PHYSICAL REVIEW C VOLUME 57, NUMBER 4 APRIL 1998

Polarized nuclear matter using a modified density dependent Seyler-Blanchard potential

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(Received 25 September 1997)

The binding energy of polarized nuclear matter with excess of neutrons, spin-up neutrons, and spin-up protons contains three symmetry energies: the spin symmetry energy, the isospin symmetry energy, and the spin-isospin symmetry energy. The potential used here for polarized nuclear matter is a modified density-dependent Seyler-Blanchard potential with an explicit dependence on the density of protons with spin-up and -down $\rho_{p\uparrow}$ and $\rho_{p\downarrow}$, also on the neutron density with spin-up and -down $\rho_{n\uparrow}$ and $\rho_{n\downarrow}$. It is found that the binding energy per particle, pressure, velocity of sound, and entropy agree very well with previous theoretical estimates. [S0556-2813(98)04103-X]

PACS number(s): 21.65.+f, 21.30.Fe

I. INTRODUCTION

The ground-state energy of nuclear matter with an excess of neutrons, spin-up neutrons, and spin-up protons was considered by Dabrowski and Haensel [1,2], using K-matrix method and applying the Brueckner-Gammel, Thaler, Hamada-Johnston, and soft-core Reid nucleon-nucleon potentials. Friedman and Pandharipande [3] reported a variational calculation of the equation of state for hot and cold nuclear and neutron matter. They covered a wide range of densities in heavy-ion collisions and astrophysics using a realistic interaction.

Another approach to study the properties of nuclear matter is by using effective interactions [4–6]. In atomic physics the mean field is produced by using electrostatic forces without any adjustable parameters. In the nuclear case the situation is different: the nucleon-nucleon interaction is not uniquely defined, and it contains adjustable parameters. That explains why there are many effective interactions and hundreds of parametrizations available. Myers and Swiatecki [7] used the Seyler-Blanchard (SB) [8] potential in nuclear matter studies which ensures saturation when used in conjunction with the Thomas-Fermi (TF) approximation. They found that it is simple enough to allow calculations without further approximations. Bandyopadhyay and Samaddar [9] proposed a modified SB potential, which contains a density-dependent term besides the old SB potential (this simulates three-body effects). This modified potential using the TF approximation reproduced successfully the symmetric nuclear matter properties and the real part of the nucleon-nucleus optical potential. Bandyopadhyay *et al.* [10] used their modified SB potential to study the thermostatic properties of finite and infinite nuclear systems. They found that the density dependence of the effective interaction is essential to reproduce the energy dependence of the single-particle potential correctly. Furthermore, they obtained a higher effective mass $m^*=0.63m$, which is more acceptable than that obtained using a density-independent SB potential. Rudra and De [11] used a density-dependent SB potential to study the equation of state of asymmetric nuclear matter and the binding energy of neutron matter. For nuclear matter their results agree with those calculated with the variational approach [3] especially in the region $5\rho_0<\rho<2\rho_0$. At higher densities their results give more repulsion. Recently, Myers and Swiatecki [12] added a reverse momentum-dependent $(1/P)$ term to the density-dependent SB potential. Such a modification gives an acceptable effective mass $m^*=0.71m$ and reproduced approximately the trend of the experimental data concerning the depth of the optical model potential. Other authors included the zero-range momentum and density-dependent Skyrme interactions, which offered some simplicity and also lead to analytic results for nuclear matter calculations. Most of the calculations using Skyrme forces on nuclear matter considered the symmetric or asymmetric case. In order to obtain the correct values for the symmetry energies of polarized nuclear matter (NM) using the Skyrme force, Dabrowski [5] and Mansour [13] proposed two different methods to add to the original density-dependent potential a term which depends on the four densities of $N\uparrow$ neutrons with spin up, $N\downarrow$ neutrons with spin down, $P\uparrow$ protons with spin up, and $P\downarrow$ protons with spin down. In this way they were able to reproduce the right values of the symmetry energies and their correct signs. In a previous work [14,15], the thermostatic properties of NM were calculated using an extended form of the SB potential which is suitable for polarized matter.

