{R}\left[1+\frac{\omega^2R^2}{6v_F^2}\left(1-\frac{r^2}{R^2}\right)\right]\frac{d}{dt}\,\delta R(t). \tag{40}$$

Substituting the velocity field (40) in the expression for the collective kinetic energy of the fluid, we can obtain the mass coefficient $B_{L=0}(\omega)$ with respect to the collective variable $\delta R(t)$. Namely,

$$B_{L=0}(\omega) = m\rho_0 \int d\vec{r}\frac{r^2}{R^2}\left[1+\frac{\omega^2R^2}{6v_F^2}\left(1-\frac{r^2}{R^2}\right)\right]^2 = \frac{3}{5}Am\left[1+\frac{2\omega^2R^2}{21v_F^2}\right] + \mathcal{O}(s^4). \tag{41}$$

Let us consider now the quantum energy-weighted sum rule

$$S_{L=0} = \sum_n (E_n - E_0)|\langle n|r^2|0\rangle|^2 = \frac{6}{5}\frac{\hbar^2}{m}AR^2. \tag{42}$$

The partial contribution $S_L(\omega_0)$ of the classical eigenexcitation with certain eigenfrequency $\omega_0$ to the quantum energy-weighted sum $S_L$ can be derived using the correspondency principle (see Ref. [27], Chap. 6, Appendix 6A). It reads

$$S_{L=0}(\omega_0) = \frac{\hbar^2}{2B_{L=0}(\omega_0)}M_{L=0}^2(\omega_0), \tag{43}$$

where $M_{L=0}(\omega_0)$ is given by

$$M_{L=0}(\omega_0) = \int d\vec{r}r^2\,\delta\rho_{L=0}(\vec{r},t)/\delta R(t) = \frac{6}{5}AR\left[1+\frac{\omega_0^2R^2}{21v_F^2}\right]. \tag{44}$$

Using Eqs. (43), (44), and (41), we obtain

$$S_{L=0}(\omega_0) = \frac{6}{5}\frac{\hbar^2}{m}AR^2 + \mathcal{O}(s^4). \tag{45}$$

Comparing this expression with the EWSR given by Eq. (42), we can see that only one resonance state at $\omega=\omega_0$ exhausts about all the EWSR for $L=0$. Thus, at least in the case of the isoscalar giant monopole resonance, the model used does not lead to an underestimate of the fraction of the giant resonance in the EWSR. The contribution of the giant quadrupole and octupole resonance regions to the EWSR will be reported in a forthcoming publication.

In Fig. 3 we show, as a function of $l'$, the energy-averaged contributions $\mathrm{Im}\,\overline{\chi^L(\omega,l')}$ to the strength of octupole vibration defined as

$$\mathrm{Im}\,\overline{\chi^L(\omega,l')} = \frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2} d\omega'\ \mathrm{Im}\,\chi^L(\omega',l'). \tag{46}$$

The energy-averaging intervals are chosen in the characteristic regions of the strength local maxima (the intervals are determined by the condition that the strength decrease by a factor of 2 compared with the value at the corresponding local maximum). We consider averaging in the characteristic regions of excitation energies: the low-lying collective states (0.5–4.5 MeV), the low-frequency octupole resonance (5–8 MeV), and the high-frequency octupole resonance (14–18 MeV). We can see that at a given excitation energy, the contribution to the strength function is coming from orbits with well-defined angular momentum $l'$. The resonance structures of $\mathrm{Im}\,\overline{\chi^3(\omega,l')}$ as a function of the particle angular momentum $l'$, displayed in Fig. 3, are generated by the poles of the intrinsic response function in a given region of frequencies; see Eqs. (46), (35), and (32). They are not related to the resonance behavior of the collective response function (26) as a function of $\omega$.

