Effective interaction in quasielastic electron scattering calculations

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The transverse nuclear response for quasifree electron scattering is discussed. The analysis is done by comparing different calculations performed in the random-phase approximation and ring approximation frameworks. The importance of the exchange terms in this energy region is investigated and the changes in the nuclear responses due to the modification of the interaction are evaluated. The calculated quasielastic responses show clear indication of their sensibility to the details of the interaction and this imposes the necessity of a more careful study of the role of the different channels of the interaction in this excitation region. [S0556-2813(98)00204-0]

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I. INTRODUCTION

An aspect of great importance in nuclear structure calculations at any excitation energy concerns the role of the effective interaction. At low energies this problem has generated a considerable body of work in the last 20 years. On the contrary, this is a question not studied yet in depth in the literature for higher energies.

Giant resonances show an intricate mixture of multipolarities and the study of how the interaction affects it is a difficult task. In the quasielastic peak region, the problem of the longitudinal and transverse separation has occupied most of the investigations carried out until now and the discussion of the effects due to changes in the effective interaction have not been considered in detail.

As an example, we mention the considerable number of random-phase approximation (RPA) type calculations performed in this energy region [1–9]. Much of these works used residual interactions which include basically a zero-range term plus meson-exchange potentials corresponding to π, ρ, and, eventually, other mesons, and then an important point concerning the interaction refers to the values chosen for the parameters entering in the zero-range piece. However, and to the best of our knowledge, only in Ref. [9] can a certain discussion relative to the effects of varying these parameters be found. In fact, the common practice is to pick an interaction from the literature, which usually corresponds to a parametrization fixed for low energy calculations, and afterwards use it to evaluate quasifree observables sometimes without taking care of the effective theory in which the interaction was adjusted. It is obvious that, to a certain level, doubtful results are possible because of the known link between effective theory and interaction.

In this work we want to address this question and investigate if different parametrizations of the interaction can produce noticeable differences in the results and the extent to which the use of an interaction fixed for a given effective theory affects the results obtained within a different one. In Sec. II we give the details about the effective theories and interactions used to perform the calculations. In Sec. III we show and discuss the results we have obtained. Finally, we present our conclusions in Sec. IV.

II. EFFECTIVE THEORIES AND INTERACTIONS

The first interaction we consider in this work is the so-called Jülich–Stony Brook interaction [10] which is an effective force widely used for calculations in the quasielastic peak region. It is given as follows:

$$V^{\mathrm{I}}_{\mathrm{res}} = V_{\mathrm{LM}} + V_{\pi} + \widetilde{V}_{\rho}. \qquad (1)$$

Here V_{LM} is a zero-range force of Landau-Migdal type, which takes care of the short-range piece of the NN interaction:

$$V_{\mathrm{LM}} = C_0[g_0 \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 + g_0' \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2]. \qquad (2)$$

On the other hand, a finite-range component generated by the $(\pi+\rho)$-meson-exchange potentials is also included. The tilde in $\widetilde{V}_{\rho}$ means that the bare ρ-exchange potential is slightly modified in order to take into account the effect of the exchange of more massive mesons. In particular, a factor of $r=0.4$ is multiplying the finite-range nontensor piece of the potential (see Ref. [10] for details). This force was fitted to reproduce low energy magnetic properties in the lead region (specifically, magnetic resonances in ^{208}Pb and magnetic moments and transition probabilities in the neighboring nuclei). The calculations were performed in the framework of the RPA and Woods-Saxon single-particle wave functions were used in the configuration space. The values $g_0=0.57$ and $g_0'=0.717$ (with $C_0=386.04$ MeV fm^3) were found to be adequate to describe the properties studied.

As previously stated, this interaction has been considered in different calculations in the quasielastic peak region (see, e.g., [9]). The problem arises because some of them have been done within the Fermi gas (FG) formalism, with a local density approximation to describe finite nuclei, in the ring approximation (RA), where the exchange terms are not taken into account, and with the full unmodified ρ-exchange potential. Under these circumstances, the possible effects in the nuclear responses due to the inconsistency between the model and the effective interaction could be non-negligible. This is precisely the first aspect we want to investigate. To do that we compare the responses obtained with the Jülich–Stony Brook interaction with those calculated with a second effective force of the form

$$V_{\rm res}^{\rm II}=V_{\rm LM}+V_{\pi}+V_{\rho}, \qquad (3)$$

by considering the same values for the zero-range parameters in both cases. The force in Eq. (3) only differs from $V_{\rm res}^{\rm I}$ in the ρ potential which, in this case, does not include any reduction factor. Both RPA and RA effective theories are used to analyze the results.

