

A generalized Gibbs equation for nuclear matter out of equilibrium

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We propose a generalized Gibbs equation for nuclear matter in the presence of a heat flux or a viscous pressure, by using the formalism of extended irreversible thermodynamics. We study the consequences on the equations of state, with special emphasis on the compressibility. The results may be useful in the analysis of nuclear collisions. [S0556-2813(98)05804-X]

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The equations of state are usually defined in equilibrium states, where thermodynamics has well-known foundations. However, in many situations, the measurements are carried out in situations rather far from equilibrium. For instance, the equations of state of nuclear matter are usually checked in nuclear collisions, which are very far from equilibrium. In such situations, nonequilibrium contributions may modify the equations of state, in such a way that what is interpreted as a given parameter in the equilibrium equation of state is, in fact, not the value of such parameter in equilibrium but a value modified by nonequilibrium contributions.

Recently, Fai and Danielewicz [1] have taken into account some nonequilibrium corrections arising in the framework of extended irreversible thermodynamics (EIT) [2–4] in order to outline their possible influence on the dispersion in the experimental values of nuclear compressibility. In their analysis, Fai and Danielewicz used for the nonequilibrium entropy an expression corresponding to ideal gases [5]. Therefore, it is normal to ask how their results would be modified by using a more suitable generalized Gibbs equation for nuclear matter.

One of the most relevant parameters in the equation of state for nuclear matter is the compressibility, defined as

$$K = 9 \left(\frac{\partial p}{\partial \rho} \right)_T, \quad (1)$$

where p is the pressure and ρ the mass density. The numerical factor is due to historical reasons. The value of K may be obtained experimentally in several ways: from the giant monopole resonance, from nuclear collisions, or from the explosion of supernovae. The corresponding results exhibit a remarkably wide dispersion, as they are typically comprised between 165 and 220 MeV [1], or between 190 and 240 MeV [6]. Thus the characteristic uncertainty is of the order of 30 MeV, i.e., $K = 190 \pm 30$ MeV [1] or $K = 210 \pm 30$ MeV [6]. This does not mean that the uncertainty in each measurement is so large. For instance, the values obtained for K by several groups in 1981 were [6] $K = 213 \pm 2$ MeV (Grenoble), $K = 231 \pm 4$ MeV (Oak Ridge), $K = 222 \pm 3$ MeV (Orsay), to mention only a few examples. Thus the uncertainty in the measurements is in fact much lower than the dispersion in the average values obtained by the different groups.

In 1981, Treiner *et al.* [6] examined this problem from the point of view of equilibrium statistical mechanics to take into account the finite size of nuclei in contrast with the infinite size corresponding to nuclear matter in the thermodynamic limit. Indeed, for finite nuclei one should take into account surface effects, several asymmetries, and, furthermore, Coulombian contributions due to the electrical charge of protons. Thus one should make a distinction between K_A , the compressibility obtained for a nucleus of massic number A , and the compressibility K of nuclear matter, which should be the limit of K_A when A tends to infinity. This model may explain a part of the discrepancies for nuclei with different A .

A different problem arises when one compares the values for K obtained from nuclei with a given value of A from different experimental methods. Again, there is a wide dispersion which cannot be explained by the former static analyses. Since in nuclear collisions nuclear matter is far from equilibrium, it is quite logical to study how nonequilibrium effects could modify the effective value of K . Fai and Danielewicz [1] have adopted the point of view of EIT to take account of some dynamical factors not included in the usual local-equilibrium equations of state.

EIT is a recent thermodynamic theory [2–4] which assumes that in nonequilibrium situations the entropy is not equal to the local-equilibrium entropy, i.e., to the local version of the entropy of equilibrium thermodynamics, but that it is also influenced by some nonequilibrium variables, in particular the dissipative fluxes (such as heat flux \mathbf{q} or the viscous pressure \mathbf{P}^ν). The generalized entropy s of EIT per unit mass in presence of nonvanishing viscous pressure tensor \mathbf{P}^ν has the form [2–4]

$$s(u, \nu, \mathbf{P}^\nu) = s_{\text{eq}}(u, \nu) - \frac{\tau_2}{4\eta\rho T} \mathbf{P}^\nu : \mathbf{P}^\nu, \quad (2)$$

with η the shear viscosity, τ_2 the relaxation time of \mathbf{P}^ν describing viscoelastic effects, T the local-equilibrium absolute temperature, and $s_{\text{eq}}(u, \nu)$ the local-equilibrium entropy. By differentiation of (2) one may obtain generalized equations of state for the nonequilibrium temperature θ or the nonequilibrium pressure π , as we will discuss in more detail below.

