Quadrupole contribution to K^+ ^{6}Li scattering

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The quadrupole cross sections for K^+ ^{6}Li elastic scattering and inelastic scattering to the 3^+ excited state are calculated using the distorted wave impulse approximation. Comparison is made to recent data to determine the importance of the quadrupole cross sections and to see if both the K^+ ^{6}Li and K^+ ^{12}C elastic scattering data can be fit using the same K^+-nucleon amplitudes. [S0556-2813(98)05603-9]

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The K^+ has proved to be a useful probe for investigating effects of the nuclear medium on the meson-baryon interaction. Due to its relatively long mean free path at low energies, the K^+ can penetrate into the interior of light nuclei. Total cross section data of K^+ with light nuclei [1] suggest that the K^+ nucleon interaction is enhanced in the nuclear medium. To explain the cross section enhancement, a rescaling of the nucleon's size [2], a decrease in the mass of the vector mesons [3], and meson exchange currents [4–6] have been considered. It is still not clear what the cause of this anomaly is. To complement the total cross section data, elastic scattering differential cross sections [7] were recently taken on ^{6}Li and ^{12}C for a K^+ laboratory momentum of 715 MeV/c. Published calculations [7] treat the nucleus as a 0^+ state, although the ground state has $J^P=1^+$. Also, the first excited state, $J^P=3^+$, is only 2.4 MeV above the ground state and is not at present resolvable experimentally. It is therefore important to know at what angles the multipole contributions to the elastic cross section become important, as well as the magnitude of the cross section to the 3^+ excited state. In this Brief Report, we consider these questions.

Calculations of the multipole cross sections were carried out using the distorted wave impulse approximation (DWIA), the details of which are described in Ref. [8]. The transition amplitude F_{if} from an initial nucleus to a final nucleus state is given by $F_{if}=\langle\Psi_f|\langle\chi_f|t_{\rm op}|\chi_i\rangle|\Psi_i\rangle$ where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial and final nuclear states, and χ_i and χ_f correspond to the distorted waves of the incoming and outgoing K^+. The distorted waves and elastic scattering amplitudes, were obtained from the optical potential of Ref. [9]. The transition operator $t_{\rm op}$ for the different multipole contributions is taken to be

$$t_{\rm op}=(\lambda_0(E)+\lambda_1(E)\mathbf{k}\cdot\mathbf{k}'+i\lambda_{sf}(E)\sigma\cdot\mathbf{k}\times\mathbf{k}')\hat{a}^\dagger\hat{a} \quad (1)$$

following Ref. [8]. The complex amplitudes, $\lambda_i(E)$, are obtained from the phase shift analysis of Ref. [10] at the appropriate center of mass energy for the reaction. Neutron and proton wave functions for the transition density were generated using a Woods-Saxon potential of radius $1.3A^{1/3}$ fm. A potential depth of 51.3 MeV (51.6 MeV) produces an $l=1$ neutron (proton) with the experimental binding energy for ^{6}Li. The transition amplitude, F_{if}, can be decomposed into spin (S), orbital (L), and total (J) angular momentum transfered to the nucleus. Using the $J(KS)$ notation of Ref. [11], we calculated the $2(20)$, $1(01)$, $1(21)$, and $2(21)$ contributions to the elastic scattering differential cross section, as well as the $2(20)$ contribution to the 3^+ excited state. The reduced matrix elements for the calculation are from Ref. [11]. The reduced matrix elements for the 3^+ inelastic transition were checked by calculating π^+ ^{6}Li inelastic scattering and comparing to the data of Ref. [12].

We find that the *only significant contribution comes from the* $2(20)$ *or quadrupole nonflip piece*, and that *both the ground state and* 3^+ *state have comparable cross sections*. Results of the calculation for ^{6}Li are shown in Fig. 1. The dotted curve corresponds to the monopole, $0(00)$, piece which is the main one. The two dashed curves correspond to DWIA calculations of the $2(20)$. The lower curve is to the ground state of ^{6}Li, and the upper curve to the 3^+ state, which is not resolvable experimentally. The solid line is the sum of all three contributions. Thus, it is seen from the figure that for angles less than 22 degrees, the higher order contributions are negligible compared to the monopole for ^{6}Li. For angles greater than 27 degrees, the quadrupole contributions to the cross section become important.

