Poincaré invariant coupled channel model for the pion-nucleon system. II. An extended model

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A previously developed Poincaré invariant, front form model of the pion-nucleon system is extended up to a pion lab kinetic energy of 1.0 GeV, which corresponds to a total c.m. energy of 1.74 GeV. The model is constructed in a space spanned by single baryon states $|B\rangle$, where B is the nucleon, or any resonance that contributes in the energy range considered, and by meson-baryon states $|\mu B\rangle$ where $|\mu B\rangle = |\pi N\rangle$, $|\pi\Delta\rangle$, or $|\eta N\rangle$. The model specifies a mass-square operator in the form $M^2 = M_0^2 + V$ where M_0 is a noninteracting mass operator and V is an interaction. The $\mu B - \mu' B'$ interactions are assumed to be separable.
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I. INTRODUCTION

Models for the pion-nucleon system fall into two broad categories; particle exchange models [1–5] and separable interaction models [6–16]. Apart from the Gross-Surya model [3], which includes coupling to an effective $\pi\Delta$ channel, none of the exchange models include the effects of inelastic channels, while many of the separable models do [6–9,11,14–16]. The Landau-Tabakin model [6] takes account of inelasticity through the use of complex form factors for the separable potentials, while several of the other separable models [7–9,11] are equivalent to a separable potential model with real form factors, but with an energy dependent potential strength which becomes complex above the inelastic threshold. A few of the separable models include inelastic channels explicitly; in particular, the Blankleider-Walker [15] model includes coupling to a $\pi\Delta$ channel, while the Bhalerao-Liu [14] and Fuda [16] models allow for coupling to both $\pi\Delta$ and ηN channels. Most of the models take account of the pole in the P_{11} elastic amplitude that occurs below the elastic threshold at a c.m. energy equal to the nucleon mass; the so-called nucleon pole. Several of the models [1–5,13,14,16] include resonances such as the $\Delta(1232)$ as single particle intermediate states.

The model developed previously by one of us [16] is an exactly Poincaré invariant, front form model of the pion-nucleon system, constructed in the space spanned by $|N\rangle$, $|\Delta\rangle$, $|\pi N\rangle$, $|\pi\Delta\rangle$, and $|\eta N\rangle$ states. This earlier model specifies a mass-square operator in the form $M^2 = M_0^2 + V$ where M_0 is a noninteracting mass operator and V is an interaction. The relative angular momentum or spin operator $\mathcal{J}$ of the system is the same as that of the noninteracting system. The model gives a good fit to the πN elastic scattering amplitudes for pion laboratory kinetic energies up to 700 MeV. Here we extend the model so as to reproduce the experimental amplitudes up to 1.0 GeV. The single baryon states $|B\rangle$ include not only the nucleon and $\Delta(1232)$ resonance, but all other resonances in the energy range considered. The interactions coupling two-particle channels to each other are taken to be separable potentials. In the S_{11} partial wave the inelastic channel is the ηN channel, while in all other partial waves it is assumed to be an effective $\pi\Delta$ channel.

Throughout we work in units for which $\hbar = c = 1$.

II. THE MODEL

Our model space is spanned by the states $|\overline{p_B}; s_B h_B\rangle$ and $|\overline{p_{\mu B}}; q_{\mu B}, lj\lambda\rangle$, where in general B denotes a baryon and μ denotes a meson. For the sake of brevity we suppress the isospin quantum numbers. Here $\overline{p} = (p^+, p_x, p_y)$ with $p^+ = (E + p_z)/\sqrt{2}$, where E is the total energy of the state and p_x, p_y, and p_z are the Cartesian components of the total three-momentum $\mathbf{p}$. The quantum numbers s_B and h_B are the spin and z component of the spin, respectively, for baryon B. The quantity $q_{\mu B}$ is the magnitude of the three-momentum of the meson μ in a particular rest frame of the μB pair, i.e., the rest frame related to a general frame by a so-called *front form boost* [16]. The angular momentum quantum numbers l, j, and λ specify the relative orbital angular momentum of the pair μB, their total angular momentum, and the z component of their total angular momentum, respectively. Since the π and η mesons are spinless, the total spin of the μB pair is simply that of the baryon.

The basic ingredient of our model is a mass-square operator of the form

$$M^2 = M_0^2 + V, \tag{2.1}$$

where M_0 is the mass operator of the noninteracting system and V is the interaction. The action of M_0 on the basis states is defined by

$$M_0|\overline{p_B}; s_B h_B\rangle = m_B|\overline{p_B}; s_B h_B\rangle, \tag{2.2a}$$

$$M_0|\overline{p_{\mu B}}; q_{\mu B}, lj\lambda\rangle = W_{\mu B}(q_{\mu B})|\overline{p_{\mu B}}; q_{\mu B}, lj\lambda\rangle, \tag{2.2b}$$

where m_B is the *physical mass* of baryon B, and $W_{\mu B}$ is the total c.m. energy of the pair μB, which is given in terms of the meson's c.m. energy $\omega_\mu(q_{\mu B})$ and baryon's c.m. energy $\varepsilon_B(q_{\mu B})$ by

$$W_{\mu B}(q_{\mu B}) = \omega_\mu(q_{\mu B}) + \varepsilon_B(q_{\mu B}). \tag{2.3}$$

The physical masses of the meson and baryon are used in Eq. (2.3).