A second question of interest to us is to determine how the change of the zero-range parameters affects the responses calculated within a given theory. This will inform us about the necessity of considering or not in detail the role of these parameters. To analyze this aspect we use the interaction $V_{\rm res}^{\rm II}$, because it is precisely the interaction used in practice in much of the calculations mentioned above. Obviously, the parameters g_0 and g_0' must be fixed for this interaction and, as in the case of the Jülich–Stony Brook interaction, this has been done in a way such that some low energy properties in the lead region are reproduced (see details in the next section).

Our analysis focuses on the transverse response functions in the quasielastic peak. We will not consider the longitudinal ones because they are strongly influenced by the spin-independent pieces of the interaction (in particular, the f_0 and f_0' channels) and these are difficult to fix at low energy because of the role played by the scalar mesons not usually taken into account.

III. RESULTS OF THE CALCULATIONS

The investigation of the various questions we are interested in has been carried out by comparing different calculations of the transverse (e,e') responses in ^{40}Ca for three different momentum transfer ($q=300$, 410, and 550 MeV/c).

First we have studied the effects produced when an effective interaction, which has been determined in a given effective theory (e.g., RPA), is used to calculate (e,e') transverse responses in a different framework (e.g., RA).

By considering the parametrization of Ref. [10] (that is, $g_0=0.57$, $g_0'=0.717$, and $C_0=386.04$ MeV fm^3), we have carried out two different calculations, the results of which are shown Fig. 1. Therein, solid curves correspond to the calculations performed in the FG approach within the RA and with the interaction $V_{\rm res}^{\rm II}$ in Eq. (3). On the other hand, dotted lines have been obtained within the RPA, also for the FG. The model used in this case is the one developed in Ref. [7], which, contrary to what happens for the RA approach, includes explicitly the exchange terms in the RPA expansion. In this case we have used the interaction $V_{\rm res}^{\rm I}$ in Eq. (1) and we have adopted the factor $r=0.4$, which is consistent with the parametrization used. Also in Fig. 1, we have plotted the free FG responses for comparison (dashed curves).

The first comment one can draw from these results is that the use of the interaction, as it was fixed at low energy, leads to transverse responses which are quite different from those obtained in the RA (with $V_{\rm res}^{\rm II}$) calculation, though the differences reduce with increasing momentum transfer. As we can see, the results obtained in the RPA are peaked at lower energies and this is clear evidence of a more attractive residual interaction. It is straightforward to check this point because the central piece of the $V_{\rm res}^{\rm I}$ is attractive, while the contrary happens for $V_{\rm res}^{\rm II}$, at least for $q\leqslant 2k_{\rm F}$. On the other hand, it is interesting to note how the RA results are more similar to the free response as long as q increases, while the same does not occur for the RPA responses.

