Analytical expressions for electromagnetic transition rates in the SU(3) limit of the *spdf* interacting boson model

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Analytical expressions for $E1$, $E2$, $E3$, $M1$, and $M2$ transition rates for low-lying negative-parity states in the SU(3) limit of the $spdf$ IBM are given. Applications to some deformed nuclei in the $A=150$ region and Uranium isotopes have yielded good agreement between calculation and data for $E1$ transitions. These formulas are useful in studying both positive- and negative-parity states of deformed nuclei. [S0556-2813(98)02305-X]

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I. INTRODUCTION

Collective negative parity states are believed to be related to octupole degrees of freedom in the nucleus in the geometrical model [1]. In the algebraic model, their description is the interacting boson model (IBM) including s, p, d, and f bosons [2–7]. One advantage of the algebraic model is the availability of analytical expressions for many physical quantities. The SU(3) limit describes a rotational spectrum [3–5]. Analytical expressions for the energy can be written explicitly in terms of the Casimir operators of the corresponding group chain. Besides the energy spectrum, electromagnetic transition properties are important in determining the structure of a nucleus. In the sd-IBM, electromagnetic transition rates are obtained analytically [6,8]. Numerical studies have shown that the low-lying positive parity states are well-described by N sd bosons, and the low-lying negative parity states are well described by coupling one pf boson with $N-1$ sd bosons. In this limit, the sd bosons and pf bosons interact only via the quadrupole interaction. This is the weak correlation case. In this study, we adopt this assumption. In the strong correlation, sd bosons and pf bosons interact strongly, and the pf bosons and sd bosons are mixed strongly. Lac and Morrison [9] have derived analytical expressions in the strong correlation case using the $1/N$ expansion technique. It is the purpose of this paper to give analytical expressions of $E1$, $E2$, $E3$, $M1$, and $M2$ transition rates involving low-lying negative parity states in the $\mathrm{SU}(3)_{sd}\times\mathrm{SU}(3)_{pf}$ limit. The paper is organized as follows. In Sec. II, we give a brief description of the method of the calculation. In Sec. III we give the results. In Sec. IV we apply the results to some deformed nuclei. In Sec. V we give a brief summary.

II. THE FORMALISM

The group chain under study is

$$\begin{array}{cccc} \mathrm{U}(16) \supset & \mathrm{U}(6) \times \mathrm{U}(10) \supset & \mathrm{SU}(3)_+ & \\ N & N_+ \quad\quad N_- & (\lambda_+,\mu_+) & \\ \times\ \mathrm{SU}(3)_- \supset & \mathrm{SU}(3) \supset & \mathrm{O}(3) & \\ (\lambda_-,\mu_-) & \omega(\lambda,\mu) & KL & . \end{array} \tag{1}$$

The numbers in the second row represent the corresponding irreducible representations of the groups. ω and K are floating index to indicate the mutiplicity in the reduction $\mathrm{SU}(3)_+\times \mathrm{SU}(3)_-\supset \mathrm{SU}(3)$ and the reduction $\mathrm{SU}(3)\supset \mathrm{O}(3)$, respectively. N is the total number of bosons, N_+ is the total number of positive-parity bosons, and N_- is the total number of negative-parity bosons. It has been shown that [3,4] the low-lying collective states are generated from N sd bosons through the SU(3) irreducible representations $(2N,0)$ and $(2N-4,2)$. The low-lying negative-parity states are generated by the coupling of one pf boson with $N-1$ sd bosons through the SU(3) irreducible representations $(2N+1,0)$ and $(2N-1,1)$.

In this paper, we give (1) $E1$ transition rates between $(2n+3,0)L_1^-$ and $(2n+2,0)L_2^+$; (2) the intraband $E2$ transition rates in the $(2n+3,0)$ negative parity state band; (3) $E3$ transitions between $(2n+3,0)L_1^-$ and $(2n+2,0)L_2^+$; (4) intraband $M1$ transitions in the $(2n+1,1)$ negative parity state band; (5) $M2$ transitions between $(2n+1,1)L_1^+$ and $(2n+2,0)L_2^+$; and (6) $M2$ transition from $(2n+1,1)L_1^-$ to $(2n-2,2)K=0,L_2^+$ (β-band) and to $(2n-4,2)K=2,L_2^+$ (γ-band) band. Here $n=N-1$, the total boson number minus 1 for the sake of simplicity. In Fig. 1, we give a illustration picture for the parity conserved transitions. In Fig. 2, a schematic picture for parity changing transitions is given.

