Structure of vacuum condensates

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It is essential to know the space-time structure of the nonlocal vacuum condensates for application to medium energy processes. Using the Dyson-Schwinger formalism in the rainbow approximation for the quark propagator, we study the nonlocal quark condensate and model forms for the nonperturbative gluon propagator constrained by fits to local condensates and deep inelastic scattering with nucleon targets. [S0556-2813(98)03103-3]

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It has long been known that chiral symmetry breaking requires a nonperturbative quark propagator with nonvanishing vacuum matrix elements of normal ordered products of quark fields [1], called quark condensates. These condensates would vanish in a perturbative vacuum but do not vanish in the QCD vacuum, and are of central importance both for the structure of hadronic matter and for the study of the early universe chiral phase transition. A systematic treatment of hadronic masses can be carried out [2–4] using operator product expansions in terms of vacuum matrix elements of local operators, the vacuum condensates, whose phenomenological values have been confirmed in lattice gauge calculations [5]. As was discussed in early work on the magnetic dipole moments of nucleons [6], for application to form factors and transition matrix elements in the low to medium momentum transfer region the operator product expansion (OPE) cannot be used, since long distance properties of nonlocal operators must be treated.

One approach to this problem of treating bilocal operators has been the use of nonlocal condensates, which have been introduced to represent the bilocal vacuum matrix elements needed for the pion wave function [7] and pion form factor [8] for low to medium momentum transfer. In this method one does not carry out an OPE for the vacuum matrix elements of the bilocal operators, but introduces new phenomenological functions needed to characterize the space-time structure of the nonlocal condensates. Both the forms of these functions and the parameters are found by fits to experiment as well as considerations of analyticity. For example, in a study of parton distribution functions [9] the space-time scale of a nonlocal condensate was determined by a fit of a monopole form in space-time to experimental data. On the other hand, in a recent use of nonlocal condensates to determine the values of vacuum susceptibilities [10], which characterize the nonperturbative quark propagation in an external field [11], it was found that the monopole form did not have suitable analytic properties, and a space-time dipole form was used to fit the low-x parton data. Although a satisfactory fit to the phenomenological pion susceptibility [12] was found, it is a good example of the importance of determining the structure of the nonlocal condensates for application to transition matrix elements over a wide range of momentum transfer.

It is the goal of the present work to study the form of the nonlocal quark condensate using the the QCD Dyson-Schwinger (DS) equations [13,14]. Using the bare gluon-quark vertex, defined as the rainbow approximation, the nonperturbative (dressed) quark propagator is determined self-consistently with a model for the nonperturbative (dressed) gluon propagator $D_{\mu\nu}^{ab}(q)$. A comprehensive review of this type of model is given in Ref. [15]. It has been shown that in the rainbow approximation the value of the quark condensate [16,17] and the mixed quark condensate [18] can be obtained with suitable choices of the gluon propagator, which also provides constraints for the present work. The gluon condensate within this approach has been studied in Ref. [19]. Other studies use the DS formalism with different approximations [20] to attempt to determine the nonperturbative quark condensates.

For hadronic properties such as the elastic and transition form factors one needs the information equivalent to the bound-state Bethe-Salpeter (BS) equation. It has also been shown that the rainbow DS model is consistent with nontopological chiral quark models [16] and low-momentum transfer meson form factors [21]. For a treatment of form factors over an extended range of momentum transfer light-cone BS studies of the pion form factor have shown [22] that nonperturbative as well as perturbative two-quark propagators are needed, and that in a QCD treatment four-quark matrix elements are also required. We plan a study of nonlocal nonperturbative four-quark matrix elements within this approach in the near future.

In the present work we use the rainbow Dyson-Schwinger equation to investigate the forms of the nonlocal quark condensate as well as the gluon propagators. Using the same phenomenological gluon propagators as were used in previous studies of the local condensates [16–18] we find that the dipole form with the parameter close to the one found from fits to the sea-quark distribution [9,10] can be obtained.