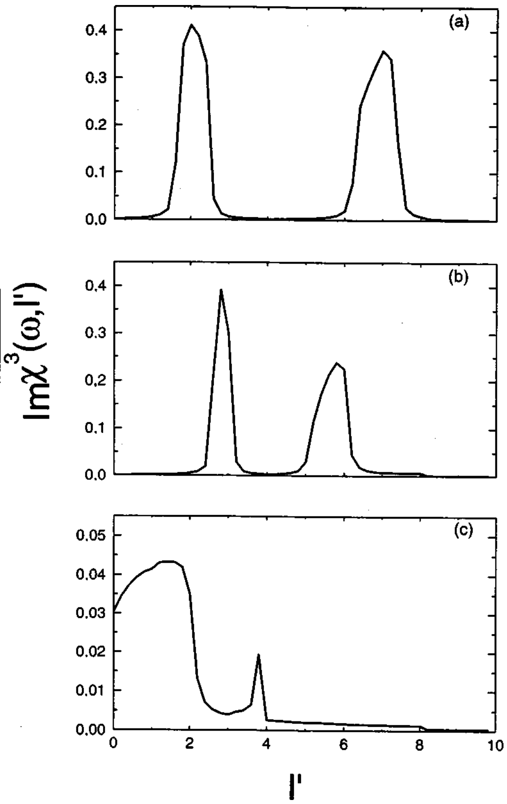


FIG. 3. The contribution to the imaginary part of the isoscalar octupole collective response function ($L=3$) [see Eq. (26)] as a function of the single-particle angular momentum $l'$ averaged over the energy in different regions of $\hbar\omega$: (a) $\hbar\omega=0.5$–4.5 MeV, (b) $\hbar\omega=5$–8 MeV, and (c) $\hbar\omega=14$–18 MeV. The smearing parameter $\eta=0.2$ MeV.

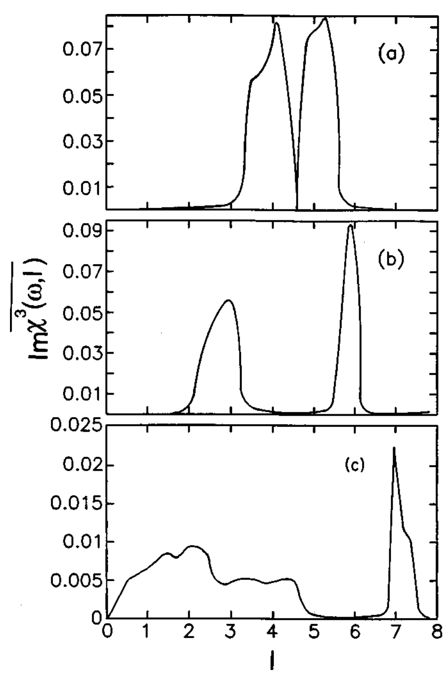


FIG. 4. The same as Fig. 3, but for the ''hole'' angular momentum $l$; see Eq. (30).

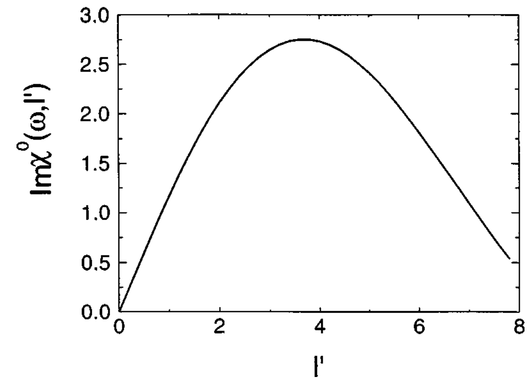


FIG. 5. The same as Fig. 3, for the isoscalar monopole collective response function ($L=0$) at the resonance excitation energy $\hbar\omega=17.2$ MeV. The smearing parameter $\eta=0.1$ MeV.

In Fig. 4 we have also plotted the energy-averaged contributions Im $\overline{\chi^L(\omega,l)}$ to the $L=3$ strength as a function of the ''hole'' angular momentum $l$; see Eq. (30). One can see from Figs. 3 and 4 that there exists a correlation in the shifts of the strength for different energy regions. We point out that the large contribution associated with the high ''hole'' angular momentum $l$, seen in Fig. 4, does not mean that the corresponding collective motion has surface character. This is because the transition density involves both ''hole'' and ''particle'' orbits and the enhancement in the ''hole'' strength can be compensated through the hindrance of the ''particle'' high angular momentum orbits [compare Fig. 3(c) and Fig. 4(c)].