The wave functions for the states involved are

$$\begin{aligned} &\text{(i) ground state band } \ |(2n,0)_+LM\rangle,\\ &\text{(ii) } \beta \text{ band } \ |(2n-4,2)_+K=0LM\rangle,\\ &\text{(iii) } \gamma \text{ band } \ |(2n-4,2)_+K=2LM\rangle,\\ &\text{(iv) } K^p=0^- \text{ -band } \ |(2n-2,0)_+(3,0)_-(2n+1,0)LM\rangle,\\ &\text{(v) } K^p=1^- \text{ -band } \ |(2n-2,0)_+(3,0)_-(2n-1,1)LM\rangle. \end{aligned} \tag{2}$$

The positive parity states in (i)–(iii) of Eq. (2) are the SU(3) limit wave functions in the sd-IBM. The negative parity states in (iv) and (v) can be written explicitly in terms of the coupling of the sd boson wave functions and the pf boson wave functions:

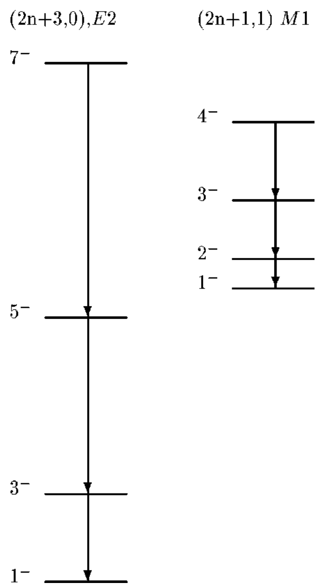

FIG. 1. Parity conserving transitions.

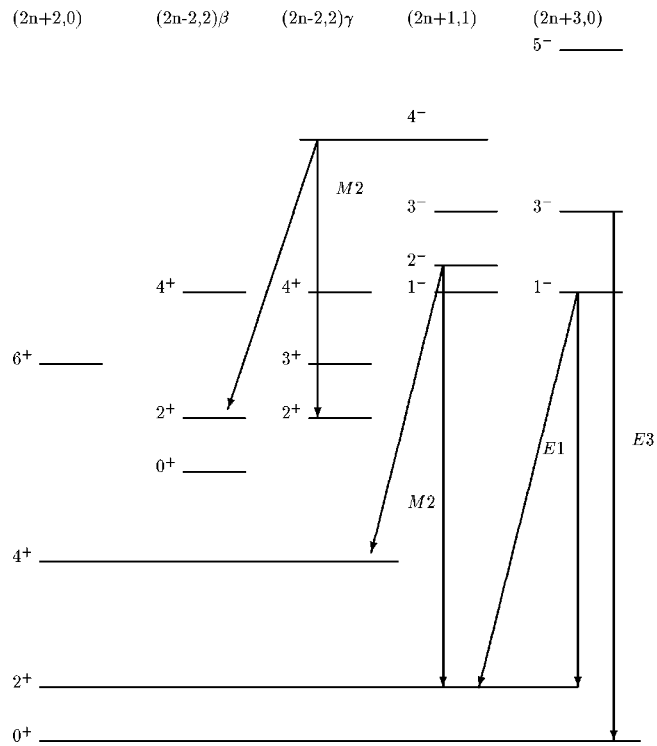

FIG. 2. Parity changing transitions.

$$|(2n-2,0)_+(3,0)_-(2n+1,0)LM\rangle = \sum_{L_+L_-} \left\langle \begin{matrix} (2n-2,0) & (3,0) \\ L_+ & L_- \end{matrix} \middle| \begin{matrix} (2n+1,0) \\ L \end{matrix} \right\rangle \times \{|(2n,0)L_+\rangle|(3,0)L_-\rangle\}^L_M, \tag{3}$$

$$|(2n-2,0)_+(3,0)_-(2n-1,1)LM\rangle = \sum_{L_+L_-} \left\langle \begin{matrix} (2n-2,0) & (3,0) \\ L_+ & L_- \end{matrix} \middle| \begin{matrix} (2n-1,1) \\ L \end{matrix} \right\rangle \times \{|(2n,0)L_+\rangle|(3,0)L_-\rangle\}^L_M, \tag{4}$$

where $\{|a\rangle|b\rangle\}^L_M$ stands for the ordinary vector coupling of two states $|a\rangle$ and $|b\rangle$. The angled bracketed quantity is the SU(3)$\supset$O(3) reduced Wigner coefficients. Most of the SU(3) Wigner coefficients for sd shell model calculation have been given by Vergados [10]. Those needed in this calculation have been given in Ref. [11]. Using these coefficients, the wave functions in the $spdf$ IBM can be written explicitly in the sd and pf wave functions.

