Q^2 dependence of deep inelastic lepton scattering off nuclear targets

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(Received 16 June 1997)

Deep inelastic scattering of leptons off nuclear targets is analyzed within the convolution model and taking into account nucleon-nucleon correlations. We show that in the nuclear medium nucleons are distributed according to a function that exhibits a sizable Q^2 dependence and reduces to the ordinary light-cone distribution in the Bjorken limit. At $Q^2<50\ (\mathrm{GeV}/c)^2$ and $x>1$ this Q^2 dependence turns out to be stronger than the one associated with the nucleon structure function, predicted by perturbative quantum chromodynamics. [S0556-2813(98)04503-8]

PACS number(s): 25.30.Fj, 13.60.Hb, 25.30.Rw

After the discovery of the European Muon Collaboration (EMC) effect [1], a number of theoretical studies of deep inelastic scattering (DIS) off nuclear targets have been carried out within the so-called convolution model (for recent reviews see [2,3]). Within this approach, the nuclear structure function $F_2^A(x,Q^2)$ ($Q^2=|\mathbf{q}|^2-\nu^2$, where $\mathbf{q}$ and ν are the three-momentum and energy carried by the exchanged virtual photon, while $x=Q^2/2m\nu$, m being the nucleon mass, is the Bjorken scaling variable) is written in terms of the nucleon structure function $F_2^N(x,Q^2)$, which can be extracted from proton and deuteron data, and the function $f_A(z)$, yielding the distribution of the nucleons in the nuclear target as a a function of the relativistic invariant variable z, defined as

$$z=\frac{M_A}{m}\frac{(kq)}{(P_Aq)}. \tag{1}$$

In Eq. (1), $P_A\equiv(M_A,0)$, M_A being the target mass, and $k\equiv(k_0,\mathbf{k})$ denote the initial four-momenta of the target nucleus and the struck nucleon in the laboratory frame, respectively. More recently, the convolution approach has been also extended to study semi-inclusive processes [4–6].

The kinematical region corresponding to DIS is defined by the Bjorken limit $Q^2,\nu\to\infty$ with x finite, implying $(|\mathbf{q}|/\nu)\to1$. In this limit the quantity z of Eq. (1) can be related to the the light-cone component of the four-momentum of the struck nucleon through $z\to k^+/m=(k_0-k_z)/m$, with $k_z=(\mathbf{k}\cdot\mathbf{q})/|\mathbf{q}|$. When the value of Q^2 is not large enough and the Bjorken limit is not yet reached, however, it is no longer possible to identify z with the light-cone variable. As a consequence, in this regime the theoretical nuclear structure function evaluated within the convolution model should in principle exhibit a Q^2 dependence coming from the distribution function $f_A(z,Q^2)$, in addition to the one associated with the nucleon structure function $F_2^N(x,Q^2)$.

In this Brief Report we report a calculation of $f_A(z,Q^2)$ in which the kinematical constraints implied by the Bjorken limits are released, and discuss the relevance of the Q^2 dependence on the resulting distribution function to the interpretation of DIS data.

The distribution function $f_A(z,Q^2)$ is defined as

$$f_A(z,Q^2)=z\int d^4k\ S(k)\,\delta\!\left(z-\frac{M_A}{m}\frac{(kq)}{(P_Aq)}\right), \tag{2}$$

where $S(k)$ is the relativistic vertex function. $S(k)$ can be approximated by the nonrelativistic spectral function $P(\mathbf{k},E)$, yielding the distribution in momentum and removal energy of the nucleons in the target nucleus, according to [7]

$$S(k)=\left(\frac{m}{k_0}\right)P(\mathbf{k},E), \tag{3}$$

with

$$k_0=M_A-[(M_A-m+E)^2+|\mathbf{k}|^2]^{1/2}. \tag{4}$$

It has to be pointed out that the above definition of $S(k)$ can be regarded as a particular implementation of the expansion

$$S(k)=P(\mathbf{k},E)\left[1+O\!\left(\frac{k^2}{m^2}\right)+\cdots\right], \tag{5}$$

originally proposed in Ref. [8], and that the distribution function obtained using $S(k)$ defined as in Eq. (3) fulfills the normalization requirement

$$\int dz f_A(z,Q^2)=1 \tag{6}$$

for any Q^2.

