Cranking model with proton-neutron correlations

Kazunari Kaneko
Joint Institute for Heavy Ion Research, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831 and Department of Physics, Kyushu Sangyo University, Matsukadai, Fukuoka 813 Japan

Jing-ye Zhang
Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996

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A cranking Hartree-Fock-BCS HFB model with $T=0$ and $T=1$ proton-neutron correlations is proposed and discussed. Numerical calculations are carried out on a single $(g_{9/2})_p(g_{9/2})_n$ shell as a first practical try. It is found that the $T=0$ proton-neutron pairing correlation is crucial for a crossing of an even-spin ground band by an odd-spin band, a so-called $T=1$ and $T=0$ band crossing in an $N=Z$ odd-odd system. A conventional spin alignment of a ground state band in an even-even system is also examined. A delay of this spin alignment due to a $T=1,J=0$ proton-neutron pairing correlation is found for nuclei with $N=Z$.
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The study of the proton-neutron (p-n) interaction has become a hot issue in recent years. The p-n interaction has been recognized as playing an important role, especially for the $N=Z$ nuclei (see Ref. [1] for a review). The nuclei in the mass 70–80 region with N nearly equal to Z are known to provide a variety of nuclear structure information [2–4]. Since they are well-deformed, one can expect that the cranking-HFB approach [5] may offer a very powerful frame for understanding their microscopic structure. This model has two average potentials, the deformation average field which is usually described by the quadrupole parameters β and γ, and pairing fields Δ_p and Δ_n which are known from BCS theory. In heavy nuclei, isovector $T=1,J=0$ proton-proton (p-p) and neutron-neutron (n-n) pairing correlations dominate. In light nuclei (e.g., in sd shell) the isoscalar ($T=0$) correlation may become stronger than isovector (including $T=1$ proton-neutron) component, and the ground state may have $T=0$ [6]. The $N=Z$ nuclei of the mass $A\sim80$ region are very interesting for studying an interplay between the $T=0$ correlation and $T=1$ correlation [6,7]. Since protons and neutrons occupy the same levels, we should expect large p-n correlations in the region with mass 70–80 and lighter. At present, however, it is not clear whether the p-n correlations are strong enough to form a static pair condensate (an average field). In this paper, we will propose and discuss a cranking-HFB model with p-n pairing average fields in addition to the usual average fields, the deformed field, and the like-nucleon pairing fields.

Recently, Rudolph *et al.* [8] have observed a band crossing of an even-spin ground state band by an odd-spin band in the odd-odd $N=Z$ nucleus ^{74}Rb. For this nucleus, the first spin alignment along the yrast line is not due to the conventional one of two-proton or two-neutron pairs. This band crossing phenomena has been studied by cranked shell model calculations [8–10]. They found that the $T=1,J=0$ p-n correlation and the $T=0,J=9$ p-n correlation are most important for the first alignment. One other interesting observation is a conventional spin alignment of two-proton or two-neutron pairs in Kr isotopes [11,12]. A delayed band crossing in ^{72}Kr has been reported [13] and may be related to the p-n correlations [10,14]. Our model may be capable of studying the microscopic structure behind these experimental findings. It is found that basic features of these two types of crossings can be explored even in a single shell approximation of this model. We find that the $T=0$ and $T=1$ band crossing (we call it first band crossing later on) in an odd-odd nucleus, and the delay of the conventional band crossing (second band crossing) in an even-even nucleus are due to the p-n correlations.

Let us start with a general effective Hamiltonian which includes the p-p, n-n, and p-n correlations with the following form:

$$\hat{H}=\sum_{\tau\alpha}\varepsilon_\alpha^\tau\hat{c}_\alpha^{\tau\dagger}\hat{c}_\alpha^\tau-\frac{1}{2}\chi\sum_{\tau\mu}\hat{Q}_{2\mu}^\dagger(\tau\tau)\hat{Q}_{2\mu}(\tau\tau)$$
$$-\sum_\tau G_\tau\hat{P}^\dagger(\tau\tau)\hat{P}(\tau\tau)$$
$$-\frac{1}{2}\sum_J k_J\sum_M \hat{A}_{JM}^\dagger(pn)\hat{A}_{JM}(pn),\quad \tau=p,n,$$
$$\hat{Q}_{\lambda\mu}(\tau\tau)=\sum_{\alpha\beta}\langle\alpha|\hat{Q}_{\lambda\mu}|\beta\rangle\hat{c}_\alpha^{\tau\dagger}\hat{c}_\beta^\tau, \tag{1}$$
$$\hat{P}(\tau\tau')=\sum_{\alpha>0}\hat{c}_{\tilde{\alpha}}^{\tau'}\hat{c}_\alpha^\tau,\quad \hat{c}_{\tilde{\alpha}}^\tau=(-1)^{j_a-m_\alpha}\hat{c}_{j_a-m_\alpha}^\tau,$$
$$\hat{A}_{JM}^\dagger(\tau\tau')=\frac{1}{\sqrt{1+\delta_{\tau\tau'}}}\sum_{\alpha\beta}\langle j_am_\alpha j_bm_\beta|JM\rangle\hat{c}_\alpha^{\tau\dagger}\hat{c}_\beta^{\tau'\dagger}.$$