The transition operators are

$$T(E1) = a_1(s^\dagger\tilde{p} + p^\dagger s)^{(1)} + b_1(p^\dagger\tilde{d} + d^\dagger\tilde{p})^{(1)} + c_1(d^\dagger\tilde{f} + f^\dagger\tilde{d})^{(1)},$$

$$T(E2) = a_2\left[(s^\dagger\tilde{d} + d^\dagger s)^{(2)} - \sqrt{\frac{7}{4}}(d^\dagger\tilde{d})^{(2)}\right] + b_2(p^\dagger\tilde{p})^{(2)} + c_2(p^\dagger\tilde{f} + f^\dagger\tilde{p})^{(2)} + d_2(f^\dagger\tilde{f})^{(2)},$$

$$T(E3) = a_3(s^\dagger\tilde{f} + f^\dagger s)^{(3)} + b_3(p^\dagger\tilde{d} + d^\dagger\tilde{p})^{(3)} + c_3(d^\dagger\tilde{f} + f^\dagger\tilde{d})^{(3)},$$

$$T(M1) = g_p(p^\dagger\tilde{p})^{(1)} + g_d(d^\dagger\tilde{d})^{(1)} + g_f(f^\dagger\tilde{f})^{(1)},$$

$$T(M2) = h_{pd}(d^\dagger\tilde{p} - p^\dagger\tilde{d})^{(2)} + h_{df}(d^\dagger\tilde{f} - f^\dagger\tilde{d})^{(2)}, \tag{5}$$

where $\tilde{b}_{lm} = (-1)^{l-m}b_{l-m}$. The signs of $(d^\dagger\tilde{p})^{(2)}$ and $(d^\dagger\tilde{f})^{(2)}$ are opposite to those of $(p^\dagger\tilde{d})^{(2)}$ and $(f^\dagger\tilde{d})^{(2)}$ in $T(M2)$. This is to ensure the hermiticity of the operator.

The wave functions can be seen as the coupling of states in the sd space with the states in the pf space. For transition operators involving only sd or pf operators, e.g., $(d^\dagger s)^{(2)}$ and $(p^\dagger\tilde{f})^{(2)}$, we can calculate the matrix elements using the following formula:

$$\langle \alpha_1 j_1 \alpha_2 j_2 J \| T_1^{(k)} \times I \| \alpha' j_1' \alpha_2' j_2' J' \rangle = (-1)^{(j_1+j_2+J'+k)}\sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} j_1 & J & j_2 \\ J' & j_1' & k \end{Bmatrix} \times \langle \alpha j_1 \| T_1^{(k)} \| \alpha' j_1' \rangle \delta(\alpha_2\alpha_2')\delta(j_2 j_2'),$$

$$\langle \alpha_1 j_1 \alpha_2 j_2 J \| I \times T_2^{(k)} \| \alpha_1' j_1' \alpha_2' j_2' J' \rangle = (-1)^{(j_1+j_2'+J+k)}\sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} j_2 & J & j_1 \\ J' & j_2' & k \end{Bmatrix} \times \langle \alpha_2 j_2 \| T_2^{(k)} \| \alpha_2' j_2' \rangle \delta(\alpha_1'\alpha_1)\delta(j_1 j_1'), \tag{6}$$

where 1 refers to pf space and 2 refers to sd space.

For operators involving one sd and one pf operators, e.g., $(s^\dagger\tilde{p})^{(1)}$, the matrix elements can be calculated from the following formula:

$$\langle \alpha_1 j_1 \alpha_2 j_2 J \| (b^\dagger_{k_1} \tilde{b}_{k_2})^{(k)} \| \alpha'_1 j'_1 \alpha'_2 j'_2 J' \rangle = \sqrt{(2J+1)(2k+1)(2J'+1)} \begin{Bmatrix} j_1 & j_2 & J \\ j'_1 & j'_2 & J' \\ k_1 & k_2 & k \end{Bmatrix} \times \langle \alpha_1 j_1 \| b^\dagger_{k_1} \| \alpha'_1 j'_1 \rangle \langle \alpha_2 j_2 \| \tilde{b}_{k_2} \| \alpha'_2 j'_2 \rangle. \quad (7)$$

The negative parity wave function involves only one pf boson, and its matrix elements can be given quite easily. Since the coupling of the pf boson relates many sd wave functions, the calculation of the matrix elements in the sd space is cumbersome. By applying Rosensteel's results [12], many of the calculations can be simplified. The reduced matrix element can be obtained through

$$(2L_b+1)^{-1/2} \langle (\lambda_b \mu_b) \chi_b L_b \| T^{(\alpha\beta)}_{\chi L} \| (\lambda_a \mu_b) \chi_a L_a \rangle = \sum_\rho \left\langle \begin{matrix} (\lambda_a \mu_a) & (\alpha\beta) \\ \chi_a L_a & \chi L \end{matrix} \middle| \begin{matrix} (\lambda_b \mu_b) \\ \chi_b L_b \end{matrix} \right\rangle_\rho \times \langle (\lambda_b \mu_b) \||T^{(\alpha\beta)}\|| (\lambda_a \mu_a) \rangle_\rho . \quad (8)$$

The triple barred quantity is the SU(3) reduced matrix element, and many of them have been given by Rosensteel [12]. Those that are not given in Ref. [12] are calculated by ourselves using a standard theoretic method as in Ref. [13].