Substitution of Eq. (3) into Eq. (2) leads to

$$f_A(z,Q^2)=\int_{E_{\min}}^{E_{\max}}dE\int d^3k\left(\frac{m}{k_0}\right)P(\mathbf{k},E)\,\delta\!\left(z-\frac{(kq)}{m\nu}\right), \tag{7}$$

where $E_{\min}$ denotes the minimum energy required to remove a nucleon from the target nucleus and $E_{\max}=\sqrt{s}-M_A$, with $s=(M_A+\nu)^2-|\mathbf{q}|^2$. Equation (7) is usually evaluated in the Bjorken limit. Here we give a general expression, applicable for any Q^2 and ν, for the case of infinite nuclear matter, in

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which $f_A(z,Q^2)$ takes a particularly simple and intuitive form. The generalization of the nuclear matter result to the case of a finite size nucleus, in which the recoil of the spectator $(A-1)$-particle system has to be taken into account, is straightforward.

Integration of Eq. (7) over $\cos\theta = k_z/|\mathbf{k}|$ using the δ function yields

$$f_A(z,Q^2) = f_A(z,\beta) = \frac{2\pi m z}{\beta}\int_{E_{\min}}^{E_{\max}} dE \int_{k_{\min}(E,z,\beta)}^{\infty} k\ dk\left(\frac{m}{k_0}\right)P(\mathbf{k},E), \tag{8}$$

where

$$\beta = \frac{|\mathbf{q}|}{\nu} = \left(1+\frac{4m^2x^2}{Q^2}\right)^{1/2} \tag{9}$$

and $k_{\min}$ is given by

$$k_{\min} = \frac{|m(1-z)-E|}{\beta}. \tag{10}$$

Equations (8)–(10) show that in the Bjorken limit, corresponding to $\beta = (|\mathbf{q}|/\nu) = 1$, the standard expression of the light-cone distribution is recovered. In general, the distribution function $f_A(z,\beta)$ depends upon β through both the factor $(1/\beta)$ in front of the integral of Eq. (8) and through the lower limit of the momentum integration, defined by Eq. (10), implying that scattering processes at different β probe the nuclear spectral function at different values of k and E. This β dependence can be readily estimated using the approximation employed in Refs. [9,6], i.e., replacing the nucleon removal energy with the average value $\langle E\rangle$ in Eq. (10):

$$k_{\min} = \frac{|m(1-z)-\langle E\rangle|}{\beta}, \tag{11}$$

where, using the nuclear matter spectral function of Ref. [10], $\langle E\rangle = 62$ MeV. Within the above approximation, one can easily show that the inclusion of the β dependence produces a shift in z, so that $f_A(z,\beta)$ of Eq. (8) can be related to $f_A(z,\beta=1)$ calculated at a different value of z:

$$f_A(z,\beta) = \beta\left(\frac{z}{\tilde{z}}\right) f_A(\tilde{z},\beta=1), \tag{12}$$

with

$$\tilde{z} = 1 + \frac{(z-1)}{\beta} - \frac{(\beta-1)}{\beta}\frac{\langle E\rangle}{m} \simeq 1 + \frac{(z-1)}{\beta}. \tag{13}$$

The calculated $f_A(z,\beta)$ are bell shaped, with a maximum around $z=1$. According to Eq. (13), $\beta>1$ implies $\tilde{z}<(>)z$ at $z>(<)1$. As a consequence, at $\beta>1$ the peak in $f_A(z,\beta)$ at $z\sim 1$ gets lower and wider, whereas the tails at $z>1$, where the contributions of the high momentum components of the nuclear spectral function dominate, are significantly enhanced. These features are clearly illustrated in Fig. 1, where $f_A(z,\beta)$, resulting from the full calculation [i.e., without replacing E with $\langle E\rangle$ in Eq. (10)], is plotted as a function of z for different values of $\beta \geqslant 1$. Figure 1 also shows that the overall β dependence of $f_A(z,\beta)$ is sizable, particularly at $z>1$.

