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# Nuclear dependence of the coherent $\eta$ photoproduction reaction in a relativistic approach

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We study the nuclear (or $A$) dependence of the coherent $\eta$ photoproduction reaction in a relativistic impulse approximation approach. We use a standard relativistic parametrization of the elementary amplitude, based on a set of four Lorentz- and gauge-invariant amplitudes, to calculate the coherent production cross section from $^4$He, $^{12}$C, and $^{40}$Ca. In contrast to nonrelativistic treatments, our approach maintains the full relativistic structure of the process. The nuclear structure affects the process through the ground-state tensor density. This density is sensitive to relativistic effects and depends on $A$ in a different manner than the vector density used in nonrelativistic approaches. This peculiar dependence results in $^4$He having a cross section significantly smaller than that of $^{12}$C—in contrast to existent nonrelativistic calculations. Distortion effects are incorporated through an $\eta$-nucleus optical potential that is computed in a simple ''$t\rho$'' approximation.
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The nuclear dependence of the coherent $\eta$ photoproduction process offers a unique opportunity to investigate medium modifications to the elementary $\gamma N \rightarrow \eta N$ amplitude and might help distinguish between different theoretical models that provide an equally good description of the elementary process. In particular, the role played by the background is very significant in spin-isospin saturated nuclei where the dominant resonance $S_{11}(1535)$ is suppressed. Furthermore, this reaction contributes significantly to our understanding of nucleon-resonance formation and sheds some light on the propagation of these resonances through the nuclear medium. The $A$ dependence of this reaction is affected by the propagation of the produced $\eta$ meson through its interaction with the nucleus. Moreover, the coherent process is sensitive to the whole nuclear volume and, thus, depends on bulk properties of the nucleus. While nonrelativistic treatments suggest that nuclear-structure effects manifest themselves through the conserved vector (or baryon) density [1–3], our recent relativistic analysis suggests that, rather, it is the tensor density that affects the process [4]. This represents an important result, since the tensor density—a quantity as fundamental as the vector density—is not well determined by experiment.

An early nonrelativistic study by Bennhold and Tanabe of the coherent $\eta$ photoproduction process predicted $^4$He to have the largest cross section of the three nuclei $^4$He, $^{12}$C, and $^{40}$Ca [1]. A recent nonrelativistic study of this process seems to confirm this earlier prediction, although important quantitative differences do emerge [3]. In nonrelativistic treatments the coherent cross section is proportional to the square of the Fourier transform of the vector density. Thus, the particular $A$ dependence predicted by these calculations emerges from a competition between $A$, which tends to increase the cross section for larger nuclei, and the vector form factor—which falls rapidly with $A$. This competition results in $^4$He having the largest cross section. Theoretical studies of this kind have motivated considerable experimental interest, which have culminated in an attempt to measure the coherent $\eta$ photoproduction cross section from $^4$He at the Mainz Microtron facility [5]. Possibilities for extensions to higher energies and other nuclei exist, both at the Bonn ELSA facility and at TJNAF.

In this report, as well as in our previous work on this subject [4], we have used a relativistic approach to study this process. At no point in our calculation do we resort to a nonrelativistic reduction of the elementary amplitude or of the nuclear-structure model. Our results are in sharp contrast with the nonrelativistic predictions. Indeed, we find the coherent cross section from $^{12}$C as the largest, while that of $^4$He as the smallest one of the three. This is due to the relativistic character of the tensor—not the vector—density, which is the fundamental nuclear-structure quantity driving the reaction. It is this peculiar dependence of the tensor density with $A$ that is novel to our approach. Although the tensor density determines the qualitative behavior of the cross section with $A$, its quantitative behavior is determined by our choice of elementary amplitude. The elementary amplitude we have used in this work [7,8] provides an excellent description of all available data on $\gamma p \rightarrow \eta p$ as well as the ones on $\gamma n \rightarrow \eta n$ as inferred from the very recent experiments on the deutron.