III. THE RESULTS

A. $E1$ transitions

The matrix element is a sum of several terms. We give each of them here.

(a) $(2n+3,0)k=0L^- \to (2n+2,0)k=0(L-1)^+$ transitions, e.g., $3^- \to 2^+$:

$$\langle (L-1)^+ \| (s^\dagger \tilde{p} + p^\dagger s)^1 \| L^- \rangle = \frac{(2n-L+3)(2n+L+2)}{2(2n+1)} \sqrt{\frac{L(2n+L+4)}{5(n+1)(2n+3)}},$$

$$\langle (L-1)^+ \| (d^\dagger \tilde{p} + p^\dagger \tilde{d})^1 \| L^- \rangle = -\frac{(2n-L+3)(4n-L+1)}{10(2n+1)} \sqrt{\frac{L(2n+L+4)}{(n+1)(2n+3)}},$$

$$\langle (L-1)^+ \| (d^\dagger \tilde{f} + f^\dagger \tilde{d})^1 \| L^- \rangle = \frac{(8n^2+L^2+4nL+4n+L-2)}{10(2n+1)} \sqrt{\frac{3L(2n+L+4)}{7(n+1)(2n+3)}}. \quad (9)$$

(b) $(2n+3,0)k=0L^- \to (2n+2,0)k=0(L+1)^+$ transitions, e.g., $1^- \to 2^+$:

$$\langle (L+1)^+ \| (s^\dagger \tilde{p} + p^\dagger s)^1 \| L^- \rangle = -\frac{(2n-L+1)(2n+L+4)}{2(2n+1)} \sqrt{\frac{(L+1)(2n-L+3)}{5(n+1)(2n+3)}},$$

$$\langle (L+1)^+ \| (d^\dagger \tilde{p} + p^\dagger \tilde{d})^1 \| L^- \rangle = \frac{(2n+L+4)(4n+L+2)}{10(2n+1)} \sqrt{\frac{(L+1)(2n-L+3)}{(n+1)(2n+3)}},$$

$$\langle (L+1)^+ \| (d^\dagger \tilde{f} + f^\dagger \tilde{d})^1 \| L^- \rangle = -\frac{(8n^2+L^2-4nL+L-2)}{10(2n+1)} \sqrt{\frac{3(L+1)(2n-L+3)}{7(n+1)(2n+3)}}. \quad (10)$$

B. $E2$ transitions

For intraband transition $(L+2)^- \to L^-$ in the $(2n+3,0)0^-$ band, the corresponding matrix element of the different $E2$ operators are

$$\langle (L+2)^- \| (s^\dagger \tilde{d} + d^\dagger s)^2 - \sqrt{\frac{7}{4}} (d^\dagger \tilde{d})^2 \| L^- \rangle = \frac{n}{(2n+3)} \sqrt{\frac{3(2n-L+3)(2n+L+6)(L+1)(L+2)}{(2L+3)}},$$

$$\langle (L+2)^- \| (p^\dagger \tilde{p})^2 \| L^- \rangle = -\frac{3(2n-L+1)(2n+L+4)}{10(2n+1)(n+1)(2n+3)} \sqrt{\frac{(2n-L+3)(2n+L+6)(L+1)(L+2)}{(2L+3)}},$$

$$\langle (L+2)^- \| (p^\dagger \tilde{f} + f^\dagger \tilde{p})^2 \| L^- \rangle = \frac{(4L^2+24n^2+12L+30n+9)}{10(2n+1)(n+1)(2n+3)} \sqrt{\frac{3(2n-L+3)(2n+L+6)(L+1)(L+2)}{7(2L+3)}},$$

$$\langle (L+2)^- \| (d^\dagger \tilde{d})^2 \| L^- \rangle = -\frac{(L^2+16n^2+3L-4)}{10(2n+1)(n+1)(2n+3)} \sqrt{\frac{2(2n-L+3)(2n+L+6)(L+1)(L+2)}{7(2L+3)}}. \quad (11)$$

When the parameters in the $E2$ transition operator take the SU(3) limit, i.e., $a_2=1$, $b_2=-\sqrt{7/4}$, $c_2=-9\sqrt{3}/10$, $d_2=3\sqrt{7}/5$, and $e_2=-3\sqrt{42}/10$, the matrix element of $T(E2)$ between the two states becomes the familiar SU(3) limit result

$$-\frac{1}{2}\sqrt{\frac{3(2n-L+3)(2n+L+6)(L+1)(L+2)}{(2L+3)}}.$$

In the large n limit, $B(E2)$ approaches the rigid rotor value

$$B(E2)\propto\frac{(L+1)(L+2)}{(2L+3)(2L+5)}.$$

C. $E3$ transitions

We give the $E3$ transition elements for transition $(2n+3,0)L^-\rightarrow(2n+2,0)L'^+$.