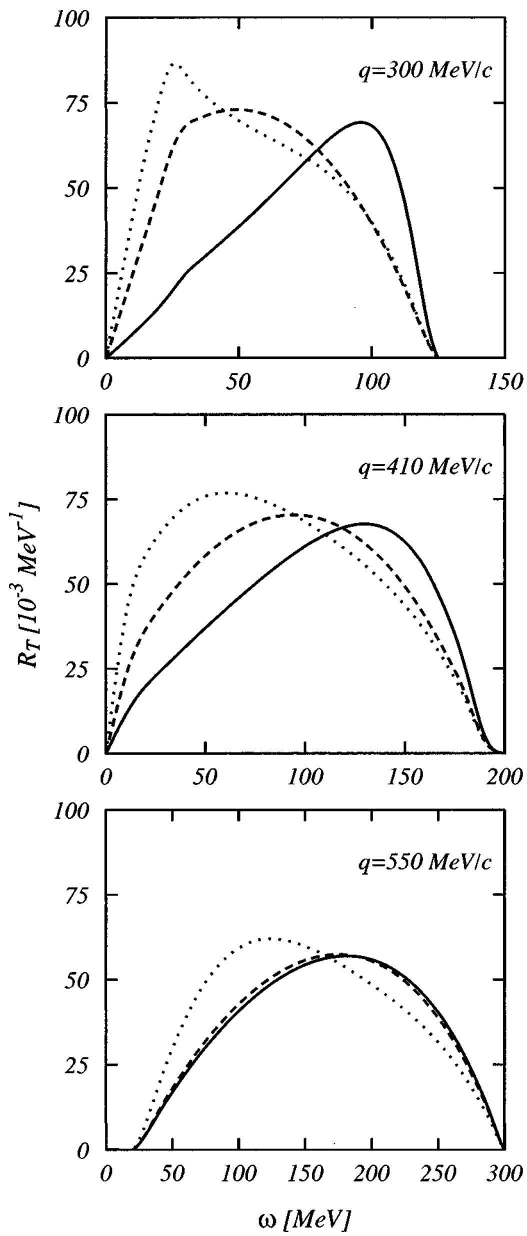

FIG. 1. Transverse nuclear responses for ^{40}Ca, calculated for the three-momentum transfers we have considered in this work. Dotted lines correspond to an RPA calculation with $V_{\rm res}^{\rm I}$, while solid curves represent the RA results for $V_{\rm res}^{\rm II}$. In both cases the values $g_0=0.57$ and $g_0'=0.717$ (with $C_0=386.04$ MeV fm^3) have been used. Dashed curves give the free FG responses. In all the calculations a value of $k_{\rm F}=235$ MeV/c has been used.

Obviously, the reason for the discrepancies between both calculations can be ascribed to the two basic ingredients of the effective theories used in each case: the exchange terms, which are included in the RPA calculations but not in the RA ones, and the reduction factor r modifying the ρ-exchange potential.

Before going deeper into this question, it is worth commenting on the nuclear wave functions used in the calculations discussed above. As in any FG-type calculation, plane waves have been considered here to describe the single-particle states. The fact that the interaction was fixed in a framework which considered microscopic RPA wave functions, based on Woods-Saxon single-particle states, is an obvious inconsistency. Despite that, it has been shown [11,12] that, in this energy region, the details concerning the nuclear wave functions are not extremely important and, at least to some extent, the shell-model response can be reasonably described with the FG model, provided an adequate value of the Fermi momentum k_F is used. In the present work, where we study the response in ^{40}Ca, we have taken $k_F = 235$ MeV/c which gives a good agreement between FG and finite nuclei calculations [11].

We come back to investigate the reasons for the large discrepancy between the RPA and RA calculations presented above. To do that we have done two new calculations: RA with $V^{\mathrm{I}}_{\mathrm{res}}$ and RPA with $V^{\mathrm{II}}_{\mathrm{res}}$. These calculations have been compared with the two previous ones by means of the two following quantities:

$$\gamma^{r}_{\mathrm{exc}}(q,\omega) = \frac{R_T^{\mathrm{RPA}(r)}(q,\omega) - R_T^{\mathrm{RA}(r)}(q,\omega)}{R_T^{\mathrm{RA}(r)}(q,\omega)} \tag{4}$$

and

$$\gamma^{\mathrm{mod}}_{r}(q,\omega) = \frac{R_T^{\mathrm{mod}(r=1.0)}(q,\omega) - R_T^{\mathrm{mod}(r=0.4)}(q,\omega)}{R_T^{\mathrm{mod}(r=0.4)}(q,\omega)}. \tag{5}$$

The first one gives us information about the effect of the consideration of the exchange terms in the calculation. The corresponding results have been plotted in Fig. 2 (left panels). The first aspect to be noted is that the exchange terms produce effects considerably larger for $V^{\mathrm{I}}_{\mathrm{res}}$ (solid lines) than for $V^{\mathrm{II}}_{\mathrm{res}}$ (dashed curves). These effects reduce with increasing momentum transfer and they are rather small for $V^{\mathrm{II}}_{\mathrm{res}}$ above $q = 410$ MeV/c.

On the other hand, the effect of the reduction factor r in the ρ-exchange potential is measured with the parameter γ_r. The values of this parameter for the two effective models considered, these are the RPA and RA, are shown in Fig. 2 (right panels), with solid and dashed curves, respectively. It is apparent that the effects of considering the r factor are much larger than those due to the exchange. In general they are more important for the RA calculations than for the RPA ones, and reduce the higher q is.