The quark propagator is defined by

$$S_q(x) = \langle 0|T[q(x)\bar{q}(0)]|0\rangle, \tag{1}$$

where $q(x)$ is the quark field and T the time-ordering operator. For the physical vacuum the quark propagator $S_q(x)$ has a perturbative and a nonperturbative part. In the case of vanishing current quark masses ($m_0 = 0$) one can write

$$S_q(x) = S_q^{\text{PT}}(x) + S_q^{\text{NP}}(x), \quad S_q^{\text{PT}}(x) = \frac{1}{2\pi^2}\frac{\gamma \cdot x}{x^4},$$

$$S_q^{\text{NP}}(x) = (-)\frac{1}{12}(\langle :\bar{q}(x)q(0): \rangle$$

$$+ x_\mu \langle :\bar{q}(x)\gamma^\mu q(0): \rangle). \tag{2}$$

It should be stressed that normal-ordered products, and therefore S_q^{NP}, do not vanish in the nonperturbative vacuum. For short distances, the OPE for the scalar part of $S_q^{\text{NP}}(x)$ gives

$$\langle :\bar{q}(x)q(0): \rangle = \langle :\bar{q}(0)q(0): \rangle$$

$$-\frac{x^2}{4}\langle 0|:\bar{q}(0)\sigma \cdot G(0)q(0):|0\rangle + \cdots, \tag{3}$$

in which the local operators of the expansion are the quark condensate, the mixed condensate, and so forth.

In Ref. [10] it is shown that with a choice of nonlocal condensate

$$\langle 0|:\bar{q}(x)q(0):|0\rangle = g(x^2)\langle 0|:\bar{q}(0)q(0):|0\rangle, \tag{4}$$

with

$$g(x^2) = \frac{1}{(1+\kappa^2 x^2/8)^2} = \int_0^\infty d\alpha f(\alpha)e^{-x^2\alpha/4},$$

$$f(\alpha) = \frac{4}{\kappa^4}\alpha e^{-2\alpha/\kappa^2}, \tag{5}$$

and $\kappa^2 = 0.15\ldots 0.20$ GeV^2, one can fit the low-x quark distributions and also the pion susceptibility.

In the Dyson-Schwinger formalism S_q^{NP} is related to the quark self-energy, Σ, by

$$S_q(p)^{-1} = i\gamma \cdot p + \Sigma(p). \tag{6}$$

Using the bare quark gluon vertex (the rainbow approximation), $\Gamma_\nu^b(q,p) = \gamma_\nu(\lambda_c^b/2)$, $\Sigma(p)$ satisfies the rainbow Dyson-Schwinger equation [14]:

$$\Sigma(p) = \int \frac{d^4q}{(2\pi)^4} g_s^2 D_{\mu\nu}^{ab}(p-q)\gamma_\mu \frac{\lambda_c^a}{2} S_q(q)\gamma_\nu \frac{\lambda_c^b}{2} \tag{7}$$

with $D_{\mu\nu}^{ab}(q)$ the gluon propagator, λ_c^a the color SU(3) matrix. In Euclidean space one can write

$$S_q(p)^{-1} = i\gamma \cdot p A(p^2) + B(p^2). \tag{8}$$

The choice of the Landau gauge for the gluon propagator

$$D_{\mu\nu}^{ab}(q) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)D(q^2) \tag{9}$$

leads to the set of coupled integral equations

$$[A(p^2)-1]p^2$$

$$= \frac{4}{3}g_s^2 \int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{A(q^2)}{q^2 A^2(q^2) + B(q^2)^2}$$

$$\times \left[p\cdot q + 2\frac{(p\cdot q - q^2)(p^2 - p\cdot q)}{(p-q)^2}\right],$$

$$B(p^2) = 4g_s^2 \int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{B(q^2)}{q^2 A^2(q^2) + B(q^2)^2}. \tag{10}$$

Mesonic bound states can be studied within this framework by solving the ladder Bethe Salpeter equations for the corresponding $\bar{q}q$ bound states. Various mesonic properties have been studied in Refs. [16]. In Ref. [17] a detailed investigation of the low energy sector was performed by deriving the general form of the effective chiral action for the SU(3) Goldstone bosons and determining f_π and most of the chiral low energy coefficients L_i, which, in turn, determines the physics of the π, K and η mesons at low energies [23].