Fai and Danielewicz [1] proposed to use in (1), instead of the local-equilibrium pressure p , the nonequilibrium pressure π , which in the simplest version is related to p as [1–5]

$$\pi = p + \rho^2 \frac{\partial \beta}{\partial \rho} \mathbf{P}^\nu : \mathbf{P}^\nu, \quad (3)$$

with $\beta = \tau_2(4\eta\rho)^{-1}$. Furthermore, they assumed that $\tau_2 \approx (\rho\sigma\nu)^{-1}$, with σ the nucleon-nucleon collision cross section and ν the average thermal speed of the particles, and $\eta \approx m\nu/3\sigma$. One then gets $\pi = p - 2\beta\rho\mathbf{P}^\nu : \mathbf{P}^\nu$. When one introduces in (1) the expression (3) for π instead of the local-equilibrium pressure p , one gets

$$K = 9 \left(\frac{\partial \pi}{\partial \rho} \right)_T = K_0 + \frac{9}{n^2 T m} P_{12}^{\nu 2}, \quad (4)$$

where we have set the Boltzmann constant equal to one, i.e., T is measured in energy units rather than in temperature units, K_0 is the equilibrium value of K , and P_{12}^ν is the pure shear component of \mathbf{P}^ν . From the results of recent simulations for the collision Au+Au at 400 MeV/nucleon, one has the following values for the parameters involved [1]: γ (shear rate) $\approx 0.07 c \text{ fm}^{-1}$, $\rho \approx 0.30 \text{ fm}^{-3}$, and $T \approx 44 \text{ MeV}$, with c being the speed of light. The value of the shear viscosity at these values of ρ and T is $\eta \approx 55 \text{ MeV/fm}^2 c$. When these values are used and when one takes into account $P_{12}^\nu = \eta\gamma$, one finds from (4) that $K - K_0 \approx 20 \text{ MeV}$. It is thus seen that the EIT corrections are not negligible, and therefore it is worthwhile to explore them carefully.

The expression of the generalized nonequilibrium entropy taking into account the contribution of the heat flux \mathbf{q} and of the viscous pressure tensor \mathbf{P}^ν up to the second order in the fluxes is [2–4]

$$s(u, \nu, \mathbf{q}, \mathbf{P}^\nu) = s_{\text{eq}}(u, \nu) - \frac{\tau_1}{2\lambda\rho T^2} \mathbf{q} \cdot \mathbf{q} - \frac{\tau_2}{4\eta\rho T} \mathbf{P}^\nu : \mathbf{P}^\nu, \quad (5)$$

with τ_1 and λ the relaxation time of the heat flux and the thermal conductivity respectively. When one introduces in (5) the information for τ_1/λ and τ_2/η for the system at hand, one has explicit expressions for the entropy of the system. Up to now, expression (5) has been checked successfully by comparison with microscopic evaluations of the entropy for ideal and nonideal gases, dilute polymer solutions, elastic solids, electromagnetic radiation, and electronic devices [2]. Here, we will use it for the first time for nuclear matter.

Fai and Danielewicz [1] used in their analysis the expression (5) evaluated for ideal gases. The results would be more faithful if one was able to compute them from a more realistic model for the entropy of nuclear matter. To do that, one needs results for the transport coefficients λ and η and for the relaxation times τ_1 and τ_2 . Danielewicz [7] has obtained from a Chapman-Enskog expansion of the Uhlenbeck-Uehling equation (the quantum version of the Boltzmann equation) the following expressions:

$$\eta = \frac{1700}{T^2} \left(\frac{n}{n_0} \right)^2 + \frac{22}{1 + 10^{-3} T^2} \left(\frac{n}{n_0} \right)^{0.7} + \frac{5.8 T^{1/2}}{1 + 160 T^{-2}}, \quad (6)$$

$$\lambda = \frac{0.15}{T} \left(\frac{n}{n_0} \right)^{1.4} + \frac{0.02}{1 + 10^{-6} (T^4/7)} \left(\frac{n}{n_0} \right)^{0.4} + \frac{0.0225 T^{1/2}}{1 + 160 T^{-2}}, \quad (7)$$

where T is given in MeV, η in $\text{MeV/fm}^2 c$, λ in c/fm^2 , and where one takes for the reference number density $n_0 \approx 0.15 \text{ fm}^{-3}$. Note that for T approaching zero, Eqs. (6) and (7) diverge (see [7] for a detailed discussion).

For the corresponding relaxation times τ_2 of the viscous pressure and τ_1 of the heat flux, Danielewicz found [7]

$$\tau_2 = \frac{850}{T^2} \left(\frac{n}{n_0} \right)^{1/3} \left[1 + 0.04 T \frac{n_0}{n} \right] + \frac{38 T^{-1/2}}{1 + 160 T^{-2}} \frac{n_0}{n}, \quad (8)$$

$$\tau_1 = \frac{310}{T^2} \left(\frac{n}{n_0} \right)^{0.4} \left[1 + 0.1 T \frac{n_0}{n} \right] + \frac{57 T^{-1/2}}{1 + 160 T^{-2}} \frac{n_0}{n}, \quad (9)$$

where τ_i are expressed in fm/c . The second term in the first brackets corresponds to binary collisions amongst nucleons, whereas the first term corresponds to collisions of nucleons with the surface of the nuclei. For $T > 6 \text{ MeV}$, the collisions amongst nucleons are dominant, while for lower temperatures the collisions with the surface dominate. A relaxation-time model has been used by Köhler [8] for the analysis of the transport coefficients of nuclear matter. His results, previous to those obtained by Danielewicz, are not so detailed as the latter ones.