The first conclusion to be noted is that when using a given interaction it is mandatory to take care of the effective theory where its parametrization was fixed. The change of the framework produces results which could not be under control.

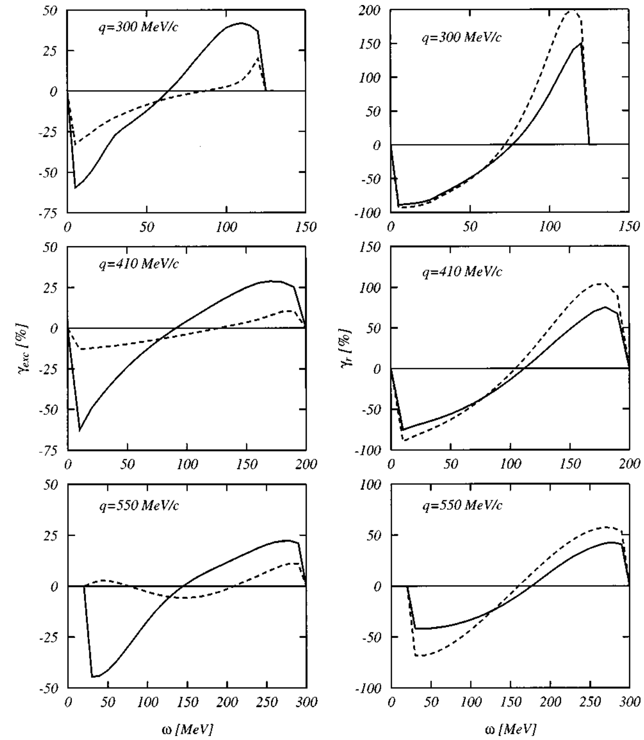

FIG. 2. Left panels: γ_{exc}, in percentage, as defined in Eq. (4). Dashed (solid) curves give the results obtained for $r = 1.0(0.4)$. Right panels: γ_r, in percent, calculated as in Eq. (5), for the RPA (solid curves) and RA (dashed curves).

The open question in this respect is how different become the responses calculated within different effective theories but with an interaction fixed consistently with the theory. This is the second aspect we investigate. To do that we have considered the interaction $V^{\mathrm{II}}_{\mathrm{res}}$. The main reason for using this force is that, as stated in Sec. II, this is the interaction used in practice in some of the calculations found in the literature and mentioned above. On the other hand, the interaction $V^{\mathrm{I}}_{\mathrm{res}}$ appears to be much too attractive for this energy region (see Fig. 1). Of course, the first point to consider is the determination of the parameters g_0 and g_0' of the Landau-Migdal piece. We have fixed them in such a way that the energies and B values of the two 1^+ states in ^{208}Pb at 5.85 and 7.30 MeV are reproduced. This has been done both in the RPA and RA. The reason for choosing these two states lies in their respective isoscalar and isovector character, which makes them particularly adequate to permit the determination of both parameters almost independently. The values obtained in this procedure are shown in Table I. The small value of g_0 needed for the RA calculation is remark-

TABLE I. Values of the Landau-Migdal parameters g_0 and g_0' obtained in the procedure of fixing the effective interaction $V^{\mathrm{II}}_{\mathrm{res}}$ (see text). The values quoted ''RPA'' (''RA'') correspond to calculations performed with (without) the consideration of the exchange terms.