The operator $\hat{c}_\alpha(\hat{c}_\alpha^\dagger)$ denotes the nucleon annihilation (creation) operator in the single-particle state characterized by a set of quantum numbers $\alpha=(n_a,l_a,j_a,m_\alpha)$. Here $\hat{P}(\tau\tau')$ is the $J=0$ pair operator, $\hat{Q}_{2\mu}(\tau\tau)$ are components of quadrupole tensor, and ε^τ_α are single-particle energies. In the mean-field approximation, the cranking Hamiltonian with p-n correlations is expressed as

$$\hat{h}'=\hat{h}_p+\hat{h}_n+\hat{h}_{pn}-\vec{\omega}\cdot\vec{\hat{j}}, \tag{2}$$

where $\hat{h}_p$ and $\hat{h}_n$ denote the proton part and the neutron part of the Hamiltonian, respectively:

$$\hat{h}_\tau=\sum_{\tau\alpha}\varepsilon^\tau_\alpha\hat{c}^{\tau\dagger}_\alpha\hat{c}^\tau_\alpha-\sum_\mu q^\tau_{2\mu}\hat{Q}_{2\mu}(\tau\tau)-\lambda_\tau\hat{N}^\tau$$
$$-\Delta_\tau(\hat{P}^\dagger(\tau\tau)+\hat{P}(\tau\tau)),$$
$$\hat{N}^\tau=\sum_\alpha\hat{c}^{\tau\dagger}_\alpha\hat{c}^\tau_\alpha,\quad \hat{j}_k=\sum_{\tau\alpha\beta}\langle\alpha|j_k|\beta\rangle\hat{c}^\tau_\alpha,\quad k=x,y,z, \tag{3}$$

and the proton-neutron part $\hat{h}_{pn}$ is given as

$$\hat{h}_{pn}=-\sum_{JM}C_{JM}[\hat{A}^\dagger_{JM}(pn)+\hat{A}_{JM}(pn)], \tag{4}$$

with

$$q^\tau_{2\mu}=\chi\langle\phi|\hat{Q}_{2\mu}(\tau\tau)|\phi\rangle,\quad \Delta_\tau=G_\tau\langle\phi|\hat{P}(\tau\tau)|\phi\rangle,$$
$$C_{JM}=k_J\langle\phi|\hat{A}_{JM}(pn)|\phi\rangle. \tag{5}$$

Here, $\hat{j}_k$ are the angular momentum operators with respect to the $k=x,y,z$ axes, and $\vec{\omega}$ is the angular frequency vector $\vec{\omega}=(\omega_x,\omega_y,\omega_z)$. The parameters λ_τ, Δ_τ, and C_{JM} are the chemical potentials, the pairing gaps, and the JM components of the p-n field strength, respectively. The quadrupole deformation parameters are defined as

$$q^\tau_{2\mu}=\beta\cos\gamma\ \ (\mu=0),\quad \frac{1}{\sqrt{2}}\beta\sin\gamma\ \ (\mu=\pm 2),$$
$$0\ \ (\mu=\pm 1). \tag{6}$$

We now consider protons and neutrons system coupled to an axially symmetric deformed prolate core so that $\gamma=0$ in the intrinsic frame. The p-n Hamiltonian $\hat{h}_{pn}$ is not invariant under a rotation of 180° around the x axis by the operator $\hat{R}_x=\exp(-i\pi\hat{j}_x)$, i.e., $\hat{R}_x\hat{h}_{pn}\hat{R}_x^{-1}\neq\hat{h}_{pn}$, and it also breaks an isospin symmetry. In fact, we can divide the above p-n Hamiltonian into two parts, signature conserved term $\hat{h}^{(\mathrm{sc})}_{pn}$ and nonconserved term $\hat{h}^{(\mathrm{nc})}_{pn}$

$$\hat{h}_{pn}=\hat{h}^{(\mathrm{sc})}_{pn}+\hat{h}^{(\mathrm{nc})}_{pn}, \tag{7a}$$