The relativistic formalism for the coherent $\eta$ photoproduction reaction, has been developed in our earlier work [4]. Thus, we will only reiterate here some of the main aspects of the formalism. The differential cross section in the center-of-momentum frame (c.m.) computed in a relativistic impulse-approximation approach is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm c.m.} = \mathcal{K}|F_0(s,t)|^2, \tag{1}$$

where

$$\mathcal{K} \equiv \left(\frac{M_T}{4\pi W}\right)^2 \left(\frac{q_{\rm c.m.}}{k_{\rm c.m.}}\right) \left(\frac{1}{2} k_{\rm c.m.}^2 q_{\rm c.m.}^2 \sin^2\theta_{\rm c.m.}\right), \tag{2}$$

 

is a kinematical factor, and $M_T$ is the mass of the target nucleus. Note that $W$, $\theta_{\text{c.m.}}$, $k_{\text{c.m.}}$, and $q_{\text{c.m.}}$ are the total energy, scattering angle, photon, and $\eta$-meson momenta in the c.m. frame, respectively. Hence, all dynamical information about the coherent process is contained in the single Lorentz-invariant form factor $F_0(s,t)$; this form factor depends on the Mandelstam variables $s$ and $t$.

The Lorentz-invariant form factor $F_0(s,t)$, is computed in a relativistic impulse-approximation approach. We use a standard, model-independent parametrization of the elementary $\gamma N \rightarrow \eta N$ amplitude. This elementary amplitude is given in terms of four Lorentz- and gauge-invariant amplitudes [1,6]. In nonrelativistic approaches it has been customary to evaluate this amplitude between on-shell Dirac spinors, thereby leading to the well-known Chew-Goldberger-Low-Nambu (CGLN) form of the elementary amplitude. In this work we do not resort to such a nonrelativistic reduction. Rather, we preserve the full relativistic content of the elementary amplitude and of the nuclear-structure model. In this way possible medium modification to the elementary process—that may arise from a different ratio of upper-to-lower components—can be examined.

For closed-shell (spin-saturated) nuclei a significant simplification occurs, as the coherent process becomes sensitive to only one component of the elementary amplitude. In addition, all the nuclear-structure information is contained in the ground-state tensor density [9]. Thus, the Lorentz-invariant form factor—computed in a relativistic plane-wave impulse approximation (RPWIA) takes the following simple form:

$$F_0^{\text{PW}}(s,t) = iA_1(\tilde{s},t)\rho_T(Q)/Q. \tag{3}$$

In this expression $\tilde{s}$ represents the effective (or optimal) value of the Mandelstam variable $s$ at which the elementary amplitude should be evaluated [10] and $Q \equiv |\mathbf{k}_{\text{c.m.}} - \mathbf{q}_{\text{c.m.}}| \simeq \sqrt{-t}$. The ground-state tensor density is defined by

$$[\rho_T(r)\hat{r}]^i = \sum_{\alpha}^{\text{occ}} \bar{\mathcal{U}}_\alpha(\mathbf{x})\sigma^{0i}\mathcal{U}_\alpha(\mathbf{x}), \tag{4}$$

where $\mathcal{U}_\alpha(\mathbf{x})$ are the relativistic Dirac spinors. Note that in a relativistic plane-wave formalism the cross section is sensitive only to the Fourier transform of the tensor density, i.e.,

$$\rho_T(Q) = 4\pi \int_0^\infty dr\ r^2 j_1(Qr)\rho_T(r). \tag{5}$$

It is this tensor density that constitutes the fundamental nuclear-structure quantity in this work. This is in contrast to nonrelativistic treatments that, instead, use the vector density [1,2,10–12]. The tensor density is a manifestation of the relativistic character of this approach.

We have computed the tensor density using a self-consistent, mean-field approximation to the Walecka QHD-I (or $\sigma$–$\omega$) model [9]. We do not expect our calculations to be very sensitive to the uncertainties in the relativistic model. Indeed, we have tested the sensitivity of our results by performing calculations with another version of the Walecka model (QHD-II) where the $NN$ interaction is mediated by the exchange of $\gamma$ and $\rho$, in addition to the exchange of $\sigma$ and $\omega$ mesons. The results of QHD-I and QHD-II are within ten percent of each other.