Effective theory	g_0	g_0'
RPA	0.470	0.760
RA	0.038	0.717

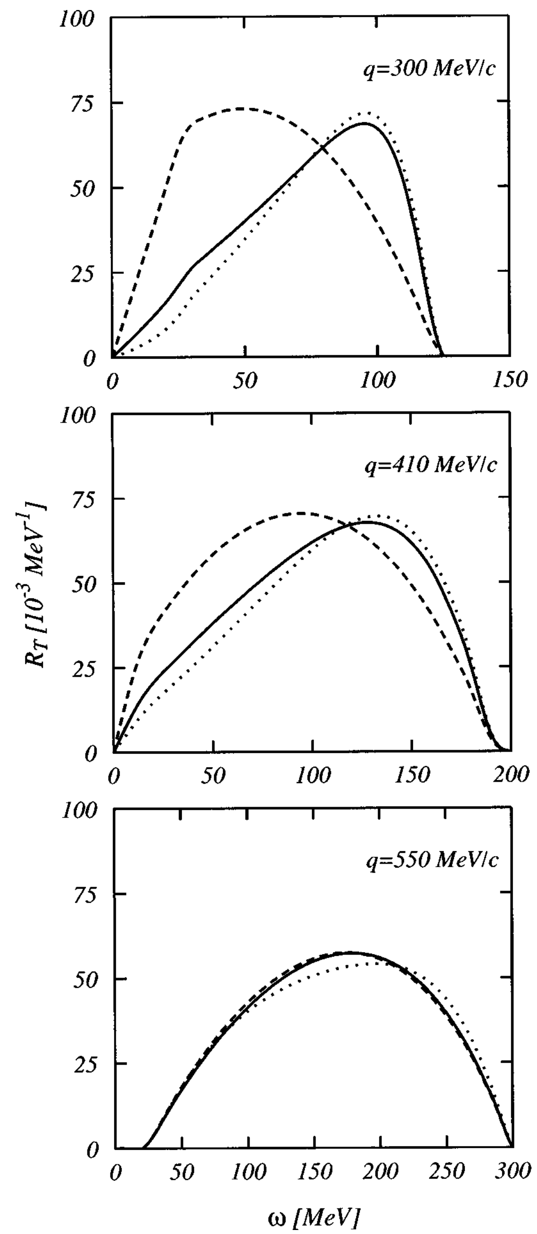

FIG. 3. Transverse nuclear responses for ^{40}Ca, calculated with the $V_{\rm res}^{\rm II}$ interaction. Dotted lines correspond to an RPA calculation while solid curves represent the RA results. The values of g_0 and g_0' in Table I have been used. Dashed curves give the free FG responses. In all the calculations a value of $k_{\rm F}=235$ MeV/c has been used.

able. A similar result is found when a pure zero-range Landau-Migdal interaction is adjusted, with the same criteria, in RPA-type calculations (see Refs. [13, 14]). This points out the importance of the exchange, at least at low energy.

With the interaction fixed in this way we have evaluated the transverse responses for the three-momentum transfer we are considering throughout this work. The results are shown in Fig. 3 where dotted (solid) curves correspond to the RPA (RA) calculations. Dashed lines represent the free FG responses. As we can see, the differences between the results obtained with the two effective theories are now much smaller than in Fig. 1.

Two points deserve a comment. First, it is clear that the large differences observed between the RPA calculation here discussed and that shown in Fig. 1 are mainly due to the presence of the reduction factor $r=0.4$ in the $V_{\rm res}^{\rm I}$ interaction. Second, the similitude of the results obtained with the two calculations done now with $V_{\rm res}^{\rm II}$, shows up the relevance of the link between effective theories and interactions.

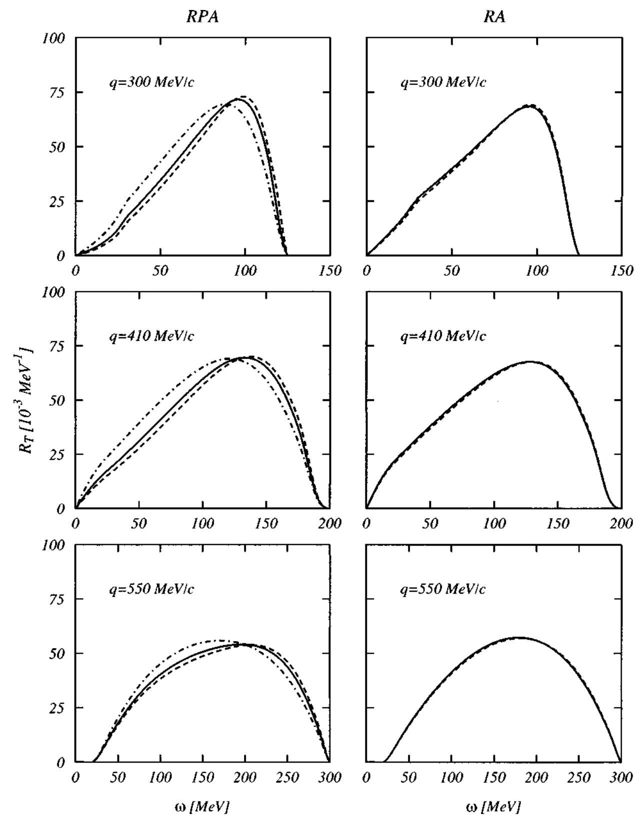

FIG. 4. R_T responses calculated in the RPA (left panels) and RA (right panels). Solid curves correspond to the parametrizations of Table I. Dash-dotted curves have been obtained with $g_0=0$, while the dashed ones correspond to $g_0=0.70$ (0.57) for the RPA (RA) calculation, with the same values of g_0' as for the solid curves.