In Fig. 2 we show the effects of multiple scattering on the

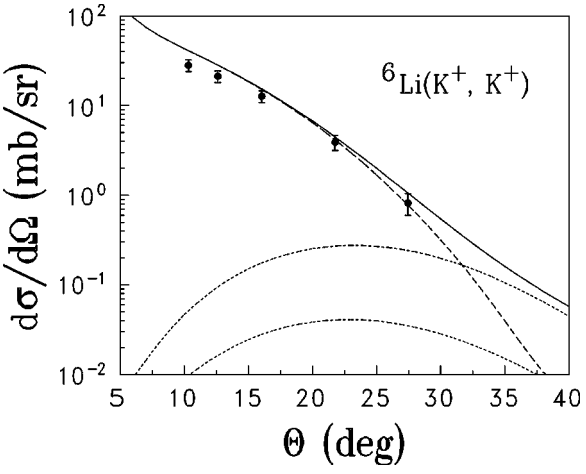

FIG. 1. K^+ ^{6}Li differential cross section for K^+ laboratory momentum of 715 MeV/c. The dashed curve corresponds to the monopole, 0(00), piece. The lower dotted curve corresponds to the elastic quadrupole, 2(20), piece, and the upper dotted curve corresponds to the quadrupole contribution to the 3^+ final state. The solid curve is the sum of these three pieces, and is to be compared with the data of Ref. [7], since the 3^+ final state is not resolvable experimentally.

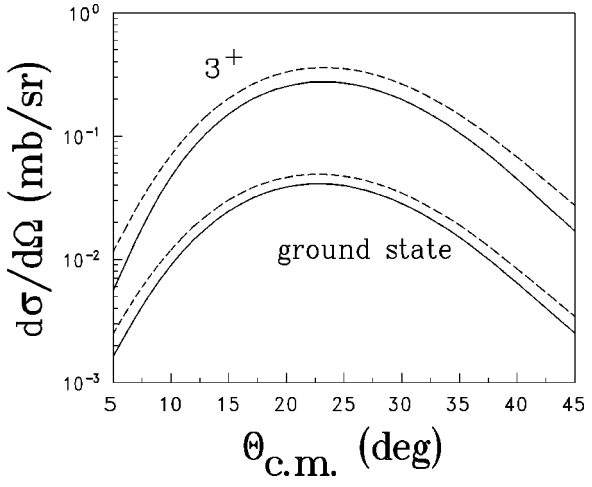

FIG. 2. The effects of multiple scattering ''distortion'' on the quadrupole calculations. The dashed curves correspond to the plane wave impulse approximation (PWIA), and the solid curves to the distorted wave impulse approximation (DWIA). The larger curves are for the 3^+ final state, and the lower curves for the ground state.

quadrupole cross section. The solid line corresponds to a DWIA calculation, and the dotted line to a plane wave impulse approximation (PWIA) calculation. Since the K^+ has a long mean free path and ^{6}Li is a small nucleus, the two cross sections are approximately equal. For angles between 10 and 50 degrees, *the DWIA cross section is approximately* 80% *that of the PWIA*. That is, multiple scattering tends to only reduce the quadrupole cross section by about 20%, and the PWIA calculation is a good approximation.