We will also consider the contribution from different particle orbits to the strength function for monopole case, $L=0$. Taking into account Eqs. (32) and (35), one finds (we point out that $l=l'$ for $L=0$)

$$\chi^0(\omega,l')=-15\frac{\omega R}{v_F l_{\max}^2}G(\omega)l'\sqrt{1-l'^2}\mathrm{ctg}\left(\frac{\omega R}{v_F}\sqrt{1-l'^2}\right). \tag{47}$$

For comparison, the partial strength function Im $\chi^0(\omega,l')$ is plotted in Fig. 5 for the isoscalar excitation at energy 17.2 MeV [see Fig. 2(a)] as a function of the particle angular momentum $l'$. In contrast to the case of $L\neq 0$ vibrations, it is seen from Fig. 5 that orbits with a wide range of orbital angular momentum $l'$ contribute to the giant monopole resonance. The intrinsic response function Im $\chi^0(\omega,l')$ has no poles at this energy (or corresponding frequency); see Eq. (47).

Two effects, the shift and the deformation of the Fermi surface in the momentum space, are responsible for the forming of the collective excitations in the Fermi liquid. We will analyze both of them as a function of the particle angular momentum $l'$ in the case of the monopole resonance. Let us rewrite Eq. (14) for $L=0$ in the following form:

$$\delta n_0(\vec{r},\vec{p},t)=\frac{dn_0}{d\epsilon}[\mathrm{Re}\, f_0(r,\epsilon,l,\omega)\cos(\omega t) + \mathrm{Im} f_0(r,\epsilon,l,\omega)\sin(\omega t)]. \tag{48}$$

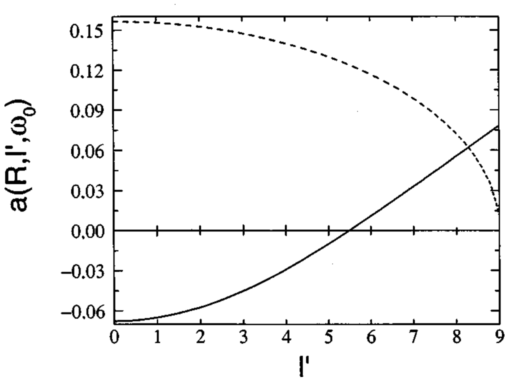


FIG. 6. The amplitudes of the Fermi surface distortions $a^{+}(r,l,\omega_0)$ (solid line) and $a^{-}(r,l,\omega_0)$ (dashed line) at $r=R$ as functions of the particle angular momentum $l=l'$. The calculation was performed for the giant monopole resonance for the nucleus $^{208}$Pb.

Here the quantities Re $f_0(r,\epsilon,l,\omega)$ and Im $f_0(r,\epsilon,l,\omega)$ represent the amplitudes of the local (at given $r$) distortions of the Fermi surface. Using the solution (15) for $L=0$, these amplitudes can be written as

$$a^{+}(r,l,\omega_0)\equiv\frac{\text{Re } f_0(r,\epsilon_F,l,\omega_0)}{\epsilon_F} =\frac{2}{\sqrt{\pi}}\frac{\delta R_{00}(\omega_0)}{R}\frac{\omega_0 R}{v_F}\sqrt{1-\left(\frac{lr}{k_F R^2}\right)^2} \times\frac{\cos[(\omega_0 r/v_F)\sqrt{1-(l/k_F R)^2}]}{\sin[(\omega_0 r/v_F)\sqrt{1-(lr/k_F R^2)^2}]} \tag{49}$$

and

$$a^{-}(r,l,\omega_0)\equiv\frac{\text{Im} f_0(r,\epsilon_F,l,\omega_0)}{\epsilon_F} =\frac{2}{\sqrt{\pi}}\frac{\delta R_{00}(\omega_0)}{R}\frac{\omega_0 R}{v_F}\sqrt{1-\left(\frac{lr}{k_F R^2}\right)^2} \times\frac{\sin[(\omega_0 r/v_F)\sqrt{1-(l/k_F R)^2}]}{\sin[(\omega_0 r/v_F)\sqrt{1-(lr/k_F R^2)^2}]}, \tag{50}$$

where $\omega_0$ is the eigenfrequency of the monopole giant resonance. The values Re $f_0(r,\epsilon,l,\omega)$ and Im $f_0(r,\epsilon,l,\omega)$ provide, respectively, a change of the particle density and the current density at the collective excitation. Correspondingly, the amplitudes $a^{+}(r,l,\omega_0)$ and $a^{-}(r,l,\omega_0)$ describe the deformation and the shift of the Fermi surface.