We may combine (6)–(9) with (5) to give an explicit expression for the nonequilibrium entropy. Since the general expression would be rather cumbersome, we specify the results for high temperature, which is the regime in which Fai and Danielewicz carried out their analysis in [1]. One has

$$s = s_{\text{eq}} - \frac{1.26}{m T^3} 10^3 \frac{n_0}{n^2} \mathbf{q} \cdot \mathbf{q} - \frac{1.64}{m T^2} \frac{n_0}{n^2} \mathbf{P}^\nu : \mathbf{P}^\nu. \quad (10)$$

Here, we study the nonequilibrium corrections to temperature and pressure due to \mathbf{P}^ν , and its consequences on the compressibility. As well as Fai and Danielewicz [1], we specialize our attention to a system submitted to a viscous pressure P_{12}^ν [in (10) we have also given the contributions of \mathbf{q} for completeness, but we neglect them here]. The generalized entropy may be written as

$$s = s_{\text{eq}} - \frac{3.28}{m T^2} \frac{n_0}{n^2} P_{12}^{\nu 2}, \quad (11)$$

where we have taken into account that in shear flow $\mathbf{P}^\nu : \mathbf{P}^\nu = 2 P_{12}^\nu P_{12}^\nu$.

The caloric equation of state for the temperature, i.e.,

$$\theta^{-1} = \left(\frac{\partial s}{\partial u} \right)_{\nu, \nu P_{12}}, \quad (12)$$

may be calculated explicitly if we take into account that $du = c' dT$, with c' the specific heat per unit mass, which is given in this limit as $c' = (3/2)m^{-1}$ (recall that we set the Boltzmann constant equal to unit). Thus we find for the nonequilibrium temperature θ the expression

$$\theta^{-1} = T^{-1} \left[1 + 4.36 \frac{n_0}{n^2 T^2} P_{12}^{\nu^2} \right]. \quad (13)$$

The conceptual difference between θ and T has been discussed in detail in [9]. We may assess the relative importance of these corrections for the collision Au+Au analyzed by Fai and Danielewicz, and we obtain that $\theta^{-1} \approx 1.065T^{-1}$. Thus the relative nonequilibrium contribution to the temperature is of the order of 6.5%, much higher than the 0.1% corrections obtained by Jou and Casas-Vázquez [9] for CO₂ at 300 K and 0.1 atm and for a heat flux of the order of 10^5 W cm^{-2} .

Furthermore, we may compute the nonequilibrium corrections to the pressure from the definition

$$\frac{\pi}{\theta} = \left(\frac{\partial s}{\partial v} \right)_{u, \nu P_{12}}. \quad (14)$$

Note that we have taken into account that the quantity to be held constant during the differentiation is νP_{12}^{ν} rather than P_{12}^{ν} itself, because of the extensive character of the former [2]. Expression (14) yields $\pi/\theta = p/T$. Therefore, from (13) and (14) we finally obtain

$$\pi = \frac{\theta}{T} p = p \left[1 + 4.36 \frac{n_0}{n^2 T^2} P_{12}^{\nu^2} \right]^{-1}. \quad (15)$$

When (15) is introduced in (4) and when we have taken into account the value $n_0 = 0.15 \text{ fm}^{-3}$ used in expressions (6)–(9) we obtain

$$K = K_0 + \frac{5.89}{mn^2 T} P_{12}^{\nu^2}. \quad (16)$$

The numerical coefficient 5.89 of the nonequilibrium correction is lower than the value 9 in expression (4). Our result corroborates qualitatively the results by Fai and Danielewicz, but it indicates that the use of a more accurate Gibbs equation is necessary to yield quantitative predictions.

The reader must be warned that the uncertainties in the nuclear properties such as viscosity and relaxation time are rather high, and that the details of the flows produced during the collisions are also uncertain. For instance, Danielewicz [10] and Köhler [11] estimated that quantum corrections to relaxation effects may be a factor of two. The purpose of the present Brief Report is not so much to provide a final, reliable value to the nonequilibrium corrections, for which a consideration of the heat flux would also be necessary, but to point out that in spite of the fact that Fai and Danielewicz used in [1] a too simplified model for the nonequilibrium entropy, the order of magnitude of their estimate is corroborated in a more detailed model. Nevertheless, for more reliable quantitative predictions, the Gibbs equations proposed in this report should be used, instead of the simplified ideal gas approximation.

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