Spin dependence of intra–ground-state-band $E2$ transitions in the SU(3) limit of the sdg interacting boson model

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$B(E2,\ L+2\rightarrow L)$ transitions in the sdg interacting boson model SU(3) limit are studied with a general $E2$ transition operator. Analytical expressions are obtained using a group theoretic method. It is found that when using transition operators of the form $(d^\dagger\tilde{g}+g^\dagger\tilde{d})^2$ or $(g^\dagger\tilde{g})^2$, the $B(E2,\ L+2\rightarrow L)$ values in the ground-state band have an $L(L+3)$ dependent term. As L increases, the $B(E2)$ values can be larger than the rigid rotor model value. Application to ^{236,238}U is discussed.
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I. INTRODUCTION

In deformed nuclei, the spectrum exhibits rotational structures. The energy levels have the $L(L+1)$ dependence. The $B(E2,\ L+2\rightarrow L)$ in the ground-state band has the form $15/32\pi e^2Q_0^2(L+2)(L+1)/((2L+3)(2L+5))$ in the rigid rotor model [1]. As L increases, $B(E2)$ approaches a constant, $15e^2Q_0^2/128\pi$. If higher order effects are considered, the $B(E2)$ formula should be modified in two ways: (1) a cutoff due to finite number effect; (2) band-mixing effect due to rotation. The latter induces an $L(L+3)$ dependence, $B(E2,\ L+2\rightarrow L)=(L+2)(L+1)(M_1+M_2L(L+3))^2/(2L+3)(2L+5)$ [1]. Before the 1980s, experimental data on these values were not accurate enough to check the $L(L+3)$ dependence. With the development of experimental techniques, data for higher spin are available. For instance in ^{236,238}U, there are data for $L\sim 30$. This effect becomes quite appreciable.

In algebraic models, rotational nuclei are described by the SU(3) group [2–4] in a first approximation. There are two effects due to the finiteness of the particle number: the cutoff and falloff effects in the $B(E2)$ values. The $B(E2)$ must vanish at a truncation angular momentum where no higher angular momentum is allowed in the model. This is the cutoff effect. As L increases from zero, $B(E2)$ increases, then $B(E2)$ reaches its peak at about one quarter of the maximum angular momentum (or cutoff angular momentum). After that $B(E2)$ decreases approximately parabolically to zero. Specifically, if the $E2$ transition operator is taken as the generator of the SU(3) group, the $B(E2,L+2\rightarrow L)$ $\propto(L+2)(L+1)(\lambda-L)(\lambda+L+3)/(2L+3)(2L+5)$, where λ is the SU(3) irreducible representation label $(\lambda 0)$. This is true for the Elliott SU(3) model [2], the sd interacting boson model (IBM) [3] and sdg-IBM [5], and the pseudosymplectic model [4]. Experimentally, some deformed nuclei indeed obey this predicted behavior [4,6]. Some nuclei do not, and the SU(3) prediction underestimates the data [6–8]. For instance in ^{236}U, as L increases, $B(E2)$ is ever increasing. $B(E2)$ increases even for L at 30. Moreover it is greater than the rigid rotor model value. In order to make the $B(E2)$ not to fall with L in SU(3) based models, one has to make the observed $B(E2)$'s to be within $L\leqslant L_{\max}/4=\lambda/4$, or in other words, to increase λ. In boson models, this can be done by putting more pairs of nucleons into the system (more boson numbers) or by including higher spin nucleon pairs (higher spin bosons), as pointed out in Ref. [4]. This means that one has to include 35 bosons in the sdg-IBM, or 14 bosons in a boson model with s, d, . . . , up to $l=10$ bosons. It is unreasonable to include a number of bosons greater than half of the valence nucleons. Besides, there has been no experimental information that bosons higher than g should be included.