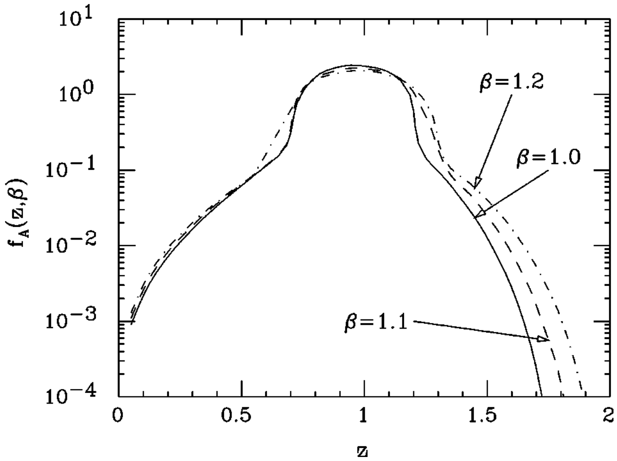

FIG. 1. z dependence of the distribution function $f_A(z,\beta)$, calculated for infinite nuclear matter at equilibrium density, at different values of β. Solid line, $\beta=1.0$; dashed line, $\beta=1.1$; dot-dashed line, $\beta=1.2$.

At $z\leqslant 0.75$ the nuclear matter distribution function $f_A(z,\beta)$ defined by Eqs. (8)–(10) can be parametrized in the form

$$f_A(z,\beta) = A_1\exp(1-\beta)\exp\left[-0.5\left(\frac{z-A_2}{A_3}\right)^2\right] + A_4 z^2, \tag{14}$$

whereas at $z>0.75$ one can use

$$f_A(z,\beta) = zA_5(1+A_7\varphi)\exp(-A_8\varphi), \tag{15}$$

where

$$\varphi = \frac{|m(1-z)-A_6\exp[2(1-\beta)]|}{\beta}. \tag{16}$$

The numerical values of the parameters appearing in Eqs. (14)–(16) are $A_1=1.74$, $A_2=0.83$, $A_3=0.1$, $A_4=0.245$, $A_5=2.28$, $A_6=0.0655$, $A_7=35.6$, and $A_8=16.6$.

The above parametrization of $f_A(z,\beta)$ can be used to study DIS within the convolution model. The nuclear structure function $F_2^A(x,Q^2)$ for an isoscalar target is written in the form

$$F_2^A(x,Q^2) = \int_x^A dz\ \ f_A(z,\beta)\ \ F_2^N\left(\frac{x}{z},Q^2\right), \tag{17}$$

where $F_2^N(x,Q^2) = [F_2^p(x,Q^2)+F_2^n(x,Q^2)]/2$, $F_2^p(x,Q^2)$ and $F_2^n(x,Q^2)$ being the proton and neutron structure functions, respectively, which can be extracted from the DIS data off hydrogen and deuterium targets.

The effect of the β dependence associated with $f_A(z,\beta)$ turns out to be very small in the region of the classical EMC effect, corresponding to $0.3<x<0.8$, which has been extensively analyzed using the convolution model in the Bjorken

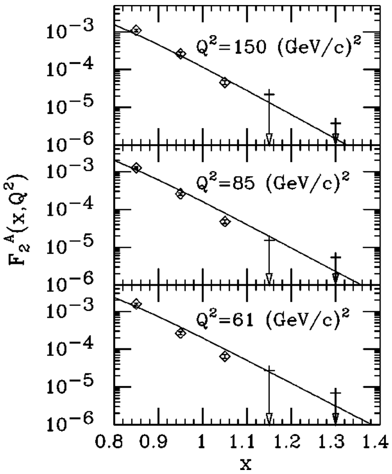

FIG. 2. $F_2^A(x,Q^2)$ for infinite nuclear matter at equilibrium density, calculated using Eqs. (8)–(10) and (17) (solid lines), compared to the carbon structure function measured by the BCDMS Collaboration [14].