In Fig. 6, the amplitudes $a^{\pm}(r,l,\omega_0)$ are shown as functions of the single-particle angular momentum near the nuclear surface ($r=R$). The deformation amplitude $a^{+}(r,l,\omega_0)$ reaches the value of 0.08 at $l=l_{\max}$ (solid line). The shift amplitude $a^{-}(r,l,\omega_0)$ has a maximum of about 0.16 at $l=0$ (dashed line) and decreases with the increase of the single-particle angular momentum. It can be seen from Figs. 5 and 6 that the region of $l'\approx 4$ where the strength function is more pronounced corresponds to the region of the hindrance of the deformation of the Fermi surface.

## IV. CONCLUSIONS

We have studied the strength distribution of collective vibrations in a finite Fermi system within a phase space approach. A simple analytical expression for small variations of the phase space distribution function was obtained and used to determine the collective response functions (23) and (26). In the present calculations we neglected the residual particle interaction in the bulk of the system, except for the monopole case, as well as effects from collisions. Introducing the moving effective surface in lieu of the nucleon-nucleon interaction, we include in a macroscopic way a complicated particle interaction in the surface region.

We found that in the case of nuclei the effective particle interaction in the surface region leads to a significant redistribution [see Figs. 1 and 2 and Eqs. (22), (23), and (27)] of the isoscalar strength in the giant resonance region as well as in the low-lying collective state region. The results obtained are in reasonable agreement with the quantum ones [33–36]. The bulk nucleon-nucleon interaction $\mathcal{F}(\hat{p},\hat{p}')$ was omitted in our consideration except for the $L=0$ excitations, taking into account that the bulk interaction is more important in the case of monopole excitations, where the nuclear interior is involved, than for the higher multipoles. A better description of the energy of the isoscalar giant monopole resonance was achieved with an appropriate choice of the Landau parameter $F_0=-0.4$. The strength distribution for quadrupole vibrations is characterized by a larger fragmentation in the energy region between the giant resonance and the low-lying collective state [see Fig. 2(b)] as compared to the one found in the quantum RPA calculations [33,34]. We have also pointed out that the strength functions in the giant quadrupole and octupole resonance regions are quite small. This is associated with the surface character of the external pressure form factor used in the present approach; see Eq. (8). We demonstrated that in our model the monopole giant resonance exhausts about all the EWSR.

We have presented an analysis of the single-particle orbit contributions to different energy regions of the collective strength function. We have observed the enhancement of certain orbits in both ''particle'' and ''hole'' channels for the quadrupole and octupole strength functions for wide intervals of the excitation energy. The corresponding $l$ dependence of the monopole strength function at the resonance energy shows rather smooth behavior. We noted that the dynamical distortion of the Fermi surface, accompanying the collective excitation of the Fermi-liquid drop, is also manifested in the $l$ dependence of the strength. This fact can be used in the analysis of the particle emission from the cold excited Fermi-liquid drop. Some results in this direction will be reported in a forthcoming publication.

In this work we focused on the applications of our approach to collective vibrations in nuclei. However, the dynamical phenomena in nuclei do not differ in a qualitative way from the ones in other finite Fermi systems. So our approach can be extended to the study of collective modes in systems such as small metal clusters or atomic systems. The present approach can be easily modified to take into account the effects of collisions as well as to study the collective motion of a finite Fermi system at finite temperature.

## ACKNOWLEDGMENTS

The reported work was partially supported by INTAS Grant No. 93-0151 and NSF Grant No. PHY-9413872. One of the authors (V.M.K.) thanks the Cyclotron Institute at Texas A&M University for the kind hospitality.