In practice, the SU(3) is broken [9,10]. One can depart from the SU(3) limit both in the Hamiltonian and in the $E2$ transition operator in the consistent Q formalism (CQF) [11], where the Q operator in the $E2$ transition is taken the same as that in the Hamiltonian. This has been done in the $1/N$ expansion technique by Kuyucak and Morrison [12]. They have shown that as the system deviates from the SU(3) limit, the resulting $B(E2)$ indeed produces an $L(L+3)$ dependent term. Li and Kuyucak [6] applied the results to various deformed nuclei with success.

If the SU(3) symmetry breaking is not big, we can use the SU(3) limit to study the effects of different $E2$ transition operators. The Hamiltonian is in the SU(3) limit, but the $E2$ transition operator is no longer the SU(3) generator. We can derive analytical expressions for them. In the sd-IBM, Van Isacker [13] has studied the $E2$ transitions in the SU(3) limit using a general $E2$ operator. In the sd-IBM, this general $E2$ transition operator induces transitions between different SU(3) irreducible representations. For the ground-state band, the L dependence is the same as that obtained using the SU(3) generator. Its effect is a renormalization of the effective charge. In the sdg-IBM, analytical expressions for the $B(E2)'$ using a general $E2$ operator is still missing due to its complexity [5]. The general $E2$ operator contains four terms: $(s^\dagger\tilde{d}+\text{H.c.})$, $(d^\dagger\tilde{d})^2$, $(d^\dagger\tilde{g})^2+\text{H.c.}$, and $(g^\dagger\tilde{g})^2$ (called sd, dd, dg, and gg term, respectively, hereafter). They can also be combined to form SU(3) tensors of rank (11), (22), (33), and (44). The (11) tensor is just the SU(3) generator, and an analytical expression is known for this tensor [5]. In this paper, we study analytically the $E2$ transition matrix elements in the ground-state band. The results show that with an

$E2$ transition operator involving the g boson, the L dependent term comes out. Applications to ^{236,238}U of the formula are also discussed. The paper is organized as follows. In Sec. II, we give the method of calculation and list the results. In Sec. III, we discuss the application to ^{236,238}U. We discuss the effects of the transition operator on ground-state-band transitions using the analytical results derived here. Effects of the operators on interband transitions are discussed in the $1/N$ expansion formalism. In Sec. IV, we give a discussion and summary. In the Appendix, we give a comparison of the exact result with the $1/N$ expansion results.

II. THE METHOD AND RESULTS

The most general $E2$ operator in the sdg-IBM can be written as

$$T(E2)=e_2[(s^\dagger\tilde{d}+d^\dagger s)^2+\chi_1(d^\dagger\tilde{d})^2+\chi_2(d^\dagger\tilde{g}+g^\dagger\tilde{d})^2 + \chi_3(g^\dagger\tilde{g})^2]. \quad (1)$$

For the SU(3) generator, $\chi_1=-\frac{11}{7}\sqrt{\frac{5}{32}}$, $\chi_2=\frac{9}{7}$, and $\chi_3=-\frac{3}{28}\sqrt{55}$. In this special case, one can apply the Elliott formula [2] to give analytical matrix elements. In order to calculate the matrix elements for each term in the general $E2$ operator, we use the standard group theoretical method. First we notice that the creation operators $s^\dagger$, $d^\dagger$, and $g^\dagger$ can be put into one symbol $b^\dagger_{lm}$, with $l=0,2,4$ for s and d and g bosons. $b^\dagger_{lm}$ is a U(15) tensor operator and transforms like the state $|[1](4\ 0)lm\rangle$. By applying the generalized Wigner-Eckart theorem, we can write the matrix element as

$$\begin{aligned}&\langle[n](\lambda\mu)L\|b^\dagger_l\|[n-1](\lambda'\mu')L'\rangle\\ &\quad=\sqrt{(2L+1)}\langle[n]\|\|b^\dagger\|\|[n-1]\rangle\\ &\qquad\times\left\langle\begin{matrix}[n-1] & [1] \\ (\lambda'\mu') & (4\ 0)\end{matrix}\middle|\begin{matrix}[n]\\ (\lambda,\mu)\end{matrix}\right\rangle\left\langle\begin{matrix}(\lambda'\mu') & (4\ 0)\\ L' & l\end{matrix}\middle|\begin{matrix}(\lambda\mu)\\ L\end{matrix}\right\rangle,\end{aligned} \quad (2)$$