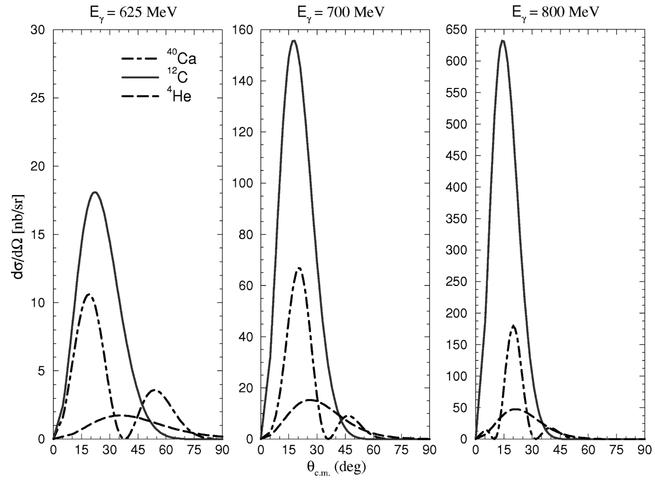


FIG. 1. The coherent $\eta$ photoproduction cross section as a function of $A$ for photon laboratory energies of $E_\gamma = 625$, 700, and 800 MeV, respectively. All results were obtained using a RPWIA approach.

Even though the use of a mean-field approximation to describe a nucleus as small as $^4$He should be suspect, we feel justified in adopting this choice, as the coherent reaction is sensitive to only its bulk properties—which can be constrained by experiment. Thus, in order to reproduce the experimental charge density of $^4$He, we have modified the mass of the $\sigma$ meson to $m_s = 564$ MeV—while maintaining constant the ratio of $g_s^2/m_s^2$. Note that we have used a standard set of parameters for the Walecka model in our calculations of the $^{12}$C and $^{40}$Ca nuclear structures: $g_s^2 = 109.63$, $g_v^2 = 190.43$, $m_s = 520$ MeV, and $m_v = 783$ MeV. Finally, to achieve a more realistic picture of this process, the plane-wave picture is modified by introducing interactions (distortions) between the outgoing $\eta$ and the nucleus. This is achieved by using a $\eta$-nucleus optical potential of the $t\rho$ form [1,4]. These distortions are sensitive to the ground-state vector density of the target nucleus.

The coherent $\eta$ photoproduction differential cross section from $^4$He, $^{12}$C, and $^{40}$Ca is shown in Fig. 1 at photon laboratory energies of 625, 700, and 800 MeV, respectively. Moreover, the total cross section as a function of the photon energy is shown in Fig. 2 for the same nuclei. No distortions have been included in these calculations. These results display significant relativistic corrections; there is a large enhancement of these cross sections relative to the nonrelativistic ones found in Refs. [1,3]. This ''$M^\star$ effect'' is a direct consequence of the enhancement of the lower component of the Dirac spinor—which is determined dynamically, rather than from a free-space relation. Moreover, there is an additional relativistic contribution for open-shell nuclei, such as $^{12}$C; note that we are treating $^{12}$C as a closed $p^{3/2}$ but open $p^{1/2}$ orbital. This can be most easily seen by assuming a free-space relation between the upper and lower components of the Dirac spinors. In this case the tensor density can be written in terms of the vector density as