limit. On the other hand, at $x>1$ the Q^2 dependence of $F_2^A(x,Q^2)$ coming from the distribution function $f_A(z,\beta)$ turns out to be larger than the one associated with the nucleon structure function $F_2^N(x,Q^2)$, predicted by the perturbative quantum chromodynamics (QCD) evolution equations [11–13]

In Fig. 2 the large x behavior of the nuclear matter structure function resulting from the approach described in this paper is compared to the ^{12}C data taken at CERN by the Bologna-CERN-Dubna-Munich-Saclay (BCDMS) Collaboration [14]. The theoretical $F_2^A(x,Q^2)$ has been calculated from Eq. (17) using the parametrized $F_2^N(x,Q^2)$ of Ref. [16] and the spectral function of Ref. [10]. Even though the structure function at $x>1$ is mostly sensitive to the short range behavior of the nuclear wave function, and is therefore not expected to be strongly affected by finite size effects, the comparison between infinite nuclear matter and ^{12}C has to be taken with some caution, since the nuclear matter equilibrium density is sizably higher than the average density in the nucleus of ^{12}C (0.16 fm^{-3}, compared to $\sim$0.12 fm^{-3}). However, since the amount of high momentum components in the nuclear matter wave function decreases as the density decreases, a more accurate calculation, carried out using the local density approximation [15], would lower the theoretical curve at large x, improving the agreement with the data.

Expanding the distribution function $f_A(z,\beta)$ in Eq. (17) it is possible to extract the x dependence of the twist-4 term, proportional to $1/Q^2$. Neglecting higher order contributions $F_2^A(x,Q^2)$ can be written as

$$F_2^A(x,Q^2) \simeq \widetilde{F}_2^A(x,\ln Q^2)\left[1+\frac{C(x)}{Q^2}\right], \qquad (18)$$

where

$$\widetilde{F}_2^A(x,\ln Q^2)=\int_x^A [f_A(z,\beta)]_{\beta=1} F_2^N\left(\frac{x}{z},Q^2\right)dz \qquad (19)$$

and

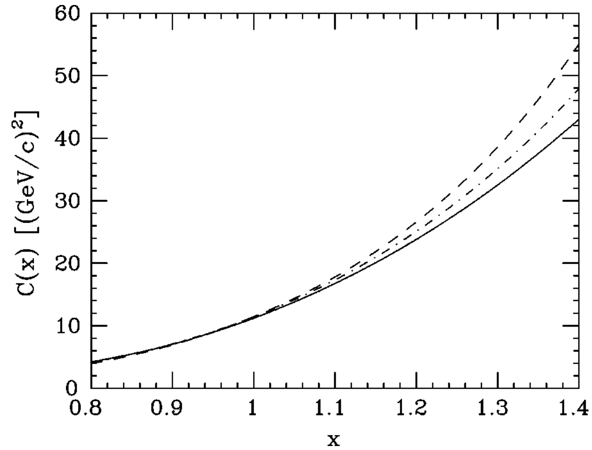

FIG. 3. x dependence of $C(x)$, defined by Eq. (18) (solid line), and $\widetilde{C}(x,Q^2)$, defined by Eq. (21), evaluated at $Q^2=61$ (GeV/c)^2 (dashed line) and $Q^2=150$ (GeV/c)^2 (dot-dashed line).

$$C(x)=\frac{2m^2x^2}{\widetilde{F}_2^A(x,\ln Q^2)}\int_x^A\left[\frac{df_A(z,\beta)}{d\beta}\right]_{\beta=1} F_2^N\left(\frac{x}{z},Q^2\right)dz. \qquad (20)$$

Expansion (18) includes corrections of order $1/Q^2$ to the nuclear structure function $F_2^A(x,Q^2)$ coming from the dependence of the nucleon distribution f_A upon x^2/Q^2.