where $\left\langle\begin{smallmatrix}[n-1] & [1] \\ (\lambda'\mu') & (4\ 0)\end{smallmatrix}\middle|\begin{smallmatrix}[n]\\ (\lambda,\mu)\end{smallmatrix}\right\rangle$ is the U(15)⊃SU(3) reduction isoscalar factor, and $\left\langle\begin{smallmatrix}(\lambda'\mu') & (4\ 0)\\ L' & l\end{smallmatrix}\middle|\begin{smallmatrix}(\lambda\mu)\\ L\end{smallmatrix}\right\rangle$ is the SU(3)⊃O(3) reduction isoscalar factor. The former can be found in Ref. [14], and the latter can be found in Ref. [15]. The triple barred quantity is the U(15) reduced matrix elements. It can be easily obtained as $\langle[n]\|\|b^\dagger\|\|[n-1]\rangle=\sqrt{n}$, whereas $\tilde{b}_{lm}=(-1)^{l-m}b_{l-m}$ behaves as (0 4) tensor. To calculate the matrix elements of $\tilde{b}_{lm}$, we use the reciprocal relation,

$$\langle L\|\tilde{b}_l\|L'\rangle=(-1)^{L-L'-l}\langle L'\|b^\dagger_l\|L\rangle. \quad (3)$$

The matrix elements of the composite operators can be calculated by

$$\begin{aligned}&\langle[n](\lambda\mu)L\|(b^\dagger_{l_1}\tilde{b}_{l_2})^2\|[n](\lambda'\mu)L'\rangle\\ &\quad=\sqrt{5}\sum_{(\lambda''\mu'')L''}\langle[n](\lambda\mu)L\|b^\dagger_{l_1}\|[n-1](\lambda''\mu'')L''\rangle\\ &\qquad\times\langle[n-1](\lambda''\mu'')L''\|\tilde{b}_{l_2}\|[n](\lambda'\mu')L'\rangle\\ &\qquad\times\begin{Bmatrix}l_1 & l_2 & 2\\ L' & L & L''\end{Bmatrix}.\end{aligned} \quad (4)$$

We have given the matrix elements of each term for the ground-state band. They are

$$\langle[n](4n0)L+2\|(s^\dagger\tilde{d}+d^\dagger s)^2\|[n](4n0)L\rangle=\sqrt{\frac{3(4n-L)(4n+L+3)(L+1)(L+2)}{280(2L+3)}\frac{(4n+L+1)(4n-L-2)(8n-5)}{(4n-3)(4n-2)(4n-1)}}, \quad (5)$$

$$\langle[n](4n0)L+2\|(d^\dagger\tilde{d})^2\|[n](4n0)L\rangle=-\sqrt{\frac{3(4n-L)(4n+L+3)(L+1)(L+2)}{7(2L+3)}\frac{(4n-L-2)(4n+L+1)(4n-4)}{7(4n-3)(4n-2)(4n-1)}}, \quad (6)$$

$$\begin{aligned}&\langle[n](4n0)L+2\|(d^\dagger\tilde{g}+g^\dagger\tilde{d})^2\|[n](4n0)L\rangle\\ &\quad=\sqrt{\frac{3(4n-L)(4n+L+3)(L+1)(L+2)}{70(2L+3)}\frac{(8n-1)[12(4n-4)^2+7L(L+3)]+8(4n-4)[4L(L+3)+9]}{42(4n-3)(4n-2)(4n-1)}},\end{aligned} \quad (7)$$

$$\langle[n](4n0)L+2\|(g^\dagger\tilde{g})^2\|[n](4n0)L\rangle=-\sqrt{\frac{2(4n-L)(4n+L+3)(L+1)(L+2)}{231(2L+3)}\frac{2(n-1)(4(4n-4)^2-16+3L(L+3))}{7(4n-3)(4n-2)(4n-1)}}. \quad (8)$$