$$\hat{h}^{(\mathrm{sc})}_{pn}=-\sum_J\sum_{M>0}C_{JM}\sum_{\alpha\beta}[\langle j_am_\alpha j_bm_\beta|JM\rangle$$
$$-(-1)^{j_a+j_b-J}\langle j_am_\alpha j_bm_\beta|J-M\rangle](\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta+\mathrm{H.c.}),$$
$$-\sum_{J=\mathrm{even}}C_{J0}\sum_{\alpha\beta}\langle j_am_\alpha j_bm_\beta|J0\rangle(\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta+\mathrm{H.c.}), \tag{7b}$$

$$\hat{h}^{(\mathrm{nc})}_{pn}=-\sum_J\sum_{M>0}\sum_{\alpha\beta}[C_{J-M}+(-1)^{j_a+j_b-J}C_{JM}]$$
$$\times\langle j_am_\alpha j_bm_\beta|J-M\rangle(\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta+\mathrm{H.c.})$$
$$-\sum_{J=\mathrm{odd}}C_{J0}\sum_{\alpha\beta}\langle j_am_\alpha j_bm_\beta|J0\rangle(\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta+\mathrm{H.c.}). \tag{7c}$$

Here, we should note that the nonconserved term may play an important role for $N=Z$ nuclei in the sd and fp shells. Especially, the $T=0$ proton-neutron correlation in time-reversed states is significant for the ground state in odd-odd nuclei near $N=Z$. As pointed out by Goodman [1], $N=Z$ nuclei in sd shell pairing correlations are important for neutrons and protons coupled to $T=0$ in time-reversed orbits, and dominate $T=1$ correlations. In the mean-field approximation, the signature nonconserved term $\hat{h}^{(\mathrm{nc})}_{pn}$ with $T=0$ gives rise to a signature breaking. Then, we may expect enhanced $M1$ transitions as an observable consequence of a signature breaking, similar to the case for the tilted-axis rotation. Furthermore, since $\hat{h}^{(\mathrm{nc})}_{pn}$ with $T=0,M\neq 0$ mixes K quantum numbers of states, this pairing p-n field may evolve to a triaxial HFB state [15]. In this paper, however, we will limit ourselves to the cases where a symmetry under a rotation by 180° about the x axis is sustained. The signature nonconserved term will be discussed in a forthcoming paper. Then, C_{JM} satisfies the following relationships:

$$C_{JM}+(-1)^{j_a+j_b-J}C_{J-M}=0\quad(\text{for all}\ \ JM),$$
$$C_{J0}=0\ \ (\text{for odd}\ \ J), \tag{8}$$

and the signature nonconserved term vanishes. This assumption is reasonable for describing axially symmetric systems. In fact, in ^{74}Rb the even-even ground band and the odd-spin band are considered to be specified by the signature $r_x=1$ and $r_x=-1$, respectively. Under the above assumption, the p-n Hamiltonian is written as

$$\hat{h}_{pn}=\hat{h}^{(\mathrm{sc})T=0}_{pn}+\hat{h}^{(\mathrm{sc})T=1}_{pn}, \tag{9a}$$

where $\hat{h}^{(\mathrm{sc})T=0}_{pn}$ and $\hat{h}^{(\mathrm{sc})T=1}_{pn}$ are isospin $T=0$ part and $T=1$ part in the signature conserved p-n Hamiltonian, respectively,

$$\hat{h}^{(\mathrm{sc})T=0}_{pn}=-\sum_{J=\mathrm{odd}}\sum_{M>0}C_{JM}\sum_{\alpha\beta}[\langle j_am_\alpha j_bm_\beta|JM\rangle$$
$$-(-1)^{j_a+j_b-J}\langle j_am_\alpha j_bm_\beta|J-M\rangle](\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta$$
$$+\mathrm{H.c.}), \tag{9b}$$

$$\hat{h}^{(\mathrm{sc})T=1}_{pn}=-\Delta^{T=1}_{pn}[\hat{P}^\dagger(pn)+\hat{P}(pn)]$$
$$-\sum_{J=\mathrm{even}>0}\sum_{M\geqslant 0}\frac{1}{1+\delta_{J0}}C_{JM}\sum_{\alpha\beta}[\langle j_am_\alpha j_bm_\beta|JM\rangle$$
$$-(-1)^{j_a+j_b-J}\langle j_am_\alpha j_bm_\beta|J-M\rangle]$$
$$\times(\hat{c}^{p\dagger}_\alpha\hat{c}^{n\dagger}_\beta+\mathrm{H.c.}), \tag{9c}$$

where $\Delta_{pn}^{T=1}$ is a parameter of the $J=0,M=0$ component. The Hamiltonian (2) can be diagonalized by the generalized Bogoliubov transformation

$$\hat{a}_i=\sum_{\tau\alpha} (U_{\alpha i}^{\tau *}\hat{c}_\alpha+V_{\alpha i}^{\tau *}\hat{c}_\alpha^\dagger). \quad (10)$$