In the present work we shall propose a potential which depends explicitly on the four nucleon densities $\rho_{p\uparrow}$, $\rho_{p\downarrow}$, $\rho_{n\uparrow}$, and $\rho_{n\downarrow}$ in the different channels of spin-isospin space rather than changing the values of the strength of the interaction for polarized nuclear matter [14,15]. This is an alternative method to study polarized NM (see, e.g., Refs. [5] and [13]) instead of that suggested previously [14,15].

Hereby, we construct an explicit density-dependent SB potential which depends on $\rho_{p\uparrow}$, $\rho_{p\downarrow}$, $\rho_{n\uparrow}$, and $\rho_{n\downarrow}$. This is a good construction which is suitable for polarized nuclear matter. Our potential contains the old SB potential plus an explicit density-dependent term.

In the next section we present the theory, and in Sec. III we present the results and discussion.

II. THEORY

A. Polarized nuclear matter at zero temperature

Hereby we refer the reader to our previous work [15] where the method of calculation is explained. Polarized nuclear matter is composed of numbers $N\uparrow$ ($N\downarrow$) of spin-up (spin-down) neutrons and $P\uparrow$ ($P\downarrow$) of spin-up (spin-down) protons, with corresponding densities $\rho_{n\uparrow}$, $\rho_{n\downarrow}$, $\rho_{p\uparrow}$, and $\rho_{p\downarrow}$, respectively; thus,

$$A=N\uparrow+N\downarrow+P\uparrow+P\downarrow \tag{1}$$

is the total number of particles and the total density ρ is given by

$$\rho=\rho_n+\rho_p=\rho_{n\uparrow}+\rho_{n\downarrow}+\rho_{p\uparrow}+\rho_{p\downarrow}. \tag{2}$$

For polarized nuclear matter, we define the following parameters: the neutron excess parameter

$$X=(\rho_n-\rho_p)/\rho, \tag{3}$$

the neutron spin-up excess parameter

$$\alpha_n=(\rho_{n\uparrow}-\rho_{n\downarrow})/\rho, \tag{4}$$

and the proton spin-up excess parameter

$$\alpha_p=(\rho_{p\uparrow}-\rho_{p\downarrow})/\rho, \tag{5}$$

$$Y=\alpha_n+\alpha_p, \tag{6}$$

and

$$Z=\alpha_n-\alpha_p. \tag{7}$$

The modified density-dependent SB potential [11] which is used for asymmetric nuclear matter is defined as

$$V(r,k)=-C_{l,u}\,\frac{e^{-r/a}}{r/a}\left[1-\frac{p^2}{b^2}-d^2(\rho_1+\rho_2)^n\right], \tag{8}$$

where the subscripts l and u refer to like-pair (nn or pp) and unlike-pair (np) interactions, respectively. Here a is the range of the two-body interaction, b is a measure of the strength of repulsion with relative momentum P, d and n are two parameters determining the strength of the density dependence, and $\rho_1(r_1)$ and $\rho_2(r_2)$ are the densities at the sites of the two interacting nucleons. In a previous work [15], we extended such a potential to describe the case of polarized nuclear matter. In such a case the parameter $C_{L,u}$ is the strength of the interaction where L and u refer to the likeness L and unlikeness u for spin and isotopic spin, respectively [14,15].

In the present work we shall take the potential

$$V(r,k)=V_1^{\mathrm{SB}}(r,k)+V_2(r,\rho_{p\uparrow},\rho_{p\downarrow},\rho_{n\uparrow},\rho_{n\downarrow}), \tag{9}$$

or explicitly V_1^{SB} is the original SB potential defined as

$$V_1^{\mathrm{SB}}(r,k)=-C\,\frac{e^{-r/a}}{r/a}\left[1-\frac{p^2}{b^2}\right] \tag{10}$$

and V_2 depends explicitly on the densities of the interacting pair, e.g., for a proton of spin up and a neutron of spin down,

$$V_2(r,\rho_{p\uparrow},\rho_{n\downarrow})=\frac{e^{-r/a}}{r/a}\,[\alpha_1(\rho_{p\uparrow}+\rho_{n\downarrow})^{\beta}+\alpha_2(\rho_{p\uparrow}-\rho_{n\downarrow})^2 +AP_{\sigma}(\rho_{p\uparrow}-\rho_{n\downarrow})^2+BP_{\tau}(\rho_{p\uparrow}-\rho_{n\downarrow})^2], \tag{11}$$

etc., where α_1, α_2, A, B, and β are parameters of the potential. P_σ and P_τ are the spin and isospin exchange operators, respectively.