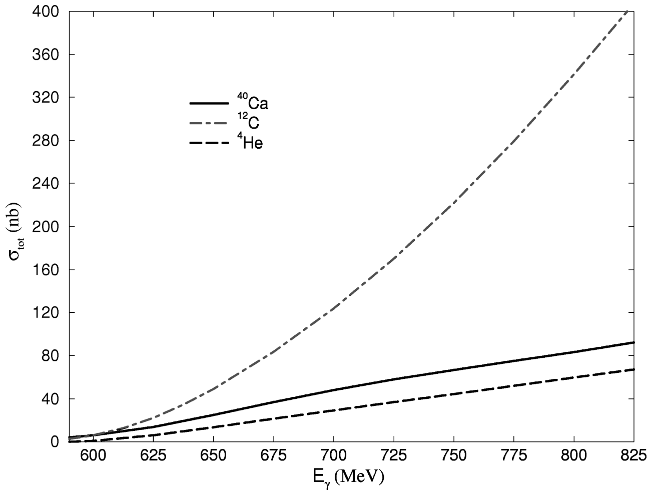


FIG. 2. The total coherent $\eta$ photoproduction cross section as a function of the incident photon laboratory energy from $^{40}$Ca, $^{12}$C, and $^{4}$He. All results were obtained using a RPWIA approach.

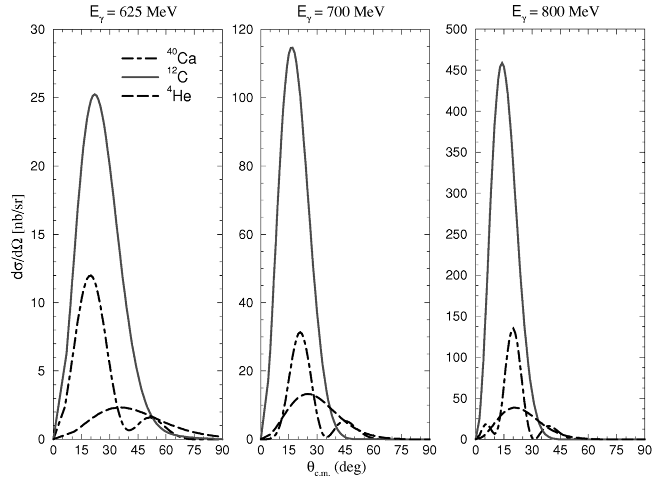


FIG. 3. The coherent $\eta$ photoproduction cross section as a function of $A$ for photon laboratory energies of $E_\gamma = 625$, 700, and 800 MeV, respectively. All results were obtained using a RDWIA approach.

$$\rho_T(Q) = -\frac{Q}{2M_N}\rho_V(Q) + \sum_{\alpha}^{\text{occ}} \frac{\kappa+1}{M_N}\int_0^\infty dr\, \frac{g_\alpha^2(r)}{r^2} j_1(Qr), \tag{6}$$

where $M_N$ is the free nucleon mass, $\kappa$ is the generalized relativistic angular momentum, $g_\alpha(r)$ is the upper component of the Dirac spinor, and $j_1(Qr)$ is the Bessel function of order 1. The second term in the above expression is negligible for closed-shell nuclei; this term is proportional to the difference between the square of the wave functions of spin-orbit partners (such as $p^{3/2}$ and $p^{1/2}$ orbitals) which is very small even in the Walecka model. Hence, for closed shell nuclei—and adopting a free-space relation—the tensor density becomes proportional to the vector density, as in the nonrelativistic approach. However, for open-shell nuclei such as $^{12}$C, the second term in Eq. (6) is no longer negligible and leads to an additional enhancement of the tensor density—above and beyond the one obtained from the $M^\star$ effect.

In Fig. 3 the cross section from the same three nuclei is displayed with distortions added to the emitted $\eta$ by using a relativistic distorted-wave-impulse approximation (RDWIA). The optical potential for the $\eta$ meson was obtained from the coupled channel calculations of Bennhold and Tanabe [1]. Since at low energy (e.g., $E_\gamma = 625$ MeV) the real part of the optical potential is attractive, its competition with the (absorptive) imaginary part produces a distorted-wave cross section that differs little from its plane-wave value. However, at higher energies the real part becomes repulsive, leading to a substantial reduction in the value of the cross section. For a small nucleus such as $^{4}$He the effect of distortions are less pronounced than in $^{12}$C and in $^{40}$Ca. This is consistent with the standard picture that emerges from nonrelativistic calculations [1].