The matrix elements of the interaction V are defined by

$$\langle \overline{p_B};s_B h_B|V|\overline{p'_{B'}};s_{B'}h'_{B'}\rangle=(2\pi)^3 2p_B^+\delta^3(\overline{p_B}-\overline{p'_B})\times\delta_{BB'}(m_{0B}^2-m_B^2), \quad (2.4a)$$

$$\langle \overline{p_{\mu B}};q_{\mu B},lj\lambda|V|\overline{p'_{B'}};s_{B'}h'_{B'}\rangle =(2\pi)^3 2p_{\mu B}^+\delta^3(\overline{p_{\mu B}}-\overline{p'_{B'}})\times\delta_{js_{B'}}\delta_{\lambda h'_{B'}}V_{cB'}(q_{\mu B}), \quad (2.4b)$$

$$\langle \overline{p_{\mu B}};q_{\mu B},lj\lambda|V|\overline{p'_{\mu' B'}};q'_{\mu' B'},l'j'\lambda'\rangle =(2\pi)^3 2p_{\mu B}^+\delta^3(\overline{p_{\mu B}}-\overline{p'_{\mu' B'}})\delta_{jj'}\delta_{\lambda\lambda'}V^j_{cc'}(q_{\mu B},q'_{\mu' B'}), \quad (2.4c)$$

where c is the set of channel labels

$$c=\{\mu,B,l\}, \quad (2.5)$$

and m_{0B} is the *bare mass* of baryon B.

The scattering amplitudes of our model are obtained by solving the Lippmann-Schwinger equation

$$T(z)=V+V(z-M_0^2)^{-1}T(z), \quad (2.6)$$

where z is a complex parameter, which when calculating physical amplitudes becomes $z=W^2+i\varepsilon$ with W the invariant mass of the scattering process. In solving the coupled integral equations that arise upon taking matrix elements of Eq. (2.6), it is possible to eliminate the single baryon channels, and thereby obtain an effective potential that acts in the subspace of meson-baryon states [16]. The resulting integral equations are

$$T^j_{cc'}(q,q';z)=U^j_{cc'}(q,q';z)+\sum_{c''}\int_0^\infty \times\frac{U^j_{cc''}(q,q'';z)q''^2dq''}{\Delta_{\mu''B''}(q'')[z-W^2_{\mu''B''}(q'')]}T^j_{c''c'}(q'',q';z), \quad (2.7)$$

TABLE I. States and particle channels of the pion-nucleon system.

πN State	Baryon and meson-baryon channels	m_Δ (MeV)
S_{11}	$S_{11}(1535),S_{11}(1650),\pi N,\eta N(l_{\eta N}=0)$	
S_{31}	$S_{31}(1620),\pi N,\pi\Delta(l_{\pi\Delta}=2)$	1402.21
P_{11}	$N,P_{11}(1440),P_{11}(1710),\pi N,\pi\Delta(l_{\pi\Delta}=1)$	1076.95
P_{13}	$P_{13}(1720),\pi N,\pi\Delta(l_{\pi\Delta}=1,3)$	1076.95
P_{31}	$P_{31}(1744),\pi N,\pi\Delta(l_{\pi\Delta}=1)$	1125.95
P_{33}	$P_{33}(1232),P_{33}(1600),\pi N,\pi\Delta(l_{\pi\Delta}=1,3)$	1076.95
D_{13}	$D_{13}(1520),\pi N,\pi\Delta(l_{\pi\Delta}=0,2)$	1076.95
D_{15}	$D_{15}(1675),\pi N,\pi\Delta(l_{\pi\Delta}=2,4)$	1076.95
D_{33}	$D_{33}(1700),\pi N,\pi\Delta(l_{\pi\Delta}=0,2)$	1076.95

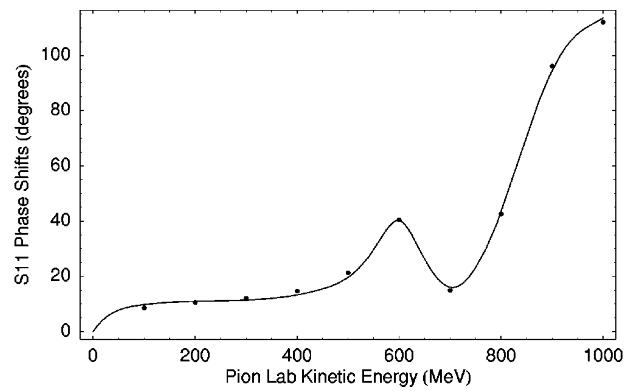

FIG. 1. Fit of the S_{11} phase shifts to the SAID-SP95 analysis.