Then the quasiparticle vacuum $|\phi_0\rangle$ is defined by $\hat{a}_i|\phi_0\rangle=0$. The above transformation is required to be unitary, and can then be inverted to express the particle operators in terms of the quasiparticle operators

$$\hat{c}_\alpha^\tau=\sum_i (U_{\alpha i}^\tau\hat{a}_\alpha+V_{\alpha i}^{\tau *}\hat{a}_\alpha^\dagger). \quad (11)$$

The unitary constraints are

$$U^\dagger U+V^\dagger V=1, \quad UU^\dagger+V^*V^T=1,$$

$$U^T V+V^T U=0, \quad UV^\dagger+V^*U^T=0. \quad (12)$$

Then, the Hamiltonian (1) is given in terms of the quasiparticle

$$\hat{h}'=E_0+\sum_i E_i\hat{a}_i^\dagger\hat{a}_i, \quad (13)$$

where E_0 is the energy of the quasiparticle vacuum and E_i are the quasiparticle energies. Then, the ground state of odd-odd system is expressed as $|\phi\rangle=\hat{a}_{i_0}^\dagger\hat{a}_{i_1}^\dagger|\phi_0\rangle$, where i_0 and i_1 denote the lowest quasiparticle state and the second lowest state, respectively. These quasiparticle states are not eigenstates of the isospin operator $\hat{T}^2$, while they are eigenstates of z component $\hat{T}_z$ of isospin. In fact, we can see that the p-n term $\hat{h}_{pn}$ does not commute with isospin operators $\hat{T}_\pm,\hat{T}^2$ and only commutes with the z component $\hat{T}_z$,

$$[\hat{h}_{pn},\hat{T}_+]=\sum_{JM} C_{JM}[\hat{A}_{JM}^\dagger(nn)-\hat{A}_{JM}(pp)], \quad (14a)$$

$$[\hat{h}_{pn},\hat{T}_-]=\sum_{JM} C_{JM}[\hat{A}_{JM}^\dagger(pp)-\hat{A}_{JM}(nn)], \quad (14b)$$

$$[\hat{h}_{pn},\hat{T}^2]=\sum_{JM} C_{JM}\{\hat{T}_+[\hat{A}_{JM}^\dagger(pp)-\hat{A}_{JM}(nn)]$$
$$+[\hat{A}_{JM}^\dagger(nn)-\hat{A}_{JM}(pp)]\hat{T}_-\}, \quad (14c)$$

$$[\hat{h}_{pn},\hat{T}_z]=0, \quad (14d)$$

where isospin operators are defined as

$$\hat{T}_+=\sum_\alpha \hat{c}_\alpha^{n\dagger}\hat{c}_\alpha^p, \quad \hat{T}_-=\sum_\alpha \hat{c}_\alpha^{p\dagger}\hat{c}_\alpha^n, \quad (15a)$$

$$\hat{T}_z=\frac{1}{2}\sum_\alpha (\hat{c}_\alpha^{n\dagger}\hat{c}_\alpha^n-\hat{c}_\alpha^{p\dagger}\hat{c}_\alpha^p), \quad (15b)$$

$$\hat{T}^2=\hat{T}_+\hat{T}_-+\hat{T}_z(\hat{T}_z-1). \quad (15c)$$

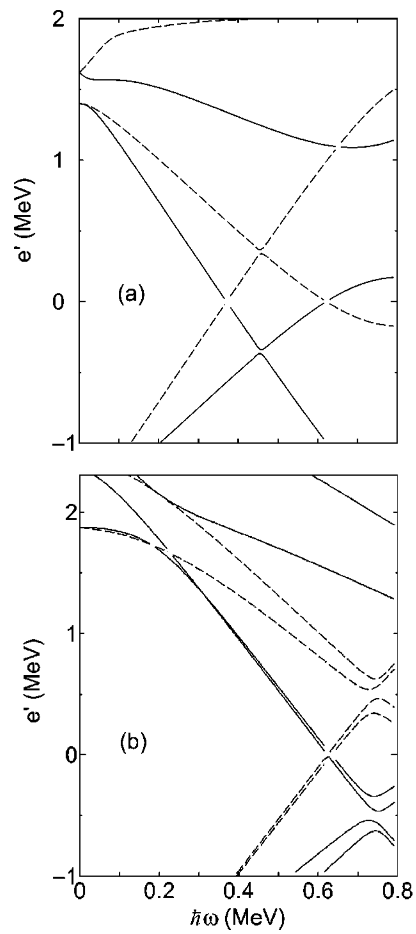

FIG. 1. (a) shows the Routhian for a $N=Z$ system without p-n interaction, and the trajectories are degenerate between quasiproton and quasineutron states. (b) is the Routhian in a $N=Z=3$ system with p-n interactions $\Delta_{pn}^{T=1}=2.3$, $C_{J=9,M=1}=1.0$. The solid lines and the dashed lines denote the signature $r_x=-i$ and $r_x=i$, respectively.