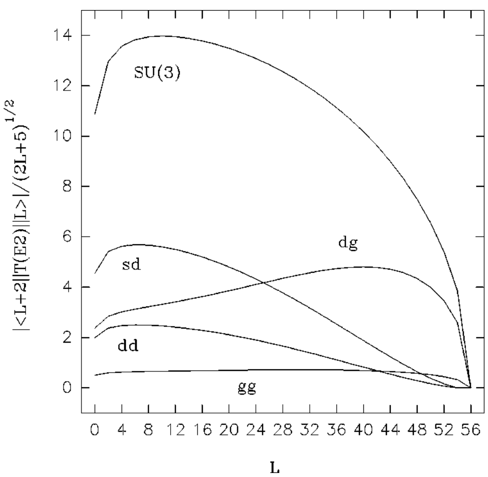

FIG. 1. $|\langle L+2\|T(E2)\|L\rangle|/\sqrt{2L+5}$ for each term (labels sd, dd, dg, gg) in the general $E2$ operator. The one labeled by SU(3) is the one for the SU(3) generator.

It is interesting to study the L dependence of each term. We have plotted the quantity $\langle L+2\|T(E2)\|L\rangle/\sqrt{2L+5} \propto \sqrt{B(E2,\ L+2\rightarrow L)}$ in Fig. 1 for each term for $N=14$. The relevant quantity for the SU(3) generator is also drawn. It is seen that the sd and dd terms increase with L rapidly for small L (about 4), then they reach their peak, and then go down with L. The $(g^\dagger\tilde{g})^2$ term is small, and the $L(L+3)$ term in it is weak. It is nearly a straight flat line. The dg term rises sharply with L. Its saturation point is much to the right, and is now at about $0.7L_{\max}$. When L is much smaller than $4n$, the dg and gg terms can be written as $\propto M_1+M_2L(L+3)$, which is the form of a Mikhailov plot.

When the contributions are combined in the SU(3) generator form, we recover the familiar known result. This is a good check on the correctness of the calculation. The terms can also be combined to form other forms of SU(3) tensors. In Table I, we give the combination coefficients for different SU(3) tensors.

TABLE I. Combination coefficients for different SU(3) tensor operators.

	$(s^\dagger\tilde{d}+d^\dagger s)^2$	$(d^\dagger\tilde{d})^2$	$(d^\dagger\tilde{g}+g^\dagger\tilde{d})^2$	$(g^\dagger\tilde{g})^2$
(11)	1	$-\frac{11}{7}\sqrt{\frac{5}{8}}$	$\frac{9}{7}$	$-\frac{3\sqrt{55}}{14}$
(22)	1	$-\frac{2\sqrt{40}}{77}$	$\frac{9}{7}$	$\frac{36}{7}\sqrt{\frac{5}{11}}$
(33)	1	$-\frac{11}{7}\sqrt{\frac{5}{8}}$	$-\frac{17}{14}$	$\frac{\sqrt{55}}{28}$
(44)	1	$\frac{4\sqrt{10}}{7}$	$-\frac{1}{7}$	$-\frac{4}{7}\sqrt{\frac{5}{11}}$

When they are combined in this way, we obtain analytical formulas for them,

$$\langle[n](4n0)L+2\|T(11)\|[n](4n0)L\rangle = \sqrt{\frac{3(L+1)(L+2)(4n-L)(4n+L+3)}{280(2L+3)}5}, \quad (9)$$

$$\langle[n](4n0)L+2\|T(22)\|[n](4n0)L\rangle = \sqrt{\frac{3(L+1)(L+2)(4n-L)(4n+L+3)}{280(2L+3)}\frac{15(8n+3)}{11(4n-1)}}, \quad (10)$$

$$\langle[n](4n0)L+2\|T(33)\|[n](4n0)L\rangle = \sqrt{\frac{3(L+1)(L+2)(4n-L)(4n+L+3)}{280(2L+3)}\frac{5(8n(4n+3)-7L(L+3)-20)}{12(4n-1)(2n-1)}}, \quad (11)$$

$$\langle[n](4n0)L+2\|T(44)\|[n](4n0)L\rangle = \sqrt{\frac{3(L+1)(L+2)(4n-L)(4n+L+3)}{280(2L+3)}\frac{5(8n+3)(2n(4n+3)-L(L+3)-5)}{33(4n-1)(2n-1)(4n-3)}}. \quad (12)$$