Using the above generalized interaction, the total energy per particle of the polarized nuclear matter is given by

$$E=E_V+X^2E_X+Y^2E_Y+Z^2E_Z, \tag{12}$$

where

$$E_V=\frac{3\hbar^2}{10m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho_0^{2/3}+2\alpha_1a^3\pi\left(\frac{1}{2}\right)^{\beta}\rho_0^{\beta+1}-2ca^3\pi\rho_0 +\frac{8ca^3}{5\pi b^2}\left(\frac{3\pi^2}{2}\right)^{5/3}\rho_0^{5/3}, \tag{13}$$

$$E_X=\frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho_0^{2/3}+\alpha_1a^3\pi(\beta+\beta^2)\left(\frac{1}{2}\right)^{\beta}\rho_0^{\beta+1} +\frac{1}{8}(2\alpha_2+A)a^3\pi\rho_0^3+\frac{8ca^3}{9\pi b^2}\left(\frac{3\pi^2}{2}\right)^{5/3}\rho_0^{5/3}, \tag{14}$$

$$E_Y=\frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho_0^{2/3}+\alpha_1a^3\pi(\beta+\beta^2)\left(\frac{1}{2}\right)^{\beta}\rho_0^{\beta+1} +\frac{1}{8}(2\alpha_2+B)a^3\pi\rho_0^3+\frac{8ca^3}{9\pi b^2}\left(\frac{3\pi^2}{2}\right)^{5/3}\rho_0^{5/3}, \tag{15}$$

$$E_Z=\frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho_0^{2/3}+\alpha_1a^3\pi(\beta+\beta^2)\left(\frac{1}{2}\right)^{\beta}\rho_0^{\beta+1} +\frac{1}{8}(2\alpha_2+A+B)a^3\pi\rho_0^3+\frac{8ca^3}{9\pi b^2}\left(\frac{3\pi^2}{2}\right)^{5/3}\rho_0^{5/3}, \tag{16}$$

where m is the nucleon mass and ρ_0 is the saturation density for normal nuclear matter.

The pressure of nuclear matter is defined as

$$P=\rho^2\,\frac{\partial E}{\partial \rho}, \tag{17}$$

the incompressibility as

$$K=9\rho^2\,\frac{\partial^2 E}{\partial \rho^2}, \tag{18}$$

and the velocity of sound is given by

$$v_S=\left(\frac{\partial P}{\partial \rho}\right)^{1/2}. \tag{19}$$

Terms higher than quadratic in X, Y, and Z are neglected in Eq. (12). The parameters a, b, C, α_1, α_2, A, and B are adjusted to fit the values of k_f, E_V, E_X, E_Y, E_Z, K, and

$(\partial E/\partial\rho)=0$ at ρ_0 for polarized nuclear matter. The parameter β was fixed at the value 1/3.

B. Polarized nuclear matter at finite temperature

The thermodynamic properties of nuclear matter are determined completely if the free energy F is known in terms of the density ρ and temperature T, where

$$F=E-TS, \tag{20}$$

E being the total energy and S is the entropy. Using the T^2 approximation [15], we obtain the entropy S, the free energy F, and the pressure P as follows:

$$S_T=\frac{T}{6}\frac{2m^*}{\hbar^2}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{-2/3}, \tag{21}$$

$$F_T=E_V-\frac{T^2}{6}\frac{2m^*}{\hbar^2}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{-2/3}, \tag{22}$$

$$P_T=P(T=0)+\frac{T^2}{9}\frac{2m^*}{\hbar^2}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{1/3}, \tag{23}$$

$$m^*=m\left[1+\frac{m}{\hbar^2}\left(\frac{4a^3Ck_f^3}{3\pi b^2}\right)\right]^{-1}, \tag{24}$$

where m^* is the effective mass and k_f is the Fermi momentum.