For a qualitative understanding of the multipole contributions, the PWIA is both relatively easy to calculate and analyze. Summing over final states and averaging over initial states, the PWIA calculation for the quadrupole $2(20)$ transition for ^{6}Li is given by

$$\frac{d\sigma}{d\Omega}=\frac{12}{5}\frac{(2J_f+1)}{(2J_i+1)}|\lambda_0(E)+\lambda_1(E)kk'\cos(\theta)|^2 \times\left|\sum_{n=1,2}\int j_2(qr)\phi_{fn}\phi_{in}r^2dr \times\langle\Psi_f||(\hat{a}^\dagger\times\hat{a})_{2(20)}||\Psi_i\rangle\right|^2, \quad (2)$$

where $n=1(2)$ correspond to the p-shell neutron (proton) in ^{6}Li. From the above equation, the large cross section to the 3^+ state compared to the ground state can be understood. First, the reduced matrix element $\langle\Psi_f||(\hat{a}^\dagger\times\hat{a})_{2(20)}||\Psi_i\rangle$ is larger for the 3^+ final state than for the ground state [11]. Second, J_f is larger for the 3^+ final state. These two factors enhance the quadrupole cross section to the 3^+ state by a factor of 6.5 over the ground state.

An important ingredient in the optical potential is the nuclear density. Following the optical potential Ref. [9], we use a two parameter Fermi density shape, $\rho(r)=1/(1+e^{(r-c)/a})$. The parameters c and a are chosen to give the correct rms charge density (after convolution of the nucleon charge form factor), and to have the correct form factor for diffractive scattering. We find values of $c=1.8$ fm and $a=0.52$ fm, satisfy these criteria, giving a charge rms radius of 2.54 fm, and good agreement with the π^+ ^{6}Li data of Ref. [12]. As a check, we used these values for c and a and calculated π^+ ^{6}Li elastic as well as inelastic scattering to the 3^+ final state. Comparison to the data of Ref. [12], gave a good fit to both the elastic scattering data and inelastic scattering to the 3^+ final state. Good agreement with the π^+ ^{6}Li data gives us confidence that the parameters used in the K^+ calculations in Fig. 1 are reliable.

As discussed in Ref. [7] it is interesting to compare the K^+ elastic scattering cross sections of ^{6}Li to ^{12}C. The dominant systematic error in the elastic cross section data for both ^{6}Li and ^{12}C, due to spectrometer acceptance, is 15% [7], but is common to both data sets as well as normalization uncertainties. In the ratio of cross sections this systematic error is greatly reduced. In Fig. 2 of Ref. [7], the elastic cross section ratio of ^{12}C to ^{6}Li is compared to model calculations. Since the form factors and Coulomb effects for ^{6}Li and ^{12}C are different, the differential cross sections changes differently with angle for these two nuclei. Thus, the ratio of elastic scattering cross sections has an angle dependence, which can mask effects of the medium. We find that a more useful quantity to compare is the ratio of experiment to theory for ^{12}C to the ratio of experiment to theory for ^{6}Li:

$$Y=\frac{\frac{d\sigma}{d\Omega}(\text{exp})\Big/\frac{d\sigma}{d\Omega}(\text{theory}) \quad \text{for } ^{12}\text{C}}{\frac{d\sigma}{d\Omega}(\text{exp})\Big/\frac{d\sigma}{d\Omega}(\text{theory}) \quad \text{for } ^{6}\text{Li}}, \quad (3)$$

where for both nuclei, the theoretical differential cross sections have the same angular dependence as the data.

Since multiple scattering is small, particularly for ^{6}Li, the forward angle elastic cross section has an angular dependence which is diffractive, similar in shape to the impulse approximation. Thus, the elastic differential cross section in the forward direction for both nuclei is mainly determined by two parameters: the rms radius of the nuclear density, and the elementary K^+-nucleon amplitude. For ^{12}C, we used a two parameter fermi density, with $c=2.274$ fm and $a=0.3979$ fm, which successfully fit pion elastic scattering from ^{12}C [9].

As seen in Fig. 3, the angular dependence of the K^+ ^{12}C cross section agrees with the data for angles less than 30 degrees. Also, for ^{6}Li, the angular dependence for forward angles in Fig. 1 matches the data well. Having determined appropriate shape parameters, and hence the angle of the first minimum for both nuclei, the K^+N amplitude is the next most important factor in the calculation of Y. The solid circles in Fig. 4 are a plot of Y for the data of Ref. [7] using the same K^+N amplitude in ^{12}C as in ^{6}Li. Note that for the last data point of the ^{6}Li data the quadrupole contributions are significant.