where

$$U^j_{cc'}(q,q';z)=V^j_{cc'}(q,q')+\sum_{B''}\delta_{js_{B''}}\frac{V_{cB''}(q)V_{c'B''}(q')}{z-m^2_{0B''}}, \quad (2.8)$$

$$\Delta_{\mu B}(q)=(2\pi)^3 2\omega_\mu(q)\varepsilon_B(q)/W_{\mu B}(q). \quad (2.9)$$

For our $\mu B-\mu' B'$ potentials we assume the separable forms

$$V^j_{cc'}(q,q')=g^j_c(q)\lambda_{cc'}g^j_{c'}(q'), \quad (2.10)$$

and take for the functions that appear here, and for the $\mu B-B'$ vertex functions the expressions

$$g^j_c(q)=C^j_c(q/\beta^j_c)^l[1+(q/\beta^j_c)^2]^{-K^j_c}, \quad (2.11)$$

$$V_{cB'}(q)=C_{cB'}(q/\beta_{cB'})^l[1+(q/\beta_{cB'})^2]^{-K_{cB'}}. \quad (2.12)$$

III. RESULTS

In calculating the pion-nucleon elastic scattering amplitudes we deal with states of well defined total angular momentum j, isospin i, and parity, labeled in the usual way, i.e., $X_{2i,2j}$, where $X=S,P,D,\ldots$, corresponding to $l_{\pi N}=0,1,2,\ldots$. Since the pion is a pseudoscalar particle the parity is $(-1)^{1+l_{\pi N}}$. The pion-nucleon states that we consider are shown in Table I, as well as the single baryon and

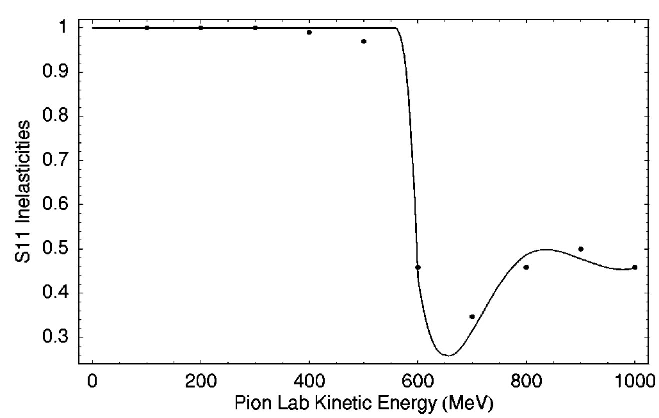

FIG. 2. Fit of the S_{11} inelasticities to the SAID-SP95 analysis.

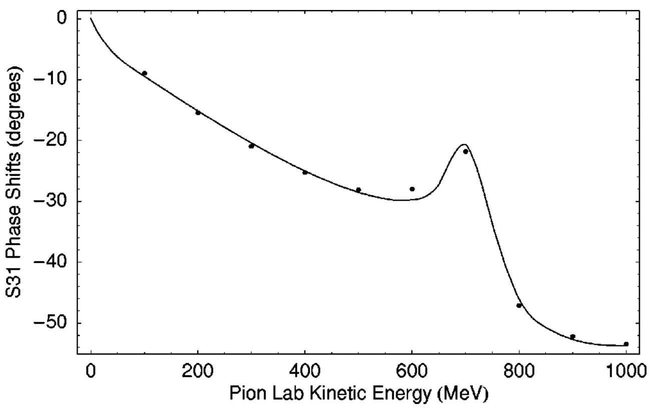

FIG. 3. Fit of the S_{31} phase shifts to the SAID-SP95 analysis.

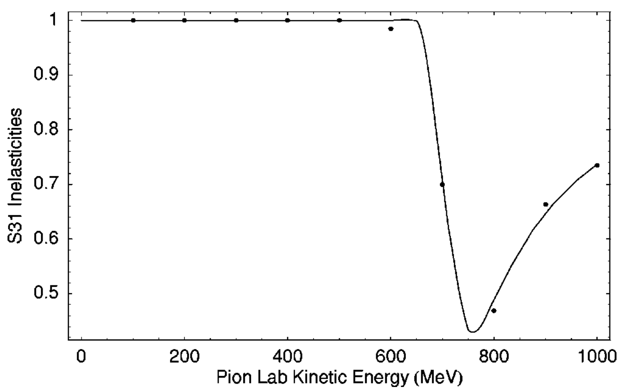

FIG. 4. Fit of the S_{31} inelasticities to the SAID-SP95 analysis.

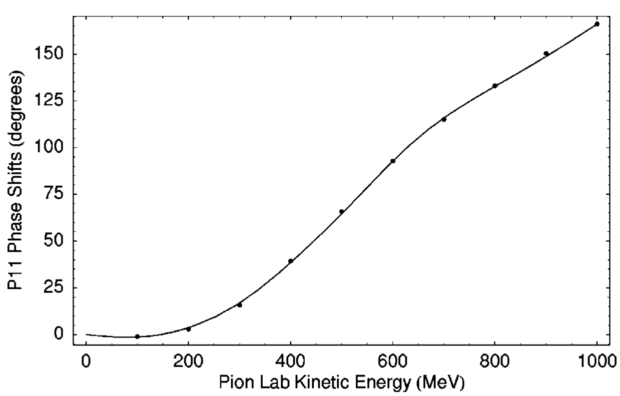

FIG. 5. Fit of the P_{11} phase shifts to the SAID-SP95 analysis.