It can be seen that (11) and (22) tensors do not have the $L(L+3)$ dependence. But (33) and (44) tensors have the explicit $L(L+3)$ dependence. This is different from Yoshinaga's numerical results [5] where no L dependence was found for intraband transitions.

III. APPLICATION TO ^{236,238}U

We applied the results to ^{236,238}U. For simplicity, we used the following $E2$ operator:

$$T(E2)=e_2[T(11)+\alpha(d^\dagger\tilde{d})^2+\beta(g^\dagger\tilde{d}+d^\dagger\tilde{g})^2], \quad (13)$$

where $T(11)$ is the SU(3) generator. The effective charge e_2 and the structure constant α and β are determined by fitting to experimental data. If only the intraband transitions are concerned, $T(11)$ and dg terms are enough for a good description of the data. However, a non-SU(3) $E2$ operator will give rise to interrepresentation transitions. Analytical expressions are absent, since its calculations involve relevant $\mathrm{SU}(3)\supset\mathrm{O}(3)$ reduction isoscalar factors which are not available. We have used the $1/N$ expansion formalism [16] to get the effects of the dd and dg operators on interband transitions (a comparison of the $1/N$ expansion and the exact results derived in Sec. II is given in the Appendix). The $T(11)$ term does not contribute to interrepresentation transi-

TABLE II. $E2$ transition matrix elements in e b.

		$\langle 2_g \| T(E2) \| 0_g \rangle$	$\langle 2_\beta \| T(E2) \| 0_g \rangle$	$\langle 2_\gamma \| T(E2) \| 0_g \rangle$
^{236}U	Calc.	3.26	0.47	0.71
	Expt.	3.26(7)		
^{238}U	Calc.	3.38	0.23	0.39
	Expt.	3.38(3)	0.23(3)	0.36(4)

tions. The dg term alone gives $\beta \rightarrow$g.s. transitions stronger than the $\gamma \rightarrow$g.s. transitions. This is in contradiction with the experiment. The dd term is necessary to reproduce the data. To see the effects more clearly, we look at the matrix elements in ^{238}U($N=15$):

$$\langle 2_g \| T(E2) \| 0_g \rangle = 24.2899 e_2 - 4.4630 e_2 \alpha + 5.2987 e_2 \beta,$$

$$\langle 2_\beta \| T(E2) \| 0_g \rangle = -0.2320 e_2 \alpha - 2.0867 e_2 \beta,$$

$$\langle 2_\gamma \| T(E2) \| 0_g \rangle = 0.8082 e_2 \alpha + 1.1925 e_2 \beta,$$

$$\langle 26_g \| T(E2) \| 24_g \rangle = 95.2220 e_2 - 13.8126 e_2 \alpha + 29.8849 e_2 \beta. \tag{14}$$