(a) $L^-\rightarrow(L-3)^+$:

$$\langle(L-3)^+\|(s^\dagger\tilde{f}+f^\dagger s)^3\|L^-\rangle=\frac{(2n+L)}{2(2n+1)}\sqrt{\frac{(2n-L+5)(2n+L+2)(2n+L+4)(L-1)(L-2)L}{3(n+1)(2n+3)(2L-3)(2L-1)}},$$

$$\langle(L-3)^+\|(d^\dagger\tilde{p}+p^\dagger\tilde{d})^3\|L^-\rangle=\frac{(2n-L+3)}{2(2n+1)}\sqrt{\frac{3(2n-L+5)(2n+L+2)(2n+L+4)(L-1)(L-2)L}{5(n+1)(2n+3)(2L-3)(2L-1)}},$$

$$\langle(L-3)^+\|(d^\dagger\tilde{f}+f^\dagger\tilde{d})^3\|L^-\rangle=-\frac{(8n+L-3)}{6(2n+1)}\sqrt{\frac{(2n-L+5)(2n+L+2)(2n+L+4)(L-1)(L-2)L}{10(n+1)(2n+3)(2L-3)(2L-1)}}. \quad (12)$$

(b) $L^-\rightarrow(L-1)^+$:

$$\langle(L-1)^+\|(s^\dagger\tilde{f}+f^\dagger s)^3\|L^-\rangle=-\frac{(2n-L+3)(2n+L+2)}{(2n+1)(2n+2)}\sqrt{\frac{(2n+L+4)(L-1)L(L+1)(n+1)}{5(2n+3)(2L-3)(2L+3)}},$$

$$\langle(L-1)^+\|(d^\dagger\tilde{p}+p^\dagger\tilde{d})^3\|L^-\rangle=-\frac{(2n+L+2)(6n+L+9)}{10(2n+1)}\sqrt{\frac{7(2n+L+4)(L-1)L(L+1)}{3(2n+3)(2L-3)(2L+3)(n+1)}},$$

$$\langle(L-1)^+\|(d^\dagger\tilde{f}+f^\dagger\tilde{d})^3\|L^-\rangle=\frac{(16n^2+28n-2nL+7L^2+2L-24)}{15(2n+1)(2n+2)}\sqrt{\frac{3(2n+L+4)(L-1)L(L+1)(n+1)}{2(2n+3)(2L-3)(2L+3)}}. \quad (13)$$

(c) $L^-\rightarrow(L+1)^+$:

$$\langle(L+1)^+\|(s^\dagger\tilde{f}+f^\dagger s)^3\|L^-\rangle=\frac{(2n-L+1)(2n+L+4)}{(2n+1)(2n+2)}\sqrt{\frac{(2n-L+3)L(L+1)(L+2)(n+1)}{5(2n+3)(2L-1)(2L+5)}},$$

$$\langle(L+1)^+\|(d^\dagger\tilde{p}+p^\dagger\tilde{d})^3\|L^-\rangle=\frac{(2n+L+4)(6n-L+8)}{5(2n+1)(2n+2)}\sqrt{\frac{7(2n-L+3)L(L+1)(L+2)}{3(2n+3)(2L-1)(2L+5)}},$$

$$\langle(L+1)^+\|(d^\dagger\tilde{f}+f^\dagger\tilde{d})^3\|L^-\rangle=-\frac{(16n^2+30n+2nL+7L^2+12L-19)}{15(2n+1)(2n+2)}\sqrt{\frac{3(2n-L+3)L(L+1(L+2))(n+1)}{2(2n+3)(2L-1)(2L+5)}}. \quad (14)$$

(d) $L^-\rightarrow(L+3)^+$:

$$\langle(L+3)^+\|(s^\dagger\tilde{f}+f^\dagger s)^3\|L^-\rangle=\frac{(2n-L-1)}{(2n+1)(2n+2)}\sqrt{\frac{(2n-L+1)(2n-L+3)(2n+L+6)(L+1)(L+2)(L+3)(n+1)}{3(2n+3)(2L+3)(2L+5)}},$$

$$\langle(L+3)^+\|(d^\dagger\tilde{p}+p^\dagger\tilde{d})^3\|L^-\rangle=-\frac{(2n+L+4)}{(2n+1)(2n+2)}\sqrt{\frac{3(2n-L+1)(2n-L+3)(2n+L+6)(L+1)(L+2)(L+3)(n+1)}{5(2n+3)(2L+3)(2L+5)}},$$

$$\langle(L+3)^+\|(d^\dagger\tilde{f}+f^\dagger\tilde{d})^3\|L^-\rangle=\frac{(8n-L-4)}{3(2n+1)(2n+2)}\sqrt{\frac{(2n-L+1)(2n-L+3)(2n+L+6)(L+1)(L+2)(L+3)(n+1)}{10(2n+3)(2L+3)(2L+5)}}. \quad (15)$$