In Ref. [7] the 5 experimental center of mass angles of ^{12}C are different than those of ^{6}Li. We used the experimental angles of ^{12}C and interpolated Y between the experimental angles of ^{6}Li. The ratio Y should be 1.0 for all angles, but as seen in the figure, the solid circles are above 1.2 for all angles measured. This demonstrates that *we were unable to*

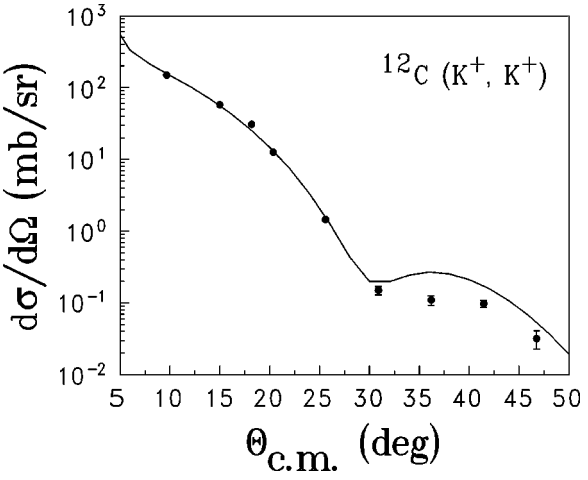

FIG. 3. K^+ ^{12}C elastic differential cross section for K^+ laboratory momentum of 715 MeV/c. The data are from Ref. [7].

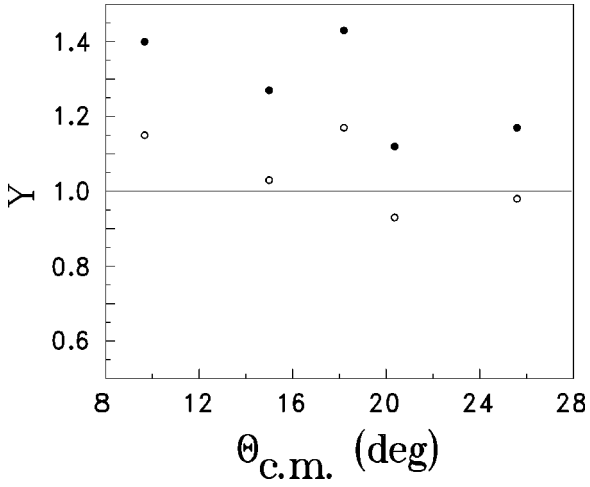

FIG. 4. The ratio Y as defined by Eq. (3) in the text is plotted as a function of angle. The solid circles correspond to using the same K^+-nucleon amplitudes in the ^{12}C and ^{6}Li optical potentials, and the open circles correspond to using K^+-nucleon amplitudes that are 15% larger in ^{12}C than in ^{6}Li.

fit both the ^{12}C and ^6Li elastic scattering cross sections using the same K^+N amplitudes in both optical potentials.

Increasing the K^+N amplitude by 15 percent in the ^{12}C calculation, but not in the ^{6}Li optical potential produced a value of Y near 1.0. These results are shown as the open circles in Fig. 4.

In conclusion, we have shown that the quadrupole contributions, $2(20)$, for both elastic and inelastic scattering to the 3^+ final state are important for angles greater than 25 degrees for the K^+ ^{6}Li scattering data of Ref. [7]. For an accurate comparison with this data, and for future calculations of K^+ ^{6}Li scattering, these quadrupole contributions must be considered. We were also not able to fit both the K^+ ^{6}Li and ^{12}C elastic scattering data using the same K^+-nucleon amplitudes for both nuclei. Increasing the K^+-nucleon amplitude in ^{12}C but not in the ^{6}Li optical potential allows both data sets to be fit. This modification does not change the shape of the elastic differential cross section for small angles, but merely increases the magnitude of the ^{12}C calculation relative to the ^{6}Li calculation.

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