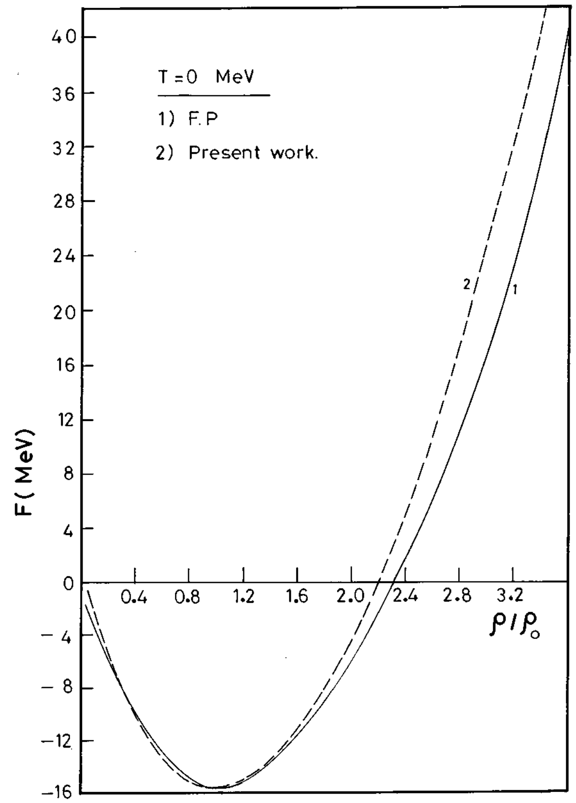

FIG. 1. Free energy for the present potential in comparison with Ref. [3] at $T=0$ MeV.

III. RESULTS AND DISCUSSION

In the present work we used a modified Seyler-Blanchard potential given by Eq. (9) which depends explicitly on the densities of the interacting nucleons $\rho_{p\uparrow}$, $\rho_{p\downarrow}$, $\rho_{n\uparrow}$, and $\rho_{n\downarrow}$ as defined by Eqs. (10) and (11). The parameters a, b, c, α_1, α_2, A, B, and β are adjusted to fit the values of k_f, E_V, E_X, E_Y, E_Z, K, and $P=0$ at ρ_0 for polarized nuclear matter. The value of the parameter $\beta=1/3$ was chosen between several other values to give the best fit.

The parameters obtained in this work are $a=0.557$ fm, $\beta=1/3$, $b=9.82$ fm^{-1}, $c=611.4$ MeV, $\alpha_1=879.08$ MeV fm, $\alpha_2=0.0$ MeV fm^6, $A=19\,761.53$ MeV fm^6, and $B=12\,796.13$ MeV fm^6. With this set of parameters with $k_f=1.33$ fm^{-1}, we were able to reproduce exactly the values of $E_X=33.4$ MeV of Ref. [16] and $E_Y=31.5$ MeV of Ref. [17]. However, for E_Z we obtained a value of 36.88 MeV to be compared with the value of 36.5 MeV of Ref. [17]. The compressibility $K=240$ MeV, which is a good value and comparable to that obtained in our previous work [15]. The free energy at temperature $T=0$ is presented in Fig. 1 in comparison with that of Friedman and Pandharipande FP [3]. Our results are very close to the FP calculation until ρ/ρ_0 ≈ 1.6, but a little bit higher for larger values of ρ/ρ_0. Figures 2 and 3 show a similar trend for the values of the free energy at temperatures $T=5$ and 10 MeV, respectively. The pressure density curves are shown in Figs. 4, 5, and 6 in comparison with FP calculations at temperatures $T=0$, 5, and 10 MeV. Here again the agreement is quite remarkable.