Thus, the p-n term gives rise to a mixing of proton and neutron quasiparticle states. Therefore, we cannot specify proton or neutron for the quasiparticles, and there no longer are separate proton or neutron Routhians. We will see this in Routhian plots of Fig. 1 later on.

In the above equations, $q_{2\mu}^\tau,\Delta_\tau$, and C_{JM} depend on the solution $|\phi\rangle$, and generally have to be self-consistently determined. However, it is too early to try a full self-consistent calculation with a microscopic model including p-n interactions at present. Theoretically, it is because, no matter one knows the existence of the p-n interaction inside nuclei for decades, there have only been a few numerical calculations published with p-n interaction treated microscopically for specifically real nuclei so far, and none of them was done with a full self-consistency. Experimentally, it is very hard to extract the p-n interaction quantitatively from the data, especially, to our knowledge, there is no known direct way to extract different components of p-n interactions, even though there is, for instance, an empirical formula to extract the average effective p-n interaction from data [16].

Therefore, it seems to us, there is still a lot of work ahead to understand basic features of different components of p-n interactions. At present, one of the meaningful and practical approaches to this issue is to examine the role of different components involved in p-n interactions under some clear and reasonable approximations. The recipe we are taking is to formulate a general model, as we have done above, and then to carry out numerical calculations in a single $(g_{9/2})_p(g_{9/2})_n$ shell with some reasonable constant parameters, and followed by checking the parameter dependence of results we obtained. We first consider an axially symmetric system with $N=Z$, odd-proton number Z, and odd-neutron number N. The deformation parameters are chosen as $\beta=0.3, \gamma=0.0$. The pairing gaps are fixed at $\Delta_p=\Delta_n=1.39$ MeV, and the single particle energies are taken to be values $\varepsilon_\alpha^\tau=0.0$.

Let us now start with the investigation of the single quasiparticle Routhian. Figure 1(a) shows the single quasiproton or quasineutron Routhians. The proton level and the neutron level are degenerate for each trajectory in the Routhian. There is not any type of p-n interaction involved, it is just the conventional Routhian plot. However, Fig. 1(b) is different. It is still the single quasiparticle Routhian, but it includes some components of p-n interaction. The chemical potentials are kept at the values which give the average proton number $N_p=\langle\hat{N}_p\rangle=3$. The average neutron number $N_n=\langle\hat{N}_n\rangle=3$ for $\hbar\omega=0.0$ and $\Delta_{pn}^{T=1}=2.3$ and $C_{J=9,M=1}=1.0$ are taken, and other components of C_{JM} set to zero. As mentioned above, we cannot specify proton states or neutron states in the Routhian due to the proton-neutron mixing by the p-n interactions [see Eq. (7)]. Therefore, in Fig. 1(b) both proton and neutron degrees of freedom are shown together. We can see a first crossing near $\hbar\omega=0.22$ MeV for the lowest-energy trajectory. Since the lowest level and the third lowest have $r_x=-i$ and the second lowest and the fourth lowest have $r_x=i$, the yrast state of odd-odd system (the lowest two quasiparticle state $\hat{a}_{i_0}^\dagger\hat{a}_{i_1}^\dagger|\phi_0\rangle$) has a signature $r_x=1$ (an even-spin) before this crossing, while it has the signature $r_x=-1$ (an odd-spin) after the crossing. Thus the signature of the yrast state changes from $r_x=1$ to $r_x=-1$ at $\hbar\omega=0.22$. The crossing at $\hbar\omega\approx0.75$ MeV is the second crossing in Fig. 1(b), the conventional one of two-proton or two-neutron pair alignment without signature change, as is well known. This crossing frequency is $\hbar\omega\approx0.45$ MeV when there is no p-n interaction as shown in Fig. 1(a). As seen later, the $T=1$, $J=0$ p-n term plays an important role in this delayed crossing frequency. Thus two kinds of crossings are found with p-n interaction presence as shown in Fig. 1(b).