[1] G. F. Bertsch, *Proceedings of the Summer School on Nuclear Physics with Heavy Ions and Mesons* (North-Holland, Amsterdam, 1978), p. 75.
[2] G. Holzwarth and G. Eckart, Nucl. Phys. **A325**, 1 (1979).
[3] P. Schuck, R. W. Hasse, J. Jaenicke, C. Gregoire, B. Remaud, F. Sebille, and E. Suraud, Prog. Part. Nucl. Phys. **22**, 181 (1989).
[4] V. M. Kolomietz, *Collective Nuclear Dynamics* (Nauka, Leningrad, 1990), p. 67.
[5] M. Di Toro, Part. Nuclei **22**, 385 (1991).
[6] A. Dellafiore and F. Matera, Phys. Rev. B **41**, 3488 (1990).
[7] A. Dellafiore and F. Matera, Phys. Rev. A **41**, 4958 (1990).
[8] V. M. Kolomietz, A. G. Magner, V. M. Strutinsky, and S. M. Vydrug-Vlasenko, Nucl. Phys. **A571**, 117 (1994).
[9] D. M. Brink, A. Dellafiore, and M. Di Toro, Nucl. Phys. **A456**, 205 (1986).
[10] V. I. Abrosimov, M. Di Toro, and V. M. Strutinsky, Nucl. Phys. **A562**, 41 (1993).
[11] V. I. Abrosimov, O. I. Davidovskaja, and V. M. Kolomietz, Phys. At. Nucl. **59**, 1130 (1996).
[12] E. M. Lifschitz and L. P. Pitajevsky, *Physical Kinetics* (Pergamon Press, Oxford, 1981).
[13] L. D. Landau, Sov. Phys. JETP **3**, 920 (1956); **5**, 101 (1957).
[14] A. B. Migdal, *Theory of Finite Fermi Systems and Application to Atomic Nuclei* (Interscience, London, 1967).
[15] K.-F. Liu, H.-D. Luo, Z.-Y. Ma, Q.-B. Shen, and S. A. Moszkowski, Nucl. Phys. **A534**, 1 (1991).
[16] K.-F. Liu, H.-D. Luo, Z.-Y. Ma, Q.-B. Shen, and S. A. Moszkowski, Nucl. Phys. **A534**, 25 (1991).
[17] G. Eckart, G. Holzwarth, and J. P. Da Providencia, Nucl. Phys. **A364**, 1 (1981).
[18] H. Krivine, J. Treiner, and O. Bohigas, Nucl. Phys. **A336**, 155 (1980).
[19] Yu. B. Ivanov, J. Exp. Theor. Phys. **81**, 977 (1981).
[20] A. Dellafiore and F. Matera, Nucl. Phys. **A460**, 245 (1986).
[21] A. Dellafiore, F. Matera, and D. Brink, Phys. Rev. A **51**, 914 (1995).
[22] R. B. Balian and C. Bloch, Ann. Phys. (N.Y.) **69**, 76 (1972); see also earlier references quoted therein.
[23] I. L. Bekarevich and I. M. Khalatnikov, Sov. Phys. JETP **12**, 1187 (1961).
[24] Yu. B. Ivanov, Nucl. Phys. **A365**, 301 (1981).
[25] P. J. Siemens and A. S. Jensen, *Elements of Nuclei: Many-Body Physics with the Strong Interaction* (Addison and Wesley, New York, 1987).
[26] L. D. Landau and E. M. Lifschitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1963).
[27] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[28] S. Shlomo and D. H. Youngblood, Phys. Rev. C **47**, 529 (1993).
[29] V. Abrosimov, M. Di Toro, and A. Smerzi, Z. Phys. A **347**, 161 (1994).
[30] A. van der Woude, *Giant Resonances* (World Scientific, Singapore, 1990), Chap. 2.
[31] M. Fujiwara, *Selected Topics in Nuclear Structure* (Nauka, Dubna, 1989), p. 133.
[32] B. Jennings and A. Jackson, Phys. Rep. **66**, 141 (1980).
[33] G. F. Bertsch and S. F. Tsai, Phys. Rep. **18**, 125 (1975).
[34] T. S. Dumitrescu, C. H. Dasso, F. E. Serr, and T. J. Suzuki, J. Phys. G **12**, 349 (1986).
[35] J. Speth, E. Werner, and W. Wild, Phys. Rep., Phys. Lett. **33C**, 127 (1977).
[36] J. Wambach, *Proceedings of the International School on Nuclear Structure* (Nauka, Dubna, 1985), p. 83.