D. $M1$ transitions

We give expressions for the intraband transitions in $(2n+1,1)$.
(a) $L^- \to (L-1)^-$, with L odd, e.g., $1^- \to 0^-$:

$$\langle (L-1)^- \| (p^\dagger \tilde{p})^1 \| L^- \rangle = -\frac{(2n+L+2)(2n-3L+3)}{10n(2n+1)} \sqrt{\frac{(2n-L+3)(L+1)(L-1)}{2(2n+3)L}},$$

$$\langle (L-1)^- \| (f^\dagger \tilde{f})^1 \| L^- \rangle = -\frac{(3L^2+4nL+3L+16n^2-40n-6)}{20n(2n+1)} \sqrt{\frac{(2n-L+3)(L+1)(L-1)}{7(2n+3)L}},$$

$$\langle (L-1)^- \| (d^\dagger \tilde{d})^1 \| L^- \rangle = \frac{(2n-3)}{(2n+1)} \sqrt{\frac{(2n-L+3)(L+1)(L-1)}{10(2n+3)L}}. \tag{16}$$

(b) $L^- \to (L+1)^-$, with L odd, e.g., $1^- \to 2^-$:

$$\langle (L+1)^- \| (p^\dagger \tilde{p})^1 \| L^- \rangle = \frac{(2n-L+1)(2n+3L+6)}{10n(2n+1)} \sqrt{\frac{(2n+L+4)(L+2)L}{2(2n+3)(L+1)}},$$

$$\langle (L+1)^- \| (f^\dagger \tilde{f})^1 \| L^- \rangle = \frac{(3L^2-4nL+3L+16n^2-44n-6)}{20n(2n+1)} \sqrt{\frac{(2n+L+4)(L+2)L}{7(2n+3)(L+1)}},$$

$$\langle (L+1)^- \| (d^\dagger \tilde{d})^1 \| L^- \rangle = -\frac{(2n-3)}{(2n+1)} \sqrt{\frac{(2n+L+4)(L+2)L}{10(2n+3)(L+1)}}. \tag{17}$$

E. $M2$ transitions

(a) $(2n+1,1)L^- \to (2n+2,0)(L-2)^+$ with L even, e.g., $2^- \to 0_g^+$ (to ground state band):

$$\langle (L-2)^+ \| (d^\dagger \tilde{p} - p^\dagger \tilde{d})^2 \| L^- \rangle = \frac{(2n-L+2)}{(2n+1)} \sqrt{\frac{(2n-L+4)(2n+L+3)(L-1)(L+1)}{30(2L-1)n}},$$

$$\langle (L-2)^+ \| (d^\dagger \tilde{f} - f^\dagger \tilde{d})^2 \| L^- \rangle = -\frac{(8n+L-2)}{2(2n+1)} \sqrt{\frac{(2n-L+4)(2n+L+3)(L-1)(L+1)}{105(2L-1)n}}. \tag{18}$$

(b) $(2n+1,1)L^- \to (2n+2,0)(L+2)^+$ with L even, e.g., $2^- \to 4_g^+$ (to ground state band):

$$\langle (L+2)^+ \| (d^\dagger \tilde{p} - p^\dagger \tilde{d})^2 \| L^- \rangle = -\frac{(2n+L+3)}{(2n+1)} \sqrt{\frac{(2n+L+5)(2n-L+2)(L+2)L}{30(2L+3)n}},$$

$$\langle (L+2)^+ \| (d^\dagger \tilde{f} - f^\dagger \tilde{d})^2 \| L^- \rangle = \frac{(8n-L-3)}{2(2n+1)} \sqrt{\frac{(2n+L+5)(2n-L+2)(L+2)L}{105(2L+3)n}}. \tag{19}$$

(c) $(2n+1,1)L^- \to (2n-2,2)k=0(L-2)^+$ with L even, e.g., $2^- \to 0_\beta^+$ (to β band):

$$\langle (L-2)^+ \| (d^\dagger \tilde{p} - p^\dagger \tilde{d})^2 \| L^- \rangle = -\frac{(2n-L+2)(2n+L-2)}{(2n+1)} \sqrt{\frac{(2n+L+1)(2n+L+3)(L-1)(L+1)}{15n(2n-1)(2L-1)(8n^2-L^2+3L-2)}},$$

$$\langle (L-2)^+ \| (d^\dagger \tilde{f} - f^\dagger \tilde{d})^2 \| L^- \rangle = \frac{(16n^2+L^2-9L+14)}{(2n+1)} \sqrt{\frac{(2n+L+1)(2n+L+3)(L-1)(L+1)}{210n(2n-1)(2L-1)(8n^2-L^2+3L-2)}}. \tag{20}$$