The values of the velocity of sound using our potential and FP potential are presented in Fig. 7. Here again we no-

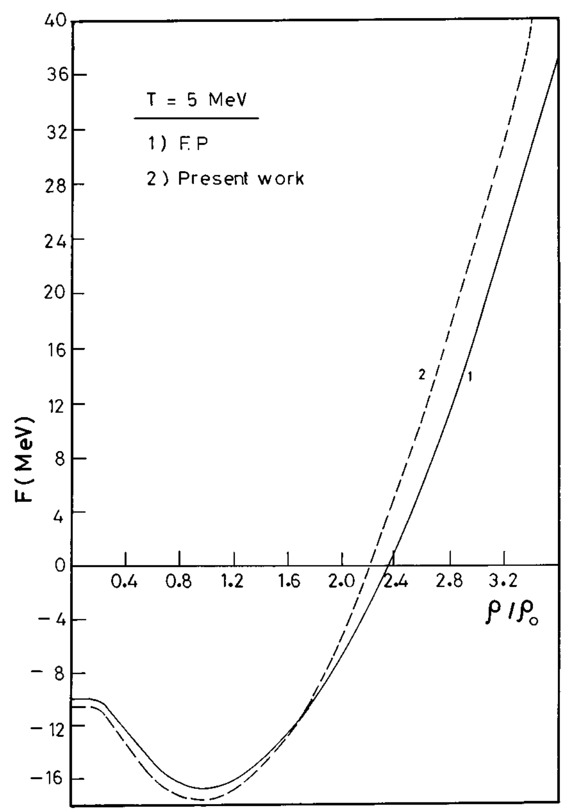

FIG. 2. Same as Fig. 1, but for $T=5$ MeV.

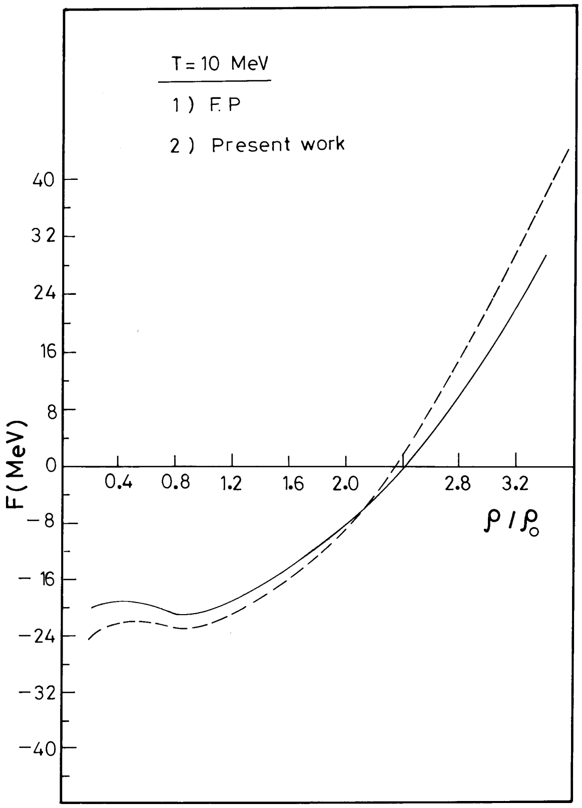

FIG. 3. Same as Fig. 1, but for $T = 10$ MeV.