We see that when $E2$ is taken as the SU(3) generator, the interrepresentation matrix elements are zero. As the dd term enters in, interrepresentation transitions appear. The transition from 2_γ to ground state is stronger than the transition from 2_β to ground state. This is the same as in the sd-IBM [17]. Different from the sd-IBM, the dd term also influences the intraband transitions. Its effect can be seen from the $2_g \rightarrow 0_g$ and $26_g \rightarrow 0_g$ matrix elements. The contributions of $T(11)$ are $24.2899 e_2$ to $2_g \rightarrow 0_g$ transition matrix element and $95.2220 e_2$ to $26_g \rightarrow 0_g$, a factor of 3.92 increase. The contributions of the dd term are $-4.4630 e_2 \alpha$ to $2_g \rightarrow 0_g$ transition matrix element and $-13.8126 e_2 \alpha$ to $26_g \rightarrow 0_g$, a factor of 3.1 increase. If $e_2 \alpha$ is positive, the dd term subtracts more from $2_g \rightarrow 0_g$ than from $26_g \rightarrow 0_g$. This effectively increases the ratio of the two matrix elements. The dd term gives $\gamma \rightarrow$g.s. transitions stronger than $\beta \rightarrow 0_g$ transitions. The dg term adds more to the $26_g \rightarrow 0_g$ matrix element than to the $2_g \rightarrow 0_g$ matrix element. This also increases the ratio of these two matrix elements. The dg term gives $\beta \rightarrow$g.s. transitions stronger than $\gamma \rightarrow 0_g$ transitions. A balanced choice of the parameters can give a good description of the $E2$ data both at low spin and high spin. We have determined the three parameters for ^{238}U by fitting 3 matrix elements: $2_g \rightarrow 0_g$, $2_\beta \rightarrow 0_g$, and $26_g \rightarrow 24_g$. This gives $e_2 = 0.1828$ e b, $\alpha = 2.0$, and $\beta = 0.36$. The resulting matrix element $\langle 2_\gamma \| T(E2) \| 0_g \rangle$ is 0.39 e b, which is in agreement with experimental data: 0.36(4) e b. We also determined the parameters for ^{236}U. There are no data for interband transitions. We require that $|\langle 2_\gamma \| T(E2) \| 0_g \rangle| / \langle 2_\beta \| T(E2) \| 0_g \rangle = 1.5$. This gives $e_2 = 0.2203$ e b, $\alpha = 2.96$, and $\beta = 0.70$. The calculated matrix elements are $\langle 2_\gamma \| T(E2) \| 0_g \rangle = 0.71$ e b and $\langle 2_\beta \| T(E2) \| 0_g \rangle = 0.47$ e b. This is a factor of 2 larger than that in ^{238}U. This remains to be checked by experiment. A summary of the matrix elements calculations is given in Table II. The calculated intraband transitions in ^{236}U and ^{238}U are compared with experiments in Fig. 2 and Fig. 3, respectively. The experimental data are taken from Refs. [18,19], respectively. It can be seen that the rising trend with L is reproduced well.

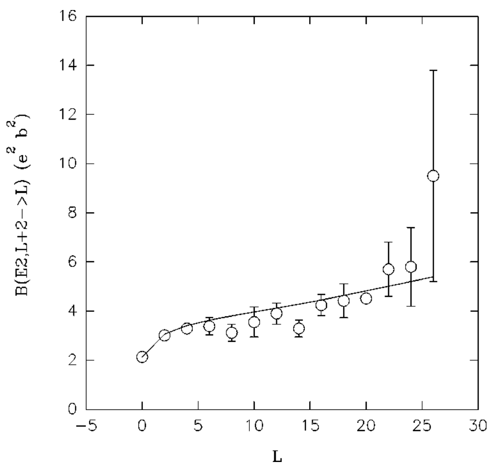

FIG. 2. Comparison of $B(E2,\ L+2 \rightarrow L)$ for ^{236}U.

The above discussion is only for illustration. In reality, SU(3) symmetry is broken. For instance in the systematic study [6], the SU(3) symmetry in ^{236,238}U is broken by deviation of Q from the SU(3) generator and the addition of ϵ_d and ϵ_g terms in the Hamiltonian. SU(3) breaking introduces mixing of the states in different bands. It brings changes to the corresponding $E2$ transitions. However, the above discussions point out that a careful choice of the parameters in the $E2$ operator can improve the agreement with data, especially for the high spin transitions.