The last aspect we want to analyze is how the responses calculated in a given approach change when the zero-range parameters are modified. In other words, we want to determine what is the role of these parameters. How g_0' affects the responses is a point which has been investigated in certain detail in different previous works (see, e.g., Ref. [7]) and then we focus here in g_0. Its influence can be seen in Fig. 4, where we compare the responses plotted in Fig. 3 (solid curves) with those obtained by changing the g_0 parameter in order to use values considered by different authors. Dash-dotted curves correspond to $g_0=0$. Dashed lines represent the responses obtained with $g_0=0.70$ (0.57) for the RPA (RA) calculation. The values of g_0' have not been changed. The first point to be noted is the insensibility of the RA responses to the changes in g_0. As we can see, strong changes in g_0 produce almost no effect on the RA result. This can be easily understood because in the ring series the g_0 contribution is weighted with the magnetic moment μ_s^2 while the g_0' piece appears with μ_v^2. That means that the g_0 contribution is $\mu_s^2/\mu_v^2\approx 1/28$ of the g_0' contribution. The situation is different in the RPA case, where the g_0 contribution is as important as the g_0' one because of the presence of the exchange terms (see Ref. [7]). This makes it such that some of the RA calculations performed by other authors can be considered as ''consistent'' in practice, of course, despite the fact that these parametrizations are unable to reproduce low energy properties. For example, in Ref. [9], the parametrization of the Jülich–Stony Brook interaction was considered and this coincides with one of those used here ($g_0 = 0.57$ and $g'_0 = 0.717$).

The results obtained in this work open a series of questions which we consider worthy for nuclear calculations in this energy region. In the following we enumerate and comment on them.

(1) It has been shown that the strength of the tensor piece of $V^{\mathrm{I}}_{\mathrm{res}}$ is too strong to describe low energy properties (see, e.g., Ref. [13]) and different mechanisms have been proposed to cure this problem [core-polarization effects [15], two-particle–two-hole (2p-2h) excitations [16], *in-medium* scaling law [17], etc.]. The role of the tensor part of the interaction in the quasielastic peak should be investigated in order to establish the effective force to be used.

(2) The presence of the exchange terms increase the sensitivity of the responses to the details of the interaction. How important the interference between these terms and other physical mechanisms basic in this energy region (such as, e.g., meson-exchange currents, final state interactions, short-range correlations, etc.) can be is a matter of relevance in order to fully understand the nuclear response. The analysis of the possible differences between the RA and RPA with respect to these effects is of special interest in view of the fact that RA calculations are the most usual in the quasielastic peak.

(3) The procedure of fixing the interaction is basic in order to deal with the possibility of having a unique framework to calculate the nuclear response at any momentum transfer and excitation energy. The problem of developing such a ''unified'' model is still unsolved, but the cross analysis of low energy nuclear properties and quasielastic peak responses could give valuable hints.

IV. CONCLUSIONS

In this work we have analyzed the role of the effective interaction in the quasielastic peak region by comparing the results obtained with different effective theories and forces previously fixed in order to give a reasonable description of several low energy nuclear properties.

Some conclusions can be drawn after our analysis. First, it has been found that the interaction plays a role that, similarly to what happens at low excitation energy, cannot be neglected. The particular point to be noted is the necessity of using effective interactions which have been fixed within an effective theory.

Second, the procedure we have followed to perform the calculations, that is, to determine the interaction at low energy before calculating at the quasielastic peak, seems to be adequate to look for a ''unique'' framework to calculate the nuclear response in different energy and momentum regimes.

The role of the tensor piece of the interaction must be investigated. At low energy is a basic ingredient of the nuclear structure calculations. Thus it is important to disentangle its contribution in other excitation energy regions. Additionally, it seems encouraging to analyze the problem by including other physical mechanisms (meson-exchange currents, short-range correlations, final state interactions, higher order (2p-2h) configuration mixing effects, etc.) which are known to be important in the description of the nuclear response and which depend on the interaction.

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