There is one more type of crossing as shown in Fig. 2 coming from nonzero $C_{J=1,M=1}$. The energy order of the second pair of signature partners is inverted, i.e., instead of the favorite signature $r_x=-i$, being lower in energy as in the first pair of the $g_{9/2}$ orbitals, the unfavored signature $r_x=i$ is lower in energy for this pair. The crossing occurs between the second level and third level with same signature $r_x=i$. As explained in detailed by Bengtsson *et al.* [17], such an energy order reversal between nearby signature partners could lead to the signature inversion which has been seen in odd-odd nuclei in mass 120 and 150 regions (e.g., see Ref. [18]). The reversal energy order is attributed to a positive γ deformation in Ref. [17], however, as mentioned above, there is no γ deformation involved in the present calculation,

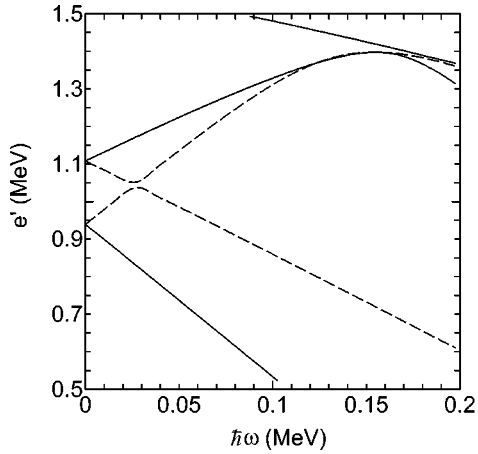

FIG. 2. The Routhian in a $N=Z=3$ system with parameters $\Delta_{pn}^{T=1}=0.0$, $C_{J=1,M=1}=1.0$.

so it is coming from some specific terms of p-n interactions. Such findings may offer a microscopic basis for those theories which attribute the signature inversion to the involvement of some type of phenomenological p-n interactions [19,20]. A detailed study of the signature inversion with present model is in progress and will be reported elsewhere.

The spin alignments due to the first band crossing in the odd-odd system for each odd neutron number $N=3,5,7,9$ and proton number $Z=3$ are shown as a function of the angular frequency $\hbar\omega$ in Fig. 3. The chemical potentials are kept at the values so that at $\hbar\omega=0.0$ the average proton number $\langle\hat{N}_p\rangle$ and the average neutron number $\langle\hat{N}_n\rangle$ give Z and N, respectively. In $N=Z=3$, we can see a sudden spin alignment at $\hbar\omega\sim0.22$ MeV due to the first level crossing, while the spins in $N\neq Z$ increase gradually. The single proton and neutron states $|j,m_z\rangle$ with large $j=9/2$ and small $m_z=3/2$ are just above the Fermi surface, then it is easy for the Coriolis force to align the angular momentum vectors of both quasiparticles in the direction of rotational axis x. We also carried out the cranking model calculations with oblate deformation for the $N=Z=3$ nucleus. However, a sudden spin alignment as mentioned above was not found in these calculations.

We also examined the Routhians with other p-n terms in Eq. (7). Similar crossings in Fig. 1(b) are seen in odd-spin

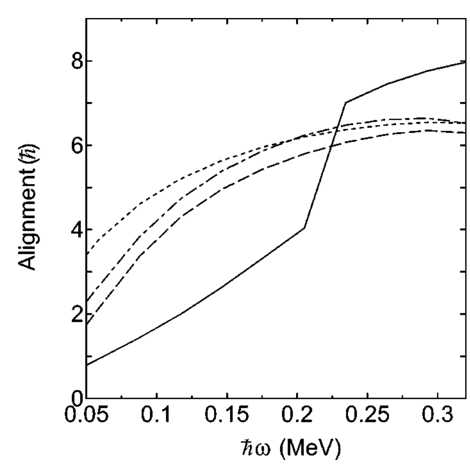

FIG. 3. Spin alignment for $Z=3$ and $N=3,5,7,9$ with the same parameters as in Fig. 1(b).