Because the form of the gluon propagator $D(s)$ in the IR region is unknown, we must use model forms as input. Our model ansatz is

$$g_s^2 D(s) = 3\pi^2 \frac{\chi^2}{\Delta^2}e^{-s/\Delta}, \tag{11}$$

which determines the quark-quark interaction through a strength parameter χ and a range parameter Δ. Its form is inspired by the δ function ansatz of Ref. [24], which it approaches for $\Delta \to 0$.

The nonlocal quark condensate $\langle :\bar{q}(x)q(0): \rangle$ is then given by the scalar part of the Fourier transformed inverse quark propagator:

$$\langle :\bar{q}(x)q(0): \rangle = (-4N_c)\int \frac{d^4p}{(2\pi)^4}\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}e^{ipx}$$

$$= (-)\frac{12}{16\pi^2}\int_0^\infty ds\, s \frac{B(s)}{sA^2(s) + B^2(s)}$$

$$\times \left[2\frac{J_1(\sqrt{sx^2})}{\sqrt{sx^2}}\right]. \tag{12}$$

At $x=0$ the expression for the local condensate $\langle :\bar{q}q: \rangle$ is recovered:

$$\langle :\bar{q}q: \rangle = (-)\frac{12}{16\pi^2}\int_0^\infty ds\, s \frac{B(s)}{sA^2(s) + B^2(s)}. \tag{13}$$

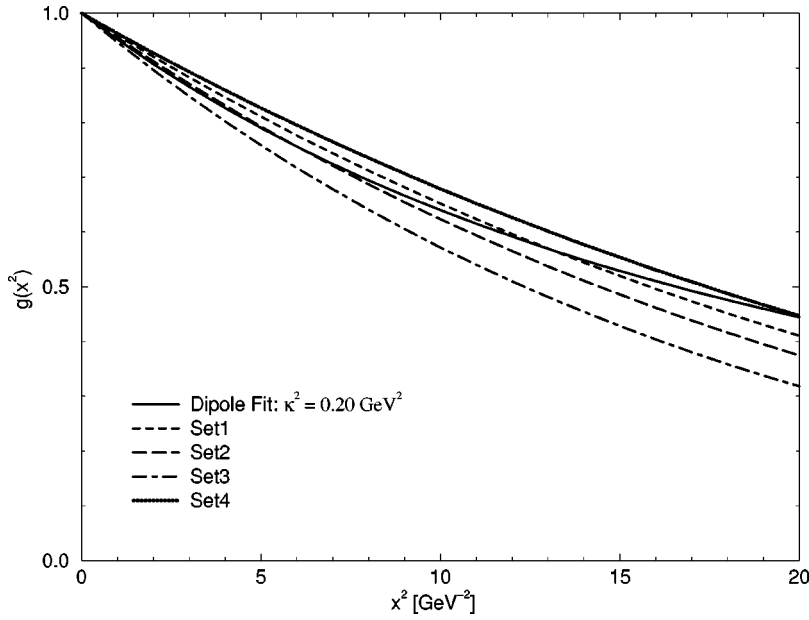The nonlocality $g(x^2)$ can be obtained immediately by dividing Eqs. (12) through Eq. (13).

FIG. 1. The nonlocal quark condensate $g(x) = \langle :\bar{q}(x)q(0): \rangle / \langle :\bar{q}(0)q(0): \rangle$ for the four sets of model gluon propagators mentioned in the text compared with the dipole fit of Ref. [10].

Because the quark-quark interaction defined by Eq. (11) has a finite range in momentum space the momentum integrals in Eqs. (12) and (13) are finite. Our analysis ignores effects from hard gluonic radiative corrections to the condensates which are connected to a possible change of the renormalization scale μ at which the condensates are defined. Those effects are of minor importance for our study of nonperturbative effects in the low and medium energy regions. It should be stressed in this context that our interaction is not renormalizable because we are using the bare quark gluon vertex. Therefore, instead of our condensates depending logarithmically on the renormalization scale μ [25], the scale at which a condensate is defined in our approach is a typical hadronic scale, which is implicitly determined by the model gluon propagator $g_s^2 D(s)$ and the solutions of the DS equations (7). The situation is very similar to the determination of vacuum condensates in the instanton liquid model [26,27] where the scale is set by the inverse instanton size.