IV. DISCUSSION AND SUMMARY

We have calculated the matrix elements for a general $E2$ operator for the ground-state band in the SU(3) limit of the sdg IBM. Analytical expressions are given for each of the four terms. It is found that the dg and gg terms can produce an $L(L+3)$ dependence. This $L(L+3)$ is consistent with the band-mixing results in the geometric model in general. This L dependence can delay the $B(E2)$ saturation point to 70% of the cutoff angular momentum, and alleviated the need to introduce more and higher bosons in the boson model.

In this study, we have departed from the consistent $Q \cdot Q$

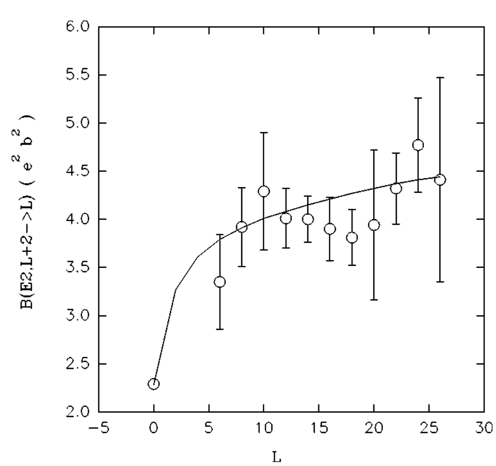

FIG. 3. The same as Fig. 2 but for ^{238}U.

formalism. The consistent $Q\cdot Q$ formalism is simple and elegant, and has gained great success in many phenomenological studies. The $B(E2)$ in many deformed nuclei indeed behaves in a way this formalism has predicted. But it is not necessarily to choose them as equal [10]. The spectrum of rotational nuclei are similar, but their $B(E2)$ behaves quite differently, especially at high spins [4]. $B(E2)$ in some nuclei agrees with the SU(3) prediction, and in some nuclei $B(E2)$ becomes smaller than the SU(3) result. While in some other nuclei, $B(E2)$ is greater than the SU(3) results. Specifically this corresponds to 0, negative and positive values for the α and β in Eq. (13). It will be interesting to study microscopically the parameters in the $E2$ transition operator.

Finally, we want to mention that the problem of $B(E2)$ reduction can also be solved by including multi-ph excitations in the core-excited IBM [20]. This model is the extension of the idea of introducing 2p-2h excitations in the model space of the IBM [21,22]. In this model, besides the ordinary space of $|(sd)^{N_0}\rangle$, there are also $|(sd)^{N_0+2}\rangle$ ... spaces. As the d boson energy for different configuration changes differently, the cutoff angular momentum can be adjusted to very high values.

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TABLE III. Comparisons of $\langle L+2\|(s^\dagger\tilde{d}+d^\dagger s)^2\|L\rangle$ between the 1/*N* expansion and the exact results.

N		$L=0$	$L=2$	$L=10$	$L=20$	$L=30$
10	1/*N*	7.4374	11.8247	19.4039	18.1519	5.6477
	Exact	7.4693	11.8659	19.3127	18.0870	8.6216
14	1/*N*	10.1419	16.1905	28.0298	32.3535	27.5049
	Exact	10.1643	16.2215	27.9954	32.1999	28.0434
20	1/*N*	14.2141	22.7400	40.4450	51.0424	53.7372
	Exact	14.1986	22.7176	40.4506	51.1477	53.7556

APPENDIX: COMPARISONS WITH THE 1/*N* EXPANSION

In the text we have applied the 1/*N* expansion for interband transitions. Intraband transitions can also be obtained from the 1/*N* expansion formalism. It is meaningful to compare the exact results obtained in this work to the results from the 1/*N* expansion to see the goodness of the 1/*N* expansion. In Table III, we give comparisons for the intraband transition matrix elements of the sd term for different values of *N* and *L*. We see even with $N=10$, the 1/*N* expansion still gives a very good value for $L=20$. As *N* goes up to 14, the 1/*N* expansion can reproduce the exact value quite accurately for $L=30$. When $N=20$, the 1/*N* expansion is nearly exact. Comparisons of other terms yield similar results. 1/*N* expansion is indeed a powerful tool for handling systems with large particle numbers.

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