The function $C(x)$ defined by Eq. (20) is shown in Fig. 3, along with the quantity

$$\widetilde{C}(x,Q^2)=Q^2\left[\frac{F_2^A(x,Q^2)}{\widetilde{F}_2^A(x,\ln Q^2)}-1\right], \qquad (21)$$

evaluated at $Q^2=61$ (GeV/c)^2 and $Q^2=150$ (GeV/c)^2. The difference between $C(x)$ and $\widetilde{C}(x,Q^2)$ can be ascribed to higher twist corrections, of order $1/Q^4, 1/Q^6, \ldots$, whose contribution becomes appreciable at $x \geqslant 1.3$.

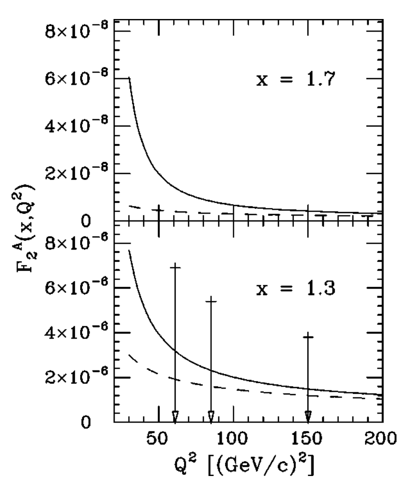

FIG. 4. Q^2 dependence of the nuclear matter $F_2^A(x,Q^2)$ at different values of x. The dashed lines correspond to the Bjorken limit, i.e., to $\beta=1$, whereas the solid lines have been obtained including the Q^2 dependence of the distribution function $f_A(z,\beta)$, parametrized as in Eqs. (14)–(16). The experimental upper bounds are from Ref. [14].

The Q^2 dependence of the nuclear matter $F_2^A(x,Q^2)$ at $x=1.3$ and $x=1.7$ is also illustrated in Fig. 4. The dashed curves correspond to the Bjorken limit, when $\beta=(1+4m^2x^2/Q^2)^{1/2}=1$. In this case the Q^2 dependence of $F_2^A(x,Q^2)$ has to be ascribed to the nucleon structure function $F_2^N(x,Q^2)$ only. The solid curves have been obtained including the Q^2 dependence implied by the β dependence of the distribution function $f_A(z,\beta)$, parametrized as in Eqs. (14)–(16). It clearly appears that this dependence produces large effects at $Q^2<50$ $(\mathrm{GeV}/c)^2$, particularly at $x=1.7$.

In conclusion we find that, primarily due to nucleon-nucleon correlations, the distribution of nucleons in the nuclear medium as a function of the relativistic invariant quantity z, defined in Eq. (1), exhibits an additional dependence upon the ratio x^2/Q^2, or Q^2/ν^2, in the kinematical regime in which the Bjorken limit is not yet reached. The effect of the β dependence of $f_A(z,\beta)$ on DIS is appreciable at $x>1$, where even small deviations of β from unity produce large effects on the nuclear structure function. For example, at $x=1.3$ and $Q^2=60$ $(\mathrm{GeV}/c)^2$, corresponding to $\beta=1.05$, the ratio of the nuclear structure functions calculated from Eq. (17) using $\beta=1.05$ and $\beta=1$, respectively, is $\sim$1.7. Figure 3 also shows that the effect becomes larger as x increases. It has to be emphasized that at $x>1$ the Q^2 dependence discussed in the present work results in a Q^2 dependence of the nuclear structure function $F_2^A(x,Q^2)$ much stronger than the one associated with the nucleon structure function. Unfortunately, the only DIS data available at $x>1$ cover the region $x<1.3$ and $Q^2>60$ $(\mathrm{GeV}/c)^2$ and only provide upper bounds of the nuclear structure function. However, the effect discussed in the present work will certainly be critical in the analysis of the planned DIS experiment at $x>1$ and moderate Q^2 [17].

This work has been encouraged and supported by the Russian Foundation of Fundamental Research. We gratefully acknowledge very helpful discussions with A. Fabrocini, E. Oset, and E. Marco. One of us (G.I.L.) wishes to thank S. Fantoni for the kind hospitality at the Interdisciplinary Laboratory of SISSA, where part of this work has been carried out.

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