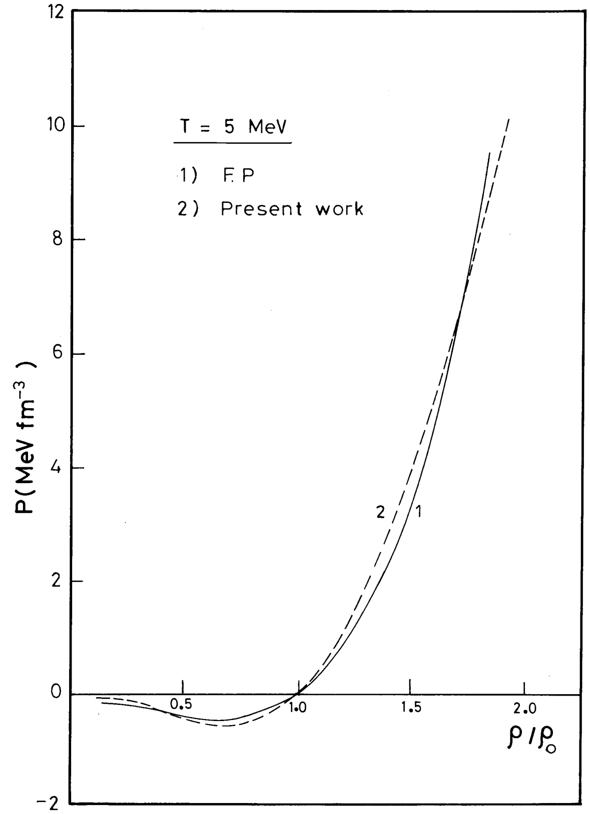

FIG. 5. Same as Fig. 4, but for $T = 5$ MeV.

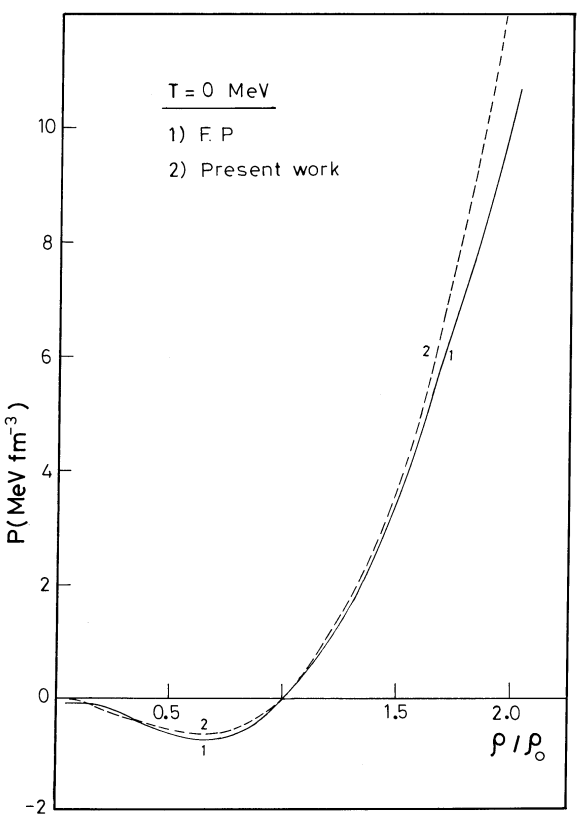

FIG. 4. Pressure dependence on density in comparison with Ref. [3] at $T = 0$ MeV.

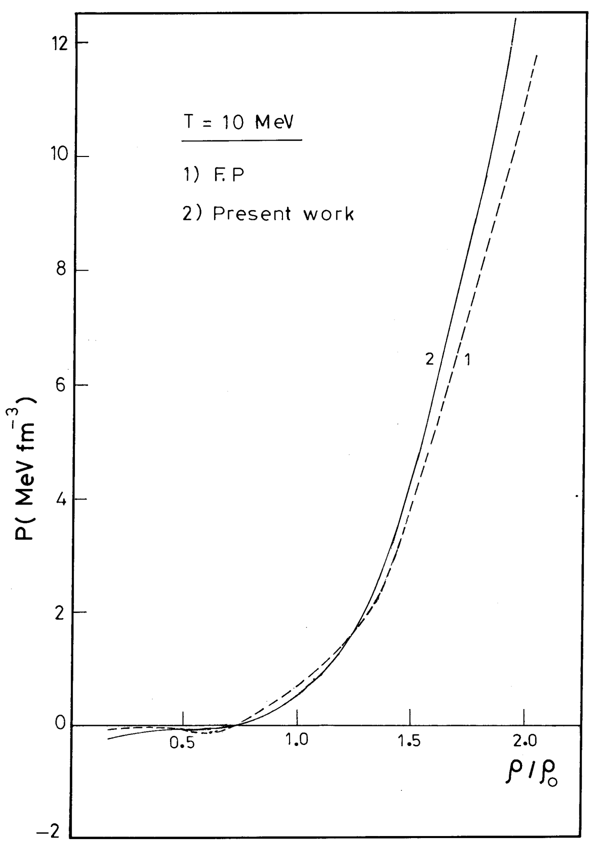

FIG. 6. Same as Fig. 4, but for $T = 10$ MeV.