(d) $(2n+1,1)L^- \to (2n-2,2)k=2(L-2)^+$ with L even, e.g., $2^- \to 0_\gamma^+$ (to γ-band):

$$\langle (L-2)^+ \| (d^\dagger \tilde{p} - p^\dagger \tilde{d})^2 \| L^- \rangle = \sqrt{\frac{2(2n-L+2)(2n+L-1)(2n+L+1)(2n+L+3)(L-3)(L-2)(L+1)}{15L(2n-1)(2n+1)(2L-1)(8n^2-L^2+3L-2)}},$$

$$\langle (L-2)^+ \| (d^\dagger \tilde{f} - f^\dagger \tilde{d})^2 \| L^- \rangle = \sqrt{\frac{12(2n-L+2)(2n+L-1)(2n+L+1)(2n+L+3)(L-3)(L-2)(L+1)}{35L(2n-1)(2n+1)(2L-1)(8n^2-L^2+3L-2)}}. \tag{21}$$

TABLE I. Comparison of absolute $B(E1)$ values.

Nucleus	I_i	I_f	$B(E1,I_i\rightarrow I_f)_{\rm cal}$(W.u.)	$B(E1,I_i\rightarrow I_f)_{\rm exp}$(W.u.)
^{152}Sm	1_1^-	0_1^+	0.0042	0.0042(4)
		2_1^+	0.0081	0.0077(7)
	3_1^-	2_1^+	0.0055	0.0081(16)
		4_1^+	0.0067	0.0082(16)
^{154}Sm	1_1^+	0_1^+	0.0046	0.0046(8)
		2_1^+	0.0089	0.0093(15)
^{156}Gd	1_1^+	0_1^+	0.0016	0.0016(12)
		2_1^+	0.0031	0.0020(15)
^{160}Gd	1_1^+	0_1^+	0.0032	0.0032(9)
		2_1^+	0.0062	0.0060(17)
^{236}U	1_1^+	0_1^+	2.59×10^{-8}	$2.59\times10^{-8}(11)$
		2_1^+	5.03×10^{-8}	$5.81\times10^{-8}(22)$

(e) $(2n+1,1)L^-\rightarrow(2n-2,2)k=2(L-2)^+$ with L odd, e.g., $5^-\rightarrow3_\gamma^+$:

$$\langle(L-2)^+\|(d^\dagger\tilde{p}-p^\dagger\tilde{d})^2\|L^-\rangle$$
$$=(2n-3L+3)$$
$$\times\sqrt{\frac{(2n+L)(2n+L+2)(L+1)(L-2)(L-3)}{15(2n+1)(2n+3)(2n)(2n-1)L(2L-1)}},$$

$$\langle(L-2)^+\|(d^\dagger\tilde{f}-f^\dagger\tilde{d})^2\|L^-\rangle$$
$$=(4n-L+6)$$
$$\times\sqrt{\frac{3(2n+L)(2n+L+2)(L+1)(L-2)(L-3)}{70(2n+1)(2n+3)(2n)(2n-1)(2L-1)L}}. \quad (22)$$

IV. APPLICATIONS: DEFORMED NUCLEI

The expressions for the matrix elements are general. In order to study the electromagnetic transitions of deformed nuclei, one has to choose appropriate coefficients for all the terms in the corresponding transition operators. This can be done by either a microscropic derivation, or by fitting to experimental data. For simplicity, one can also take the transition operators as generators of the group. This will simplify the expressions quite a lot. We must emphasize that this is merely for the sake of simplicity. Sometimes this approach does not bring meaningful results. For example, in the $M1$ transition case, if one takes the transition operator as the generator of $O(3)$, the angular momentum operator, there will be no $M1$ transitions since angular momentum is a good quantum number. Nevertheless this property does provide a cross check on the correctness of the calculation. Sometimes it offers a simple formula that contains meaningful results. For instance, if $E2$ operator is taken as the SU(3) generator Q, one obtains the familiar results.

We have applied the results in Sec. III to some deformed nuclei with negative parity states. In this section, we give two applications. In the first application, we compare directly $B(E1)$ values. For simplicity, we choose the generator of D_μ^1 of $O(10)$:

TABLE II. Comparisons of relative intensities.