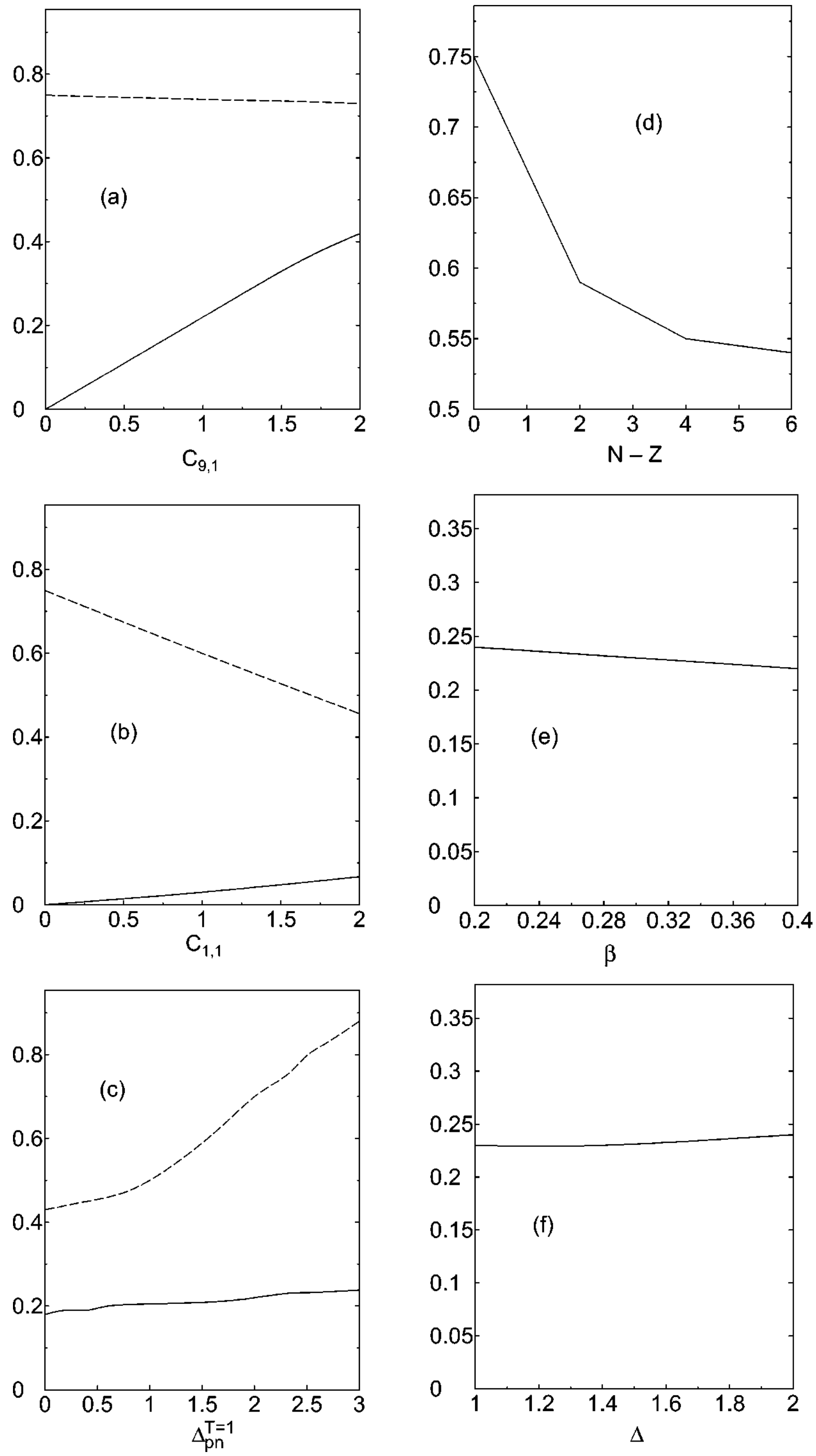

FIG. 4. The parameter dependence of the three types of band crossing: dependence on (a) $C_{J=9,M=1}$ for $\hbar\omega_c^{(1)}$ (solid) and $\hbar\omega_c^{(2)}$ (dashed); (b) $C_{J=1,M=1}$ for $\hbar\omega_c^{(3)}$ (solid) and $\hbar\omega_c^{(2)}$ (dashed); (c) $\Delta_{pn}^{T=1}$ for $\hbar\omega_c^{(1)}$ (solid) and $\hbar\omega_c^{(2)}$ (dashed); (d) N-Z of even system for $\hbar\omega_c^{(2)}$, the conventional crossing with $\Delta_{pn}^{T=1}=2.3$, $C_{J=9,M=1}=1.0$, the same p-n interaction as in Fig. 1(b); (e) the deformation parameter β for $\hbar\omega_c^{(1)}$; (f) the gap parameter $\Delta(=\Delta_p=\Delta_n)$ for $\hbar\omega_c^{(1)}$.