In order to check the sensitivity of our results on the model gluon 2 point function (11) we try various sets of parameters χ and Δ and investigate the x^2 dependence of the function $g(x^2)$ for these forms. We solve the set of integral equations (10) self-consistently for a given model form for $g_s^2 D(q^2)$ obtaining the quark propagator functions $A(p^2)$ and $B(p^2)$, which, in turn will allow us to calculate $g(x^2)$ from Eq. (12). The result can then be compared to the dipole fit of Ref. [10] with $\kappa^2 = 0.15 \ldots 0.20$ GeV^2. The parameter sets we are using are

Set 1: $\Delta = 2.0 \times 10^{-3}$ GeV^2; $\chi = 1.40$ GeV,

Set 2: $\Delta = 1.0 \times 10^{-2}$ GeV^2; $\chi = 1.56$ GeV,

Set 3: $\Delta = 2.0 \times 10^{-2}$ GeV^2; $\chi = 1.58$ GeV.

$$(14)$$

These parameters have been chosen so that they reproduce the correct value for the pion decay constant in the chiral

limit $f_\pi = 88$ MeV. Moreover the values of the chiral low energy coefficients L_i [23] are compatible with the phenomenological values in both cases. Following Ref. [17] one finds $L_1 = 0.86$, $L_3 = -4.53$, $L_5 = 0.78$, $L_8 = 0.84$ ($\times 10^{-3}$) for set 1; $L_1 = 0.84$, $L_3 = -4.48$, $L_5 = 0.88$, $L_8 = 0.84 (\times 10^{-3})$ for set 2; and $L_1 = 0.83$, $L_3 = -4.42$, $L_5 = 0.92$, $L_8 = 0.78 (\times 10^{-3})$ for set 3.

Many of the works in Refs. [15–18] have used a model ansatz for the gluon propagator

$$D_{\mu\nu}^{ab}(q) = \delta^{ab} \delta_{\mu\nu} D(q^2), \qquad (15)$$

which is often referred to as the Feynman-like gauge. It is however not identical to the Feynman gauge QCD, in which the dressed gluon propagator would have different longitudinal and transverse components. Therefore the ansatz (15) should be regarded merely as a model form for the gluon 2 point function. For our purpose it is, however, interesting to ask if and how the nonlocal quark condensate depends on choosing either Landau gauge (9) or the Feynman-like gauge (15). Therefore we perform the calculation for another parameter set:

Set 4: $\Delta = 2.0 \times 10^{-3}$ GeV^2; $\chi = 1.23$ GeV, $\qquad (16)$

while using Eq. (15) instead of Eq. (9) for the gluon propagator. Set 4 has the same range parameter Δ than set 1. The strength parameter χ is slightly smaller in order to obtain the correct value of $f_\pi = 88$ MeV. The chiral low energy coefficients are $L_1 = 0.85$, $L_3 = -4.46$, $L_5 = 0.82$, $L_8 = 0.93 (\times 10^{-3})$, values rather close to those obtained with set 1.

Figure 1 shows the results for $g(x^2)$ for the four parameter sets and compares with the dipole fit of Ref. [10] with $\kappa^2 = 0.20$ GeV^2 (solid line). The result of Ref. [10] is best reached for a gluon propagator with a small range parameter

$\Delta = 0.002$ GeV^2 in the infrared. Larger values for the width parameter Δ lead to stronger deviations from the form of Ref. [10].

By comparing the curves for set 1 and set 4 we can demonstrate that the nonlocal condensate is very robust with respect to using Landau gauge (9) or the Feynman-like gauge (15). The change between the two forms for the gluon 2 point function can be easily made up by a slight readjustment of the parameters of the IR model ansatz without any significant change of the final result.

We conclude that the DS formalism is a valuable tool for the study of the nonlocal quark condensate, and expect that the BS formalism will also prove to be useful for the study of the nonlocal four-quark condensates, which provide nonperturbative QCD effects for hadron couplings and form factors.

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