Nucleus	$E_{\rm level}$(keV)	I_i	I_f	Cal	Expt.
^{152}Sm	963	1_1^1	0_1^+	78	82.3(7)
			2_1^+	100	100(2)
	1041	3_1^-	2_1^+	100	100(4)
			4_1^+	48	40.4(19)
	1221	5_1^-	4_1^+	100	100(4)
			6_1^+	23	24(3)
	1505	7_1^-	6_1^+	100	100(3)
			8_1^+	11	12
	1879	9_1^-	8_1^+	100	100(3)
			10_1^+	4	15
^{154}Sm	921	1_1^-	0_1^+	68	65(2)
			2_1^+	100	100(2)
	1012	3_1^-	2_1^+	100	100(2)
			4_1^+	63	60(1)
	1181	5_1^-	4_1^+	100	100(3)
			6_1^+	36	29.0(7)
^{156}Sm	804	1_1^-	0_1^+	69	100(18)
			2_1^+	100	82(18)
	876	3_1^-	2_1^+	100	100(11)
			4_1^-	59	17(3)
	1021	5_1^-	4_1^+	100	100(12)
			6_1^+	30	12(4)
^{158}Gd	1264	1_2^-	0_1^+	63	67(4)
			2_1^+	100	100(6)
	1403	3_2^-	2_1^+	100	100(6)
			4_1^+	80	85(5)
^{230}U	367	1_1^-	0_1^+	82	81(13)
			2_1^+	100	100(13)
	435	3_1^-	2_1^+	100	100(8)
			4_1^+	41	39(6)
^{232}U	563	1_1^-	0_1^+	67	67(4)
			2_1^+	100	100(4)
	629	3_1^-	2_1^+	100	100(4)
			4_1^+	66	69(3)
^{234}U	786	1_1^-	0_1^+	61	58(2)
			2_1^+	100	100(2)
	849	3_1^-	2_1^+	100	100(7)
			4_1^+	84	90(5)
	963	5_1^-	4_1^+	100	100(6)
			6_1^+	58	62(4)
	1125	7_1^-	6_1^+	100	100(30)
			8_1^+	42	66(12)
^{238}U	680	1_1^-	0_1^+	63	74(4)
			2_1^+	100	100(2)
	732	3_1^-	2_1^+	100	100(2)
			4_1^+	77	83(2)
	826	5_1^-	4_1^+	100	100(4)
			6_1^+	49	57(6)

$$D_\mu^1=\sqrt{\frac{1}{2}}(s^\dagger\tilde{p}+p^\dagger s)_\mu^1-\sqrt{\frac{4}{5}}(p^\dagger\tilde{d}+d^\dagger+d^\dagger\tilde{p})_\mu^1$$
$$+\sqrt{\frac{7}{10}}(d^\dagger\tilde{f}+f^\dagger\tilde{d})_\mu^1, \quad (23)$$

as our $E1$ transition operator, $T(E1)^1_\mu = e_1 D^1_\mu$. e_1 is determined by fitting to one of the experimental data. In Table I, we compare the calculated $B(E1)$ with those of the experimental data in ^{152,154}Sm [14,15], ^{156,160}Gd [16,17], and ^{236}U [18].

Absolute $B(E1)$ data are rare, we compare relative intensities in the second application. If one assumes that $E1$ is dominant in parity changing transitions and ignore higher order transitions, then the transition probabilities and hence the intensities are

$$\text{int} \propto E_\gamma^3 B(E1). \tag{24}$$

Using formulas obtained in the previous section for the $B(E1)$'s, we have calculated the relative intensities in ^{152,154,156}Sm [14–16], ^{158}Gd [19], and $^{230-238}$U [20,21]. The experimental data are taken from the references after each nucleus and/or from Ref. [20]. The calculated results are compared with experimental data in Table II.

It is seen from Table I that the agreement between calculation and experiment is very good for absolute $B(E1)$'s for all the five nuclei studied. One interesting observation is that the absolute $B(E1)$ in ^{236}U is 4–5 orders less than those in the rare earth.

The agreement for the relative intensities is also very good with the exception of ^{156}Sm. In all other nuclei listed in Table II, the intensity from 1_1^- to 0_1^+ is weaker than that from 1_1^- to 2_1^+. But in ^{156}Sm, this situation is reversed. This can not be explained in the present model. Further efforts are needed to understand this inversion. Apart from this nucleus, the agreement is good, and in some nuclei, excellent.

V. SUMMARY

We have given analytical expressions for $E1$, $E2$, $E3$, $M1$, and $M2$ transitions involving low-lying negative parity states in the SU(3) limit. In real nuclei, the SU(3) symmetry will be more or less broken. Then a detailed calculation should take into account the symmetry breaking. This can be done numerically or by perturbation method. When the breaking is not big, the analytical results derived here can be used in the first order approximation. A preliminary application of $E1$ transition formula to some deformed nuclei yields promising results. With more experimental knowledge of E/M transitions, these SU(3) transition prediction can be checked.

One question is the role of the g bosons. As is well known, in the uranium isotopes, the g boson is necessary in order to describe the $E2$ transitions and to remove the early cutoff of band in the sd IBM for the positive parity states (for instance, see, [22,23]). If the g boson is included, the low-lying positive parity states are from N sdg bosons. The low-lying negative parity states are the coupling of one pf boson with $N-1$ sdg bosons. The formulas of the E/M transitions in the $sdgpf$ IBM are beyond the scope of the present study. In the present study, the $spdf$ IBM calculation gave good agreement with the data for $E1$ transitions. It is presumed that because of the common SU(3) algebraic structure in the two models ($spdf$ and $sdgpf$ IBM), it is probably that the results for the low-lying states are similar.

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