Our relativistic results differ significantly from those obtained in nonrelativistic calculations (for a quantitative comparison between the models see Ref. [4]). Indeed, Bennhold and Tanabe [1] have predicted that $^{4}$He would have the largest cross section of the three nuclei, due to its largest charge form factor. This, we believe, might have been an important reason to select $^{4}$He for the first experimental measurement of the coherent process. However, this finding is at odds with our relativistic results, which instead show $^{4}$He to have the smallest cross section as can be seen in Figs. 1 and 2. There are two main reasons for these differences. First, in relativistic calculations the ratio of upper-to-lower components is determined dynamically in the Walecka model, rather than from its free-space relation. The Walecka model is characterized by strong scalar and vector potentials that generate an enhancement in the lower component of the wave function and a corresponding enhancement in the tensor density. Second, the elementary $\eta N$ interaction used in this work [7,8] is different from the one used by Bennhold and Tanabe [1], in particular the nonresonant contributions were not considered in the latter. Although both models seem to give an adequate description of the elementary process, important differences emerge in the calculation of the coherent reaction. This is primarily due to the fact that the coherent process from spin-saturated nuclei becomes insensitive to the dominant $S_{11}(1535)$ intermediate-resonance contribution, and therefore quite sensitive to the details of other resonant and nonresonant background contributions such as the $D_{13}(1520)$ and vector mesons. Note that our calculations for $^{4}$He are similar to the nonrelativistic ones reported recently by Fix and Arenhövel [3]. However, this agreement seems to be fortuitous, since neither their nuclear-structure model nor their elementary amplitude are similar to ours; their coherent process is dominated by $\omega$-meson exchange, while ours contains, in addition, a significant contribution from the $D_{13}(1520)$ resonance.

In conclusion, the main goal of our present work was to elucidate the $A$ dependence of the coherent $\eta$ photoproduction cross section in a relativistic impulse-approximation approach. We found the cross section sensitive to two nuclear-structure quantities: (i) the ground-state vector density and (ii) the ground-state tensor density. The tensor density is as fundamental as the vector density used in the nonrelativistic treatment, although it is not as well constrained by experiment.

We have found important discrepancies vis-a-vis nonrelativistic results. Part of these discrepancies stem from the fact that we have used a fully relativistic approach—with no resort to a nonrelativistic reduction. Moreover, the elementary amplitude used in our model is different from the ones used in other theoretical calculations [1,3]. Our relativistic approach suggests the use of the tensor density as the fundamental nuclear-structure quantity driving the reaction. Although our results are also sensitive to the vector density (through distortion effects) for a small nucleus such as $^4$He, or at low energies (where the real part of the optical potential is attractive) distortion effects become small and the relativistic cross section becomes dominated by the the tensor density. The tensor density, as opposed to the vector density, is sensitive to the relativistic corrections arising in the nuclear medium. The use of the tensor density represents one of the central results of our treatment.

Many challenges remain. First, one should try to study possible violations to the impulse-approximation picture. Second, there are off-shell ambiguities in the elementary amplitude. The form of the elementary amplitude used here is standard but not unique. There are many other choices which are equivalent on shell, but can give vastly different results off shell [6,12]. Although there are some attempts to deal with this issue [3], a detailed microscopic model is needed to take the amplitude off shell. Finally, as the coupling to the intermediate $S_{11}(1535)$ resonance dominates the elementary $\gamma N \to \eta N$—but not the coherent process—the coupling to additional resonances is poorly determined. Indeed, while Fix and Arenhövel [3] suggest a negligible $D_{13}(1520)$ contribution to the coherent process, our elementary model predicts a significant one.

Undoubtedly, there is still a lot of work to be done both experimentally and theoretically. We hope that with the advent of new powerful and sophisticated facilities, such as TJNAF and MAMI, the validity of the different theoretical models could be tested. This could help us elucidate the underlying mechanism behind the coherent $\eta$ photoproduction process.

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