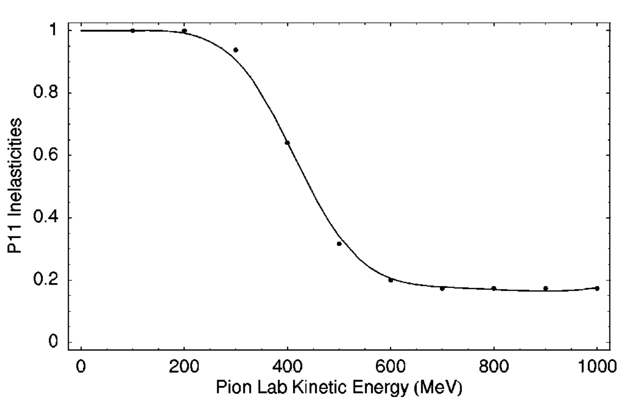

FIG. 6. Fit of the P_{11} inelasticities to the SAID-SP95 analysis.

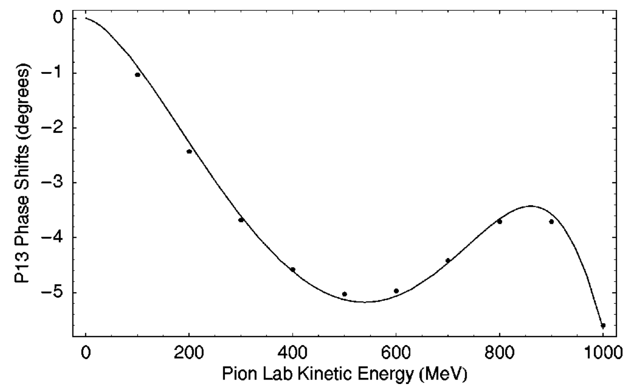

FIG. 7. Fit of the P_{13} phase shifts to the SAID-SP95 analysis.

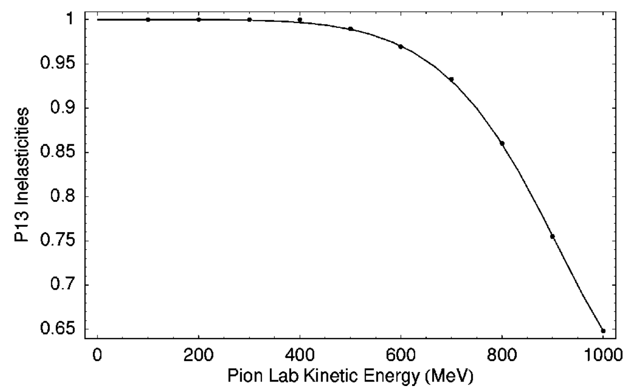

FIG. 8. Fit of the P_{13} inelasticities to the SAID-SP95 analysis.

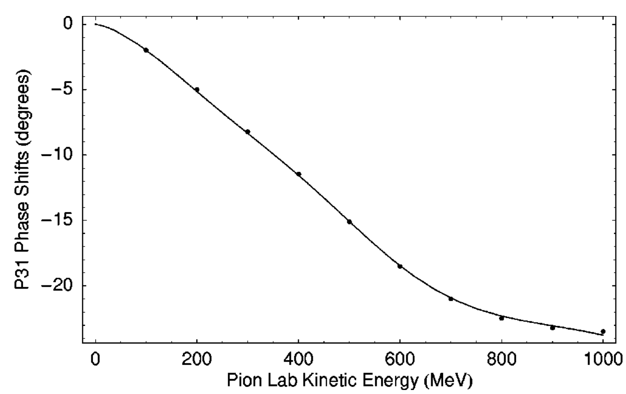

FIG. 9. Fit of the P_{31} phase shifts to the SAID-SP95 analysis.

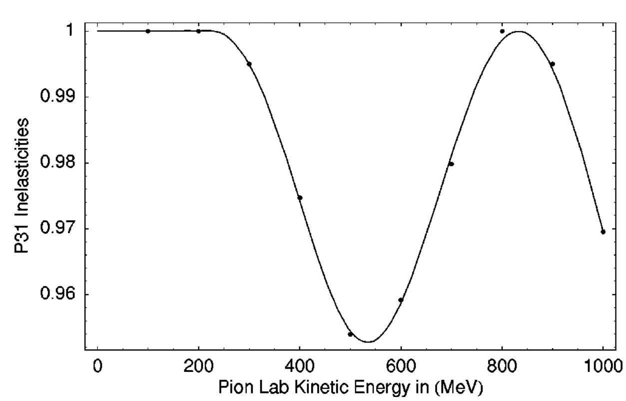

FIG. 10. Fit of the P_{31} inelasticities to the SAID-SP95 analysis.