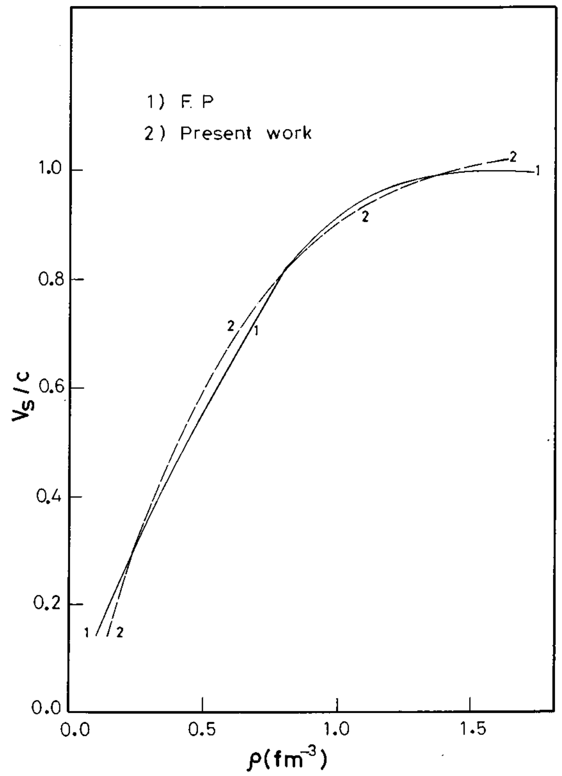

FIG. 7. Comparison of the sound velocity of the present work with Ref. [3].

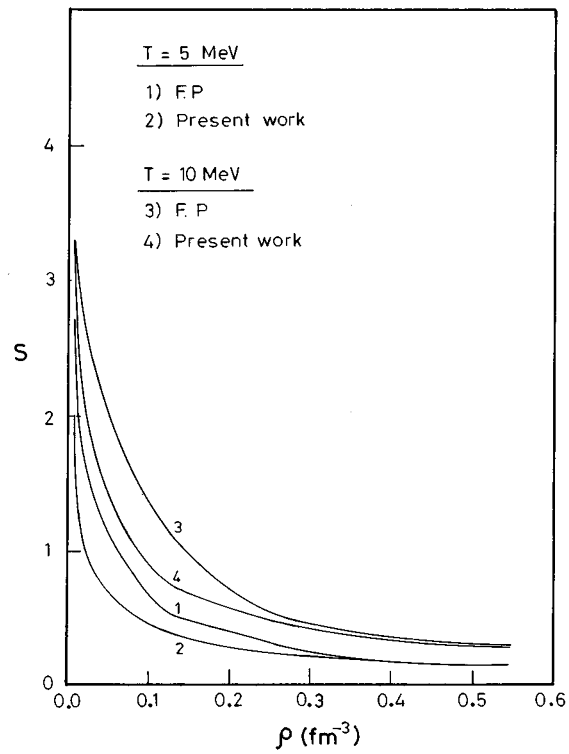

FIG. 8. Entropy of polarized nuclear matter using our potential in comparison with Ref. [3] at $T=5$ and 10 MeV.

tice that the agreement is good. The entropy per nucleon is given in Fig. 8 in comparison with FP calculations at $T=5$ and 10 MeV. Our results coincide with those of FP in the low density region, but at higher densities it is lower. We notice that the entropy decreases as the density increases and the values become larger at higher temperatures. Our results show that the modified SB potential of the present work gives a soft equation of state. This is clear from the value of $K=240$ MeV and $m^*/m=0.974$ in our case. The method and potential proposed here are quite satisfactory for the purpose of the present work, showing that with suitably adjusted parameters it can come close to the FP nuclear matter results even without adding the $1/P$ term. In our previous work [15], the results were at their best agreement with FP by adding to the density-dependent term a $1/P$ term. However, the present work can be extended to include the $1/P$ momentum-dependent term of the potential [12] or to include other physical quantities (in the process of fitting) to obtain the parameters of the potential.

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