$J=3,5,7$ and $M=1,3$. However, as examples, we will only focus on the role of three terms of the p-n interaction $\Delta_{pn}^{T=1}$, $C_{J=1,M=1}$, and $C_{J=9,M=1}$. Figure 4 shows the two band crossing frequencies as a function of $C_{J=9,M=1}$, $C_{J=1,M=1}$, $\Delta_{pn}^{T=1}$, and N-Z. In Figs. 4(a)–4(c), the chemical potentials are fixed at $\lambda_p=\lambda_n=-2.21$ which is just below the single particle state $|j=9/2,m_z=3/2\rangle$ at $\hbar\omega=0.0$. We can see in Fig. 4(a) that under constant value $\Delta_{pn}^{T=1}=2.3$ the first crossing frequency $\hbar\omega_c^{(1)}$ (solid line) increases monotonically as $C_{J=9,M=1}$ increases, while the second crossing frequency $\hbar\omega_c^{(2)}$ (dashed line) is almost constant. On the other hand, Fig. 4(b) shows the crossing frequencies as a function of $C_{J=1,M=1}$. The parameters are fixed at $\Delta_{pn}^{T=1}=2.3$ and $C_{J=9,M=1}=0.0$. The crossing frequency $\hbar\omega_c^{(3)}$ (the one described in Fig. 2) has a small value (solid line) in the frequency region discussed, and increases slightly with increasing $C_{J=1,M=1}$. The frequency $\hbar\omega_c^{(2)}$ decreases as $C_{J=1,M=1}$ increases. Thus the $J=1,M=1$ p-n term affects the frequency $\hbar\omega_c^{(2)}$ of the second type of crossing, and results in a small value of $\hbar\omega_c^{(3)}$. Figure 4(c) shows that, under constant value $C_{J=9,M=1}=1.0$, the second crossing frequency $\hbar\omega_c^{(2)}$ increases as $\Delta_{pn}^{T=1}$ increases, while the first crossing frequency $\hbar\omega_c^{(1)}$ is almost constant. Let us next discuss the N-Z dependence of the second crossing frequency $\hbar\omega_c^{(2)}$ in the even-even systems. The crossing frequencies as a function of N-Z are shown in Fig. 4(d) with the same p-n interactions as in Fig. 3. We can see a large crossing frequency at $N=Z$. The $T=0,J=0$ p-n correlation in $N=Z$ becomes stronger than that in $N\neq Z$. As the neutron excess increases, this correlation becomes less favorable.

In this paper, we carried out on a single $(g_{9/2})_p(g_{9/2})_n$ shell as a first practical try, and assumed constant values for the parameters $\beta,\gamma,\Delta_p,\Delta_n$, and C_{JM} for simplicity. For more realistic calculation, however, we have to force the self-consistency conditions [Eq. (5)]. Then, the pairing correlations will be weakened by the Coriolis force, and the deformation parameters β,γ may vary with the angular frequency. Figure 4(e) shows that the first crossing frequency $\hbar\omega_c^{(1)}$ has very small β dependence. This agrees with the result [8] of no sensitivity for deformation parameter β. The gap parameter $\Delta(=\Delta_p=\Delta_n)$ dependence of $\omega_c^{(1)}$ is also very small as seen from Fig. 4(f). Furthermore, as shown in Fig. 4(c) the first crossing frequency $\omega_c^{(1)}$ is constant. Therefore, we can expect that even if we force self-consistency conditions (5) the crossing frequencies will have about the same values. Thus, the first crossing frequency $\omega_c^{(1)}$ strongly depends only on the $T=0,J=9$ parameter $C_{9,1}$ [see Fig. 4(a)]. In the whole, one can see from Fig. 4 that the parameters we use in the calculation are reasonable, especially, our results are not sensitive to the choice of the β value in our single-j shell approach. All three types of band crossing we examined vary with rotational frequency smoothly. In the future, certainly, one will increase the self-consistency level step by step to deepen our understanding of the p-n interaction.

In conclusion, we here proposed a cranking model with $T=0$ as well as $T=1$ p-n fields, and analyzed the band structure of $N\approx Z$ systems using the $(g_{9/2})_p(g_{9/2})_n$ shell as a first practical try. In our analysis, we have shown that the p-n correlations play important roles for the mechanism of the band crossings in the $N=Z$ system. The $T=0,J=$odd p-n correlation gives rise to the first crossing along the yrast line, with change of the signature from $r_x=1$ of the ground band with even-spin to $r_x=-1$ of an excited band with odd-spin. In the example calculated presently with $N=Z=3$ in the $g_{9/2}$ shell (approximately corresponding to the case of ^{74}Rb), the isoscalar $J=9$ component plays the key role in causing such a transition as shown in Fig. 1(b). This result is in agreement with the finding reported in Ref. [9]. On the other hand, the $T=1,J=0$ p-n correlation results in a delay of the second (conventional) crossing. The results of our cranking calculation are consistent with the experimental data of the band crossing in ^{74}Rb and the delayed crossing frequency in ^{72}Kr as mentioned in the introduction. The basic features of p-n interaction explored in our single j shell calculation are very interesting, and tell that the cranking model with $T=0$ and $T=1$ p-n fields may be a powerful tool for describing microscopic structure in the nuclei with $N\approx Z$ for mass $70-80$ and lighter. Thus, we can expect that the p-n correlations are strong enough to form a static pair condensate in the nuclei of this region. We considered only the signature conserved terms for the p-n Hamiltonian in the present calculations. As mentioned before, the signature nonconserved terms, however, may also play an important role for $N=Z$ systems. This will be discussed in a forthcoming paper.

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