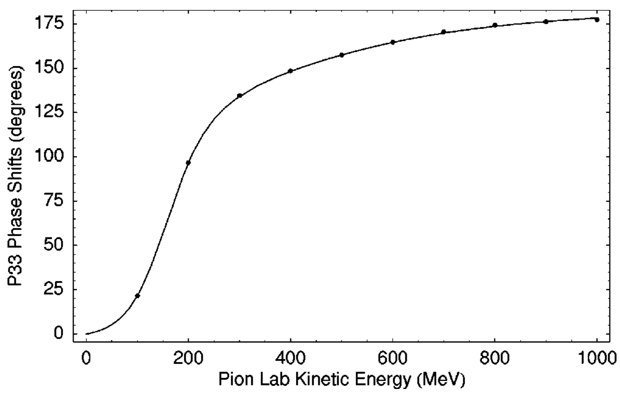

FIG. 11. Fit of the P_{33} phase shifts to the SAID-SP95 analysis.

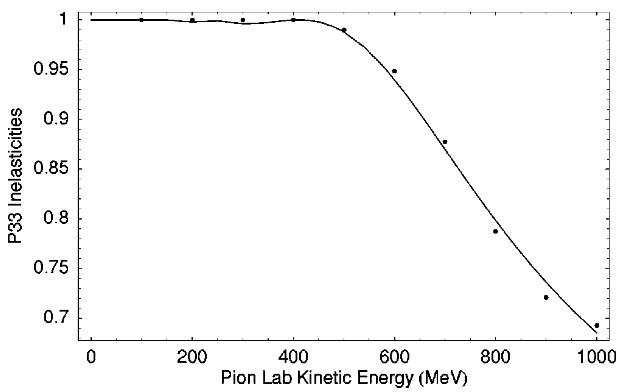

FIG. 12. Fit of the P_{33} inelasticities to the SAID-SP95 analysis.

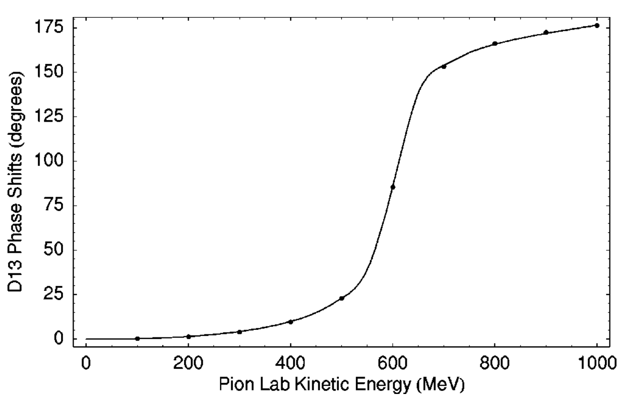

FIG. 13. Fit of the D_{13} phase shifts to the SAID-SP95 analysis.

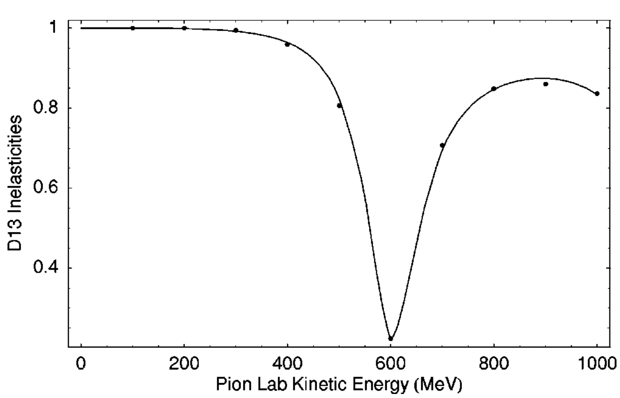

FIG. 14. Fit of the D_{13} inelasticities to the SAID-SP95 analysis.

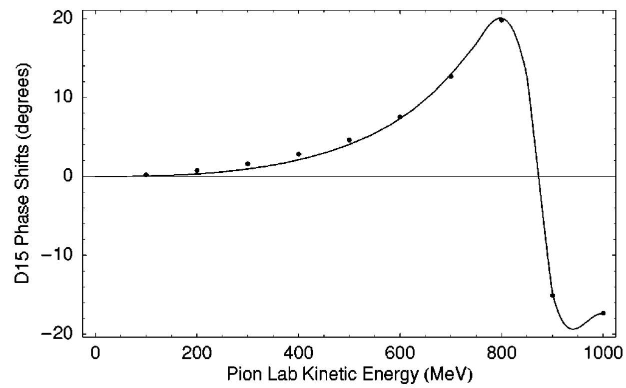

FIG. 15. Fit of the D_{15} phase shifts to the SAID-SP95 analysis.

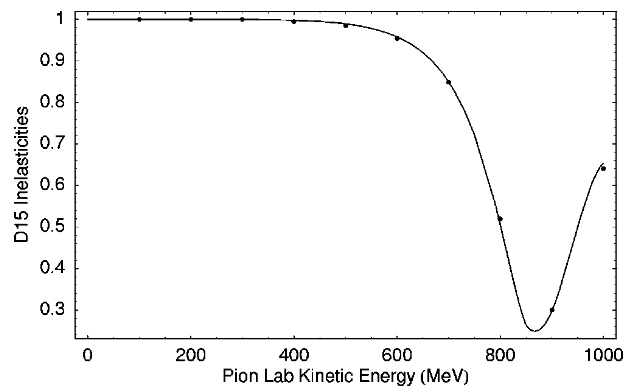

FIG. 16. Fit of the D_{15} inelasticities to the SAID-SP95 analysis.

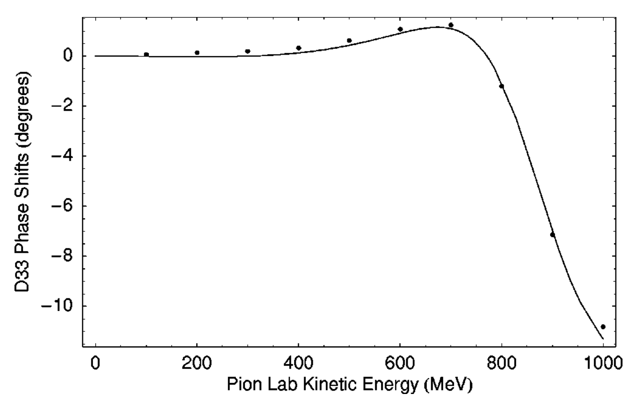

FIG. 17. Fit of the D_{33} phase shifts to the SAID-SP95 analysis.

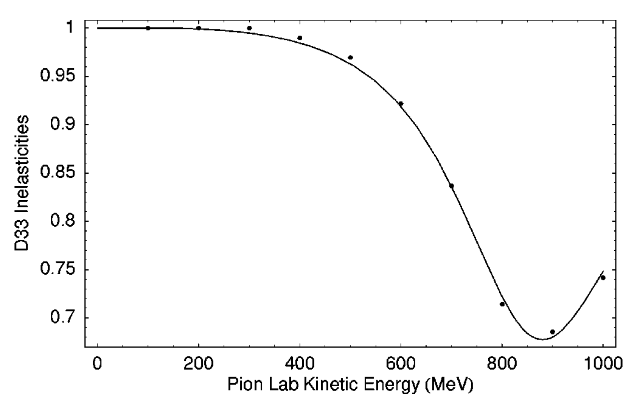

FIG. 18. Fit of the D_{33} inelasticities to the SAID-SP95 analysis.

TABLE II. Potential parameters.

States	C_1	β_1 (fm^{-1})	K_1	C_2	β_2 (fm^{-1})	K_2	λ_{11}	λ_{22}	λ_{12}
S_{11}	2.16469	3.20151	1	68.2495	1.37367	1	1	1	−46.8376
S_{31}	186.477	5.87154	1	−49.0307	124.436	2	1	1	−5.26329
P_{11}									
P_{13}	51.1975	4.47382	2	11.0837	382.613	2	1	1	4.37850
P_{31}	70.318	4.70084	2	3.85991	2.5995	2	1	1	−9.19168
P_{33}									
D_{13}	−41.1344	45.9792	4	219.111	27.5447	2	1	1	−7.04494
D_{15}									
D_{33}									

meson-baryon channels that are coupled in each state. The quantities in parentheses are the relative orbital angular momenta in the meson-baryon channel other than the pion-nucleon channel. In those cases for which there are two possible relative orbital angular momenta we retain only the smaller one.

The parameters in Eqs. (2.10)–(2.12) were determined by a least squares fit to the phase shifts δ_c^j and inelasticities η_c^j of the SAID-SP95 analysis of the pion-nucleon scattering data [17], where these quantities are related to the on-shell T-matrix elements by

$$\eta_c^j \exp[2i\delta_c^j(W)] = 1 - iqT_{cc}^j(q,q,W^2+i\varepsilon)/(16\pi^2 W),$$
$$W = W_{\pi N}(q), \quad c = \{\pi,N,l\}. \tag{3.1}$$

The resulting parameters are given in Tables II and III, while the phase shifts and inelasticities are shown in Figs. 1–18. In Tables II and III, the subscripts 1 and 2 on C,β, etc., refer to the πN channel and the inelastic channel, respectively. The masses of the particles were taken to be $m_\pi = 138.03$ MeV, $m_N = 938.92$ MeV, and $m_\eta = 547.45$ MeV. Since the $\pi\Delta$ channel is an effective channel the mass of the Δ was either set equal to $m_\pi + m_N = 1076.95$ MeV, or adjusted to improve the fit in a particular partial wave. In fitting the P_{11} channel, the amplitude was constrained to have a pole at $W_{\pi N} = m_N$ with the residue properly related to the pion-nucleon coupling constant $g_{\pi NN}$; explicitly we required that

$$T_{\pi N,\pi N}[q,q,W_{\pi N}^2(q)] \rightarrow -\frac{12 m_\pi^2 g_{\pi NN}^2}{W_{\pi N}^2(q) - m_N^2} \quad (P_{11}\text{channel}), \tag{3.2}$$

with $g_{\pi NN}^2/4\pi = 13.5$.

IV. DISCUSSION

It is clear that the model presented here gives a good description of the pion-nucleon elastic scattering amplitude up to a pion lab kinetic energy of 1.0 GeV. The model easily accounts for the rapid variation of the amplitudes due to the presence of the various resonances, as well as the opening of the inelastic channels. Even though the model presented here is based on a formalism developed in the front form of relativistic quantum mechanics [16], the *internal parts* of the mass operator matrix elements can be used in an instant form version of the model. By the *internal parts* we mean, e.g., the expressions to the right of the $(2\pi)^3 2p^+ \delta^3(\overline{p} - \overline{p'})$ factors in Eq. (2.4). The final Lippmann-Schwinger equations, i.e., Eq. (2.7), are exactly the same in the corresponding instant form model.

The model presented here has many more parameters than the version given in Ref. [16], which of course is partly due to the fact that the present model extends the upper limit of the fit from $T_{\pi\text{lab}} = 0.7$ GeV to $T_{\pi\text{lab}} = 1.0$ GeV. Also, in the 0–0.7 GeV range there are four resonances, while in the

TABLE III. Resonance parameters.

Resonance	m_{0B} (MeV)	C_{1B} (fm^{-1})	β_{1B} (fm^{-1})	K_{1B}	C_{2B} (fm^{-1})	β_{2B} (fm^{-1})	K_{2B}
$S_{11}(1535)$	1152.32	2326.58	26.8145	2	23562.6	0.787567	2
$S_{11}(1650)$	1637.43	234.645	2.95343	1	1233.88	0.143372	1
$S_{31}(1620)$	1880.38	442.095	4.4564	3	2131.29	1.43694	4
$P_{11}(938)$	1031.42	186.246	2.88064	4	−453.457	10.2076	4
$P_{11}(1440)$	1157.73	235.996	37.8333	2	1759.86	1.62646	2
$P_{11}(1710)$	2522.56	−1032.87	12.0482	4	1991.89	2.5982	4
$P_{13}(1720)$	2036.37	293.686	13.9169	2	−341.716	5.43274	2
$P_{31}(1744)$	1774.31	166.97	21.5643	2	−375.29	3.47007	2
$P_{33}(1232)$	1302.96	−221.928	2.66403	3	619.428	2.07256	3
$P_{33}(1600)$	1686.34	594.243	2.42704	1	2959.12	0.604585	1
$D_{13}(1520)$	1528.38	456.701	4.29899	4	92.6454	3.58040	2
$D_{15}(1675)$	1725.81	321.648	4.69012	3	437.185	3.90871	3
$D_{33}(1700)$	1286.32	865.209	8.48629	3	1220.29	0.933036	1

0.7–1.0 GeV range there are eight resonances. Besides these obvious reasons for an increase in complexity, there is another reason having to do with a shift in approach. In Ref. [16] it was possible to account for the $S_{11}(1535)$, $P_{11}(1440)$, and $D_{13}(1520)$ resonances through coupling to the inelastic channels, i.e., ηN, $\pi\Delta$, and $\pi\Delta$, respectively. No single-baryon states corresponding to these resonances were included. Upon attempting to extend this approach to higher energies, we found that we were unable to do so. For example, in the S_{11} partial wave it is also necessary to account for the $S_{11}(1650)$ resonance. The coupling to the $\pi\Delta$ channel does not lead to this second resonance, so it was necessary to include a single-baryon state corresponding to this resonance. This, however, led to a deterioration of the fit at lower energies, which necessitated introducing a single baryon state corresponding to the $S_{11}(1535)$ resonance. Also it was found that a rather delicate interplay between the two S_{11} single-baryon states was necessary to account for the S_{11} inelasticity above 0.7 GeV. Upon comparing Fig. 1 of the present work with Fig. 1 of Ref. [16], it is seen that the cusplike behavior of the S_{11} phase shift near the threshold for the ηN channel that occurs in the model of Ref. [16], does not appear in the present model. This threshold occurs at a total c.m. energy of 1487 MeV, which is very close to the location of the $S_{11}(1535)$ resonance. This probably accounts for the sensitivity of the phase shifts near this threshold to the mechanism used to produce the $S_{11}(1535)$ resonance.

We also found that coupling to the inelastic channel could not produce both the $P_{11}(1440)$ and $P_{11}(1710)$ resonances, and therefore found it necessary to introduce single-baryon states corresponding to these resonances. Here the interplay between the two P_{11} single-baryon states played an important role in accounting for the flatness of the inelasticity above 0.7 GeV. Our experience suggests that is necessary to include explicitly the Roper resonance, i.e., the $P_{11}(1440)$, in order to get a good fit to the data, but further analysis is necessary to arrive at a firm conclusion. In all of the other partial waves it was also found necessary to include single-baryon states corresponding to the resonances in order to obtain high quality fits. In fact, as Table II shows, it was possible to obtain good results in the P_{11}, P_{33}, D_{15}, and D_{33} partial waves using only the resonances.

Another reason for the increase in the number of parameters has to do with the fact that all inelastic processes in a particular partial wave are taken into account by a single, two-particle channel. This is clearly an oversimplification. In an isobar model analysis of $\pi N \to \pi\pi N$ for total c.m. energies in the range 1.32 to 1.93 GeV, Manley *et al.* [18] found it necessary to treat the inelasticity as arising from a coherent superposition of the two-body channels $\pi\Delta$, ρN, ϵN, and πN^*. Here ϵ denotes the strong s-wave isoscalar $\pi\pi$ interaction, and N^* denotes the Roper resonance. In a more recent multichannel resonance parametrization of πN scattering, Manley and Saleski [19] extended this set of inelastic two-body channels to include ηN, $K\Lambda$, ωN, and $\rho\Delta$ channels. In the model of Ref. [16] it was only necessary to account for inelasticity in the S_{11}, S_{31}, P_{11}, and D_{13} partial waves, and over a significantly smaller energy range than here. In Ref. [16] an effective $\pi\Delta$ channel was used to account for the inelasticity in the S_{31}, P_{11}, and D_{13} partial waves, and the effective Δ mass was simply taken to be $m_\Delta = m_\pi + m_N$, so as to put the inelastic threshold at the correct place, i.e., $2m_\pi + m_N$. Here, as the work of Manley *et al.* [18,19] shows, we are in an energy range for which in principle, several inelastic, two-body channels come into play. Since it was not practical to include them all in our model, we compromised in the S_{31} and P_{31} partial waves by adjusting the effective m_Δ so as to optimize our fits. The effective P_{31} m_Δ came out rather close to $m_\pi + m_N$, but the S_{31} m_Δ came out significantly larger. The analysis of Manley and Saleski [19] finds branching fractions for the decay of the $S_{31}(1620)$ resonance to πN, $\pi\Delta$, and ρN channels of 9, 62, and 29 %, respectively. So our high value for the S_{31} m_Δ may be compensating for our omission of the ρN channel. It is worth noting that in an analysis of $\pi N \to \eta N$ data, Batinić *et al.* [20] used an effective two-particle channel to account for all inelastic channels other than the ηN channel, and they also found it necessary to vary the threshold energy for the effective two-particle channel in order to obtain decent fits.

An important lesson that we have learned from fitting our model to the πN elastic scattering amplitudes, is that it is difficult to pin down the nature of the πN resonances. It is worth noting that this problem is an old one. In the original Chew-Low model [21] it was possible, by an appropriate choice of the cutoff function, to reproduce the $P_{33}(1232)$ resonance without introducing a single-baryon state corresponding to this resonance. Nowadays, most people agree that it is three-quark state, and would approve of our including a single-baryon state corresponding to this resonance. At higher energies the importance of the inelastic channels makes it even more difficult to pin down the nature of the resonances, i.e., are they essentially dressed versions of the single-baryon states, or are they the result of coupling to the inelastic channels? Clearly both mechanisms come into play, but it is difficult to establish their relative importance. An analysis of the inelastic cross sections, as well as calculations of meson photoproduction and electroproduction, should help to minimize the ambiguities.

As it stands our model can be used in calculations of meson photoproduction from nucleons, by a relatively straightforward extension of the method of Nozawa *et al.* [13]. The present model should lead to an improvement over the earlier work in that it includes coupling to inelastic channels, as well as an accurate description of the many resonances in the energy range considered. We are presently pursuing this application of our model.

Some time ago Betz and Coester [22] developed an exactly Poincaré invariant model of the $NN\pi$ system which takes into account cluster separability. The model was subsequently applied by Betz and Lee [23], and shown to give a satisfactory description of pion absorption by deuterons, and of elastic pion-deuteron scattering for pion lab kinetic energies up to about 300 MeV. The formalism [22] allows for the treatment of a vertex such as $N\pi \Leftrightarrow \Delta$, so it can accommodate the type of model for the πN system that has been presented here. The original development of the formalism [22] was in the *instant form* of relativistic quantum mechanics, but it can be easily adapted to the *front form* as well. In general three-particle models such as that of Betz and Coester [22], which are based on a Bakamjian-Thomas construction of a mass operator [24,25], lead to somewhat cumbersome square root operators that can be difficult to treat numerically. Separable models for the two-particle subsystems, which of course can be solved analytically, can alleviate this difficulty to a large extent.

It is of course desirable to see if it is possible to develop an exchange model of the πN system within the three-dimensional framework employed here. Since the coupling constants and the masses of the exchanged particles appear in other contexts, there are fewer arbitrary parameters than in the type of model presented here. The method for constructing exchange models in the various forms of relativistic quantum mechanics exists. One of us (M.G.F.) has shown that Okubo's formalism [26] can be used to develop instant and front form mass operators starting from field theory vertices [27,28]. The method has been successfully applied to the NN system [27], as well as to a limited model of the πN system [28], and we are now applying it to the πN system in the energy range considered here. We are hoping that the more constrained nature of an exchange model will help in determining the nature